MAE 289A: Mathematical Analysis for Applications (F15)

Homework #7

Due on 11/19/15

1. Let f, g be continuous maps from (S_1, d) to (S_2, d). Let

$$A = \{p \in S_1 \mid f(p) = g(p)\}.$$

   Show that A is closed.

2. Let A and B be compact sets in $\mathbb{R}^d$ such that $A \cap B = \emptyset$. Show that

$$\inf\{\|p - q\| \mid p \in A, q \in B\}$$

   is positive.

3. Let f be a uniformly continuous function on the bounded set E in $\mathbb{R}$. Prove that f is bounded on E. Show that the conclusion is false if boundedness of E is omitted from the hypothesis by providing a counterexample.

4. I descended a trail in the Great Canyon in six hours (7 am to 1 pm). I camped there overnight. Then, at 7 am the next day, I started ascending following the same trail. I noticed that my compass was missing after half an hour, so I turned around and descended a short distance, where I found the compass. I climbed for another 2 hours and sat on a rock to admire the view. Then I ascended the rest of the way resting several times. The entire ascend took about 12 hours. I thought that I remembered there was a place on the trail where I was at the same place at the same time on both days. Is this right? Why?

5. Consider $\mathbb{R}^2$ with the Euclidean metric. Is the projection map onto the first component $\pi : \mathbb{R}^2 \to \mathbb{R}$, $\pi(x_1, x_2) = x_1$ continuous? Use this fact to show that, if S is a compact subset of $\mathbb{R}^2$, then the set

$$X(S) = \{x_1 \in \mathbb{R} \mid \exists x_2 \text{ such that } (x_1, x_2) \in S\}$$

   is a compact subset of $\mathbb{R}$ (with the Euclidean metric).