9-1. $f(t) = 10\left[e^{-50t} - 2e^{-100t}\right]u(t)$

$$F(s) = 10\left[\frac{1}{s+50} - \frac{2}{s+100}\right]$$

$$10\left[\frac{s+\cancel{100} - 2s - \cancel{100}}{(s+50)(s+100)}\right] = \boxed{\frac{-10s}{(s+50)(s+100)} = F(s)}$$

Zeros: $s = 0$
Poles: $s = -50$, $s = -100$

9.8) a) $f_1(t) = 4\delta(t) + \left[20e^{-20t} + 40e^{-40t}\right]u(t)$

$$F_1(s) = 4 + \frac{20}{s+20} + \frac{40}{s+40}$$

b) $f_2(t) = \left[15 + 15\cos 500t\right]u(t)$

$$= 15u(t) + 15\cos 500t$$

$$= \frac{15}{s} + \frac{15s}{s^2 + 500^2}$$

9.10

(a) Since $f_1(t) = 3\delta(t-2) = 3\delta(t-2)u(t-2)$, the Laplace transform of $f_1(t)$ is $F_1(s) = 3e^{-2s}$ (to graders: explanation not necessary).

(b) The Laplace transform of $f_2(t) = 10e^{-50(t-1)}u(t-1)$ is $F_2(s) = \frac{10e^{-s}}{s+50}$.

(c) The Laplace transform of $f_3(t) = 10e^{-50(t-2)}u(t-2)$ is $F_3(s) = \frac{10e^{-2s}}{s+50}$.

9-16

a) $F_1(s) = \dfrac{s+100}{s(s+50)}$

$$\frac{s+100}{(s+50)s} = \frac{A}{s} + \frac{B}{s+50}$$

$s+100 = A(s+50) + B(s)$ let $s=0$

$100 = A(50) + 0$ $A = 2$

$50 = 0 + B(-50)$ let $s = -50$

$B = -1$

$F_1(s) = \dfrac{2}{s} + \dfrac{-1}{s+50}$

$f(t) = 2u(t) - e^{-50t}u(t) = \boxed{(2 - e^{-50t})u(t)}$

b) $F_2(s) = \dfrac{s+10}{s(s+50)(s+100)}$

$s+10 = A(s+50)(s+100) + B(s)(s+100) + C(s)(s+50)$

let $s=0$ $1\cancel{0} = A(50)(10\cancel{0}) + 0 + 0$ $A = \frac{1}{500}$

let $s=-50$ $-\overset{2}{\cancel{4}}\cancel{0} = 0 + B(-\overset{25}{\cancel{50}})(5\cancel{0}) + 0$ $B = \frac{-2}{125}$

let $s=-100$ $-9\cancel{0} = 0 + 0 + C(-100)(-5\cancel{0})$

$C = \frac{-9}{500}$

$F_2(s) = \dfrac{\frac{1}{500}}{s} + \dfrac{\frac{-2}{125}}{s+50} + \dfrac{\frac{-9}{500}}{s+100}$

$\boxed{F(t) = \left(\frac{1}{500} - \frac{2}{125}e^{-50t} - \frac{9}{500}e^{-100t}\right)u(t)}$

9.25) a) $F_1(s) = \dfrac{4(s^2+16)}{s(s^2+8s+32)}$

$$\frac{4(s^2+16)}{s(s^2+8s+32)} = \frac{k_1}{s} + \frac{k_2}{s+4+4j} + \frac{k_3}{s+4-4j}$$

$$k_1 = sF(s)\Big|_{s=0} = \frac{4(16)}{32} = 2$$

$$k_2 = (s+4+4j)F(s)\Big|_{s=-4-4j} = \frac{4\left((-4-4j)^2+16\right)}{(-4-4j)(\cancel{-4}-4j\cancel{+4}-4j)}$$

$$= \frac{4\left(16+16j^2+32j+\cancel{16}\right)}{(-4-4j)(-8j)}$$

$$= \frac{2+4j}{(-1-j)(-j)} = \frac{2+4j}{-1+j}\frac{(1+j)}{(-1+j)} = \frac{2+2j+4j+4j^2}{2} = 1-3j$$

$$K_3 = (s+4-4j)F(s)\Big|_{s=-4+4j}$$

$$= \frac{4\left[(-4+4j)^2+16\right]}{(-4+4j)(\cancel{-4}+4j+\cancel{4}+4j)}$$

$$= \frac{4\left[16+16j^2-32j+\cancel{16}\right]}{(-4+4j)(8j)} = \frac{\left[2-4j\right]}{(-1+j)(j)}$$

