MAE140 - Linear Circuits - Fall 09
Final, December 7

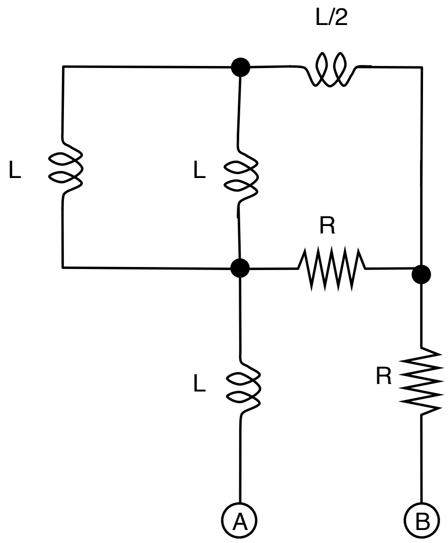
Instructions

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities

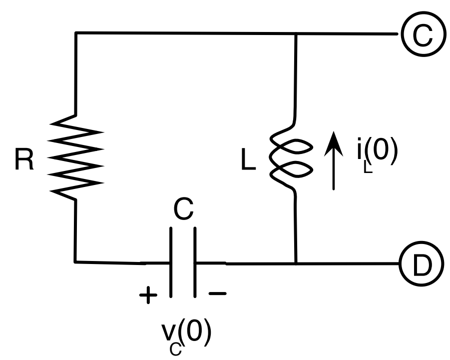
(ii) You have 170 minutes

(iii) Do not forget to write your name, student number, and instructor

(iv) On the questions for which we have given the answers, please provide detailed derivations.



(a) Question 1, Part I



(b) Question 1, Part II

Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

Part I: [5 points] Assuming zero initial conditions, find the impedance equivalent to the circuit in Figure 1(a) as seen from terminals A and B. The answer should be given as a ratio of two polynomials.

Part II: [5 points] Assuming that the initial conditions of the inductor and capacitor are as indicated in the diagram, redraw the circuit shown in Figure 1(b) in the s-domain. Then use source transformations to find the s-domain Norton equivalent of this circuit as seen from terminals C and D. (**Hint:** Use an equivalent model for the inductor in which the initial condition appears as a current source, and an equivalent model for the capacitor in which the initial condition appears as a voltage source)

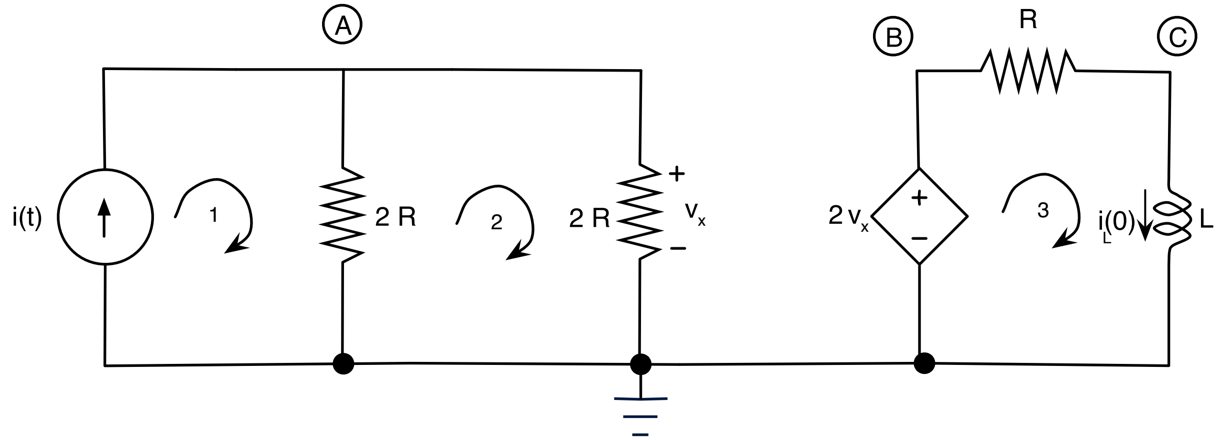


Figure 2: Nodal and Mesh Analysis Circuit

2. **Nodal and Mesh Analysis**

Part I: [5 points] Formulate node-voltage equations in the s-domain for the circuit in Figure 2. Use the reference node and other labels as shown in the figure. Do not assume zero initial conditions! Transform the initial condition on the inductor into a current source.

Part II: [5 points] Formulate mesh-current equations in the s-domain for the circuit in Figure 2. Use the mesh currents shown in the figure. Do not assume zero initial conditions! Transform the initial condition on the inductor into a voltage source.

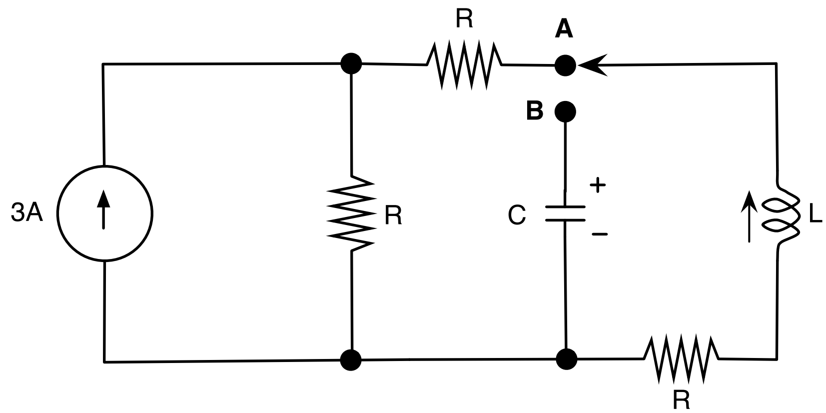


Figure 3: RCL circuit for Laplace Analysis

3. **Laplace Domain Circuit Analysis**

Part I: [3 points] Consider the circuit depicted in Figure 3. The current source is constant. The switch is kept in position **A** for a very long time. At $t = 0$ it is moved to position **B**. Show that the initial capacitor voltage and inductor currents are given by

$$v_C(0^-) = 0V, \quad i_L(0^-) = -1A.$$

[Show your work]

Part II: [2 points] Use these initial conditions to transform the circuit into the s-domain for $t \geq 0$. Use equivalent models for the capacitor and the inductor in which the initial conditions appear as current sources.

[Show your work]

Part III: [5 points] Use domain circuit analysis and inverse Laplace transforms to show that the inductor current when $C = 1F$, $L = 1H$, and $R = 2\Omega$ is

$$i_L(t) = (-e^{-t} + te^{-t})u(t).$$

4. **Frequency Response**

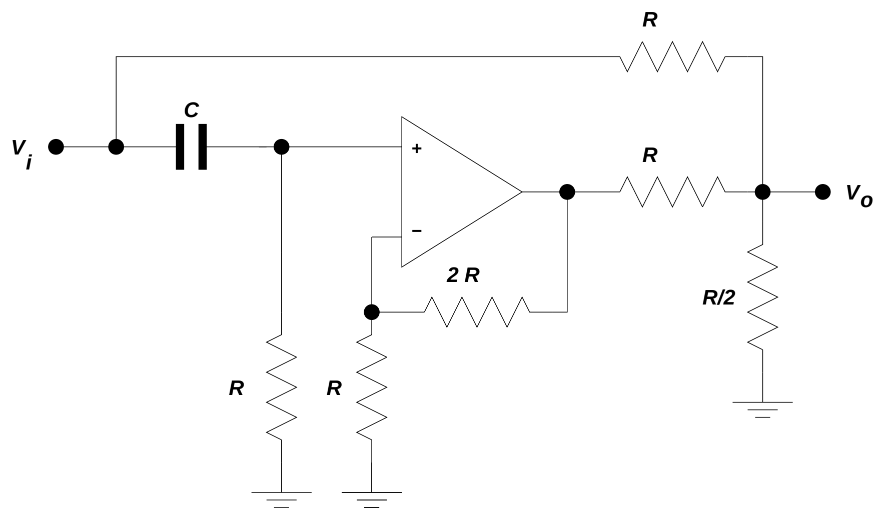


Figure 4: Circuit for Question 4.

Part I [4 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ for the circuit in Figure 4 is given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{s + 1/(4RC)}{s + 1/(RC)}$$

Show your work.

Part II [4 points] Let $R = 10$ KΩ, $C = 100$ nF and compute the magnitude and phase of $T(s)$ at $s = j\omega$ where $\omega = \{0, 250, 1000, \infty\}$ rad/s. Use these values to sketch the magnitude and phase response of the circuit.

(**Hint:** $4 + j \approx 4.1\angle 14°$, $\quad 1 + 4j \approx 4.1\angle 76°$)

Part III [2 points] Using what you know about frequency response, compute the steady state response $V_o^{SS}(t)$ of this circuit when $V_i(t) = 2\cos(1000t)$ using the same values of R and C as in Part II.

5. **Op-Amp Circuit Analysis and Design**

When you take MAE 143 B you will study a very popular controller called a *proportional-integral-derivative controller* (PID-Controller). As the name suggests, a PID-Controller transforms a signal $y(t)$ into the control signal $u(t)$ through the formula

$$u(t) = b\,y(t) + c\int_0^t y(\tau)d\tau + a\,\frac{d}{dt}y(t)$$

where the parameters a, b and c are chosen by an engineer.

Part I [4 points] Assuming zero initial conditions, show that the transfer function from $Y(s)$ to $U(s)$ is given by

$$T(s) = \frac{U(s)}{Y(s)} = \frac{as^2 + bs + c}{s}$$

Show your work, indicating the simplifications implied by the assumption of zero initial conditions.

Part II [4 points] Now let $a = 0$ and $b < 1$. Design a circuit using one capacitor, one OpAmp and multiple resistors that realizes $T(s)$. If you don't know how to do that using a single OpAmp, try doing it with two OpAmps still using one capacitor for 3 points. Clearly indicate the relationship between your circuit components and b and c, making sure $b < 1$.

Part III [2 points] Find values for your components so that $a = 0$, $b = .5$ and $c = 0.05$.