$$= \frac{2-4j}{-j+j^2} = \frac{2-4j\,(-1+j)}{-j-1\,(-1+j)}$$

$$= \frac{-2+2j+4j-4j^2}{(-1)^2-j^2}$$

$$= \frac{+2+6j}{2} = 1+3j$$

$$= \sqrt{10}\,e^{j71.5}$$

$$= \sqrt{10}\,e^{-j1.25}$$

$$F_1(s) = \frac{2}{s} + \frac{1-3j}{s+4+4j} + \frac{1+3j}{s+4-4j}$$

$$f_1(t) = 2u(t) + 2|\sqrt{10}|\,e^{-4t}\cos(4t+\tan^{-1}(3))\,u(t)$$

9.25b) $F_2(s) = \dfrac{2(s^2+30s+800)}{s(s^2+50s+400)} = \dfrac{k_1}{s} + \dfrac{k_2}{s+10} + \dfrac{k_3}{s+40}$

$$k_1 = sF(s)\Big|_{s=0} = \frac{2(800)}{400} = 4$$

$$k_2 = (s+10)F(s)\Big|_{s=-10} = \frac{2(100-300+800)}{(-10)(-10+40)} = \frac{2(600)}{-300} = -4$$

$$k_3 = (s+40)F(s)\Big|_{s=-40} = \frac{2(1600-1200+800)}{(-40)(-40+10)}$$

$$= \frac{2(1200)}{+1200} = +2$$

$$F_2(s) = \frac{4}{s} - \frac{4}{s+10} + \frac{2}{s+40}$$

$$= [4 - 4e^{-10t} + 2e^{-40t}]u(t)$$

9.32

To solve the following ordinary differential equation,

$$\frac{di(t)}{dt} + 500i(t) = .100e^{-200t}u(t), \quad i(0^-) = 0,$$

begin by taking the Laplace transform;

$$sI(s) + 500I(s) = \frac{.100}{s+200} \quad \text{(we have used the fact that } i(0^-) = 0\text{)}.$$

We have

$$\begin{aligned} I(s) &= \frac{.100}{(s+200)(s+500)} \\ &= \frac{c_1}{s+200} + \frac{c_2}{s+500}, \end{aligned}$$

where the terms c_1 and c_2 are

$$\begin{aligned} c_1 &= \lim_{s\to -200}(s+200)I(s) \approx \frac{1}{3000}, \\ c_2 &= \lim_{s\to -500}(s+500)I(s) \approx -\frac{1}{3000} \end{aligned}$$

Now, taking the inverse Laplace transform of $I(s)$, we have that

$$i(t) \approx \frac{1}{3000}\left(e^{-200t} - e^{-500t}\right)u(t).$$

(This is an approximate solution due to the term .100 having only three significant digits.)

9-47

a) $F_1(s) = \dfrac{50(s^2+3s+6)}{(s+2)(s+6)(s+12)} = \dfrac{50(s+3)}{(s+6)(s+12)}$

initial value $\lim\limits_{s\to\infty} \dfrac{s\,50(s+3)}{(s+6)(s+12)} = \boxed{50}$

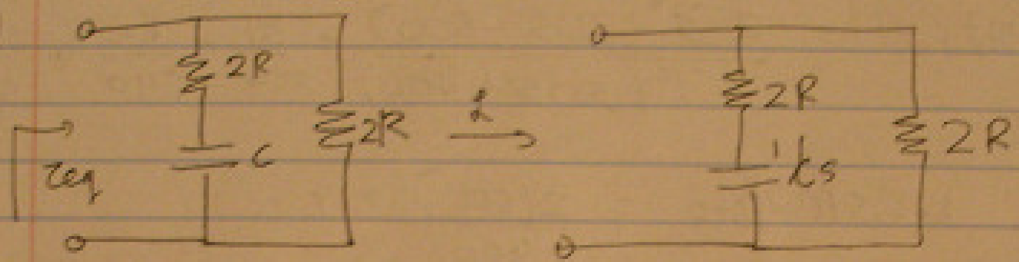
final value $\lim\limits_{s\to 0} \dfrac{s\,50(s+3)}{(s+6)(s+12)} = \boxed{0}$

b) $F_2(s) = \dfrac{10(s^2+10s+40)}{s(s+25)(s-25)}$

initial value $\lim\limits_{s\to\infty} \dfrac{\cancel{s}10(s^2+10s+40)}{\cancel{s}(s+25)(s-25)} = \boxed{10}$

final value doesn't exist. There is a pole in the Right-half plane so it's unstable.

10.1



$$Z_{eq} = \frac{(2R + 1/Cs)2R}{2R + \frac{1}{Cs} + 2R}$$

$$= \frac{(2RCs+1)2R}{4RCs+1} = \frac{4R^2Cs + 2R}{4RCs+1}$$

$$= \frac{Rs + \frac{2R}{4RC}}{s + \frac{1}{4RC}} = \frac{Rs + \frac{1}{2C}}{s + \frac{1}{4RC}}$$

pole: $\frac{-1}{4RC}$

zero: $\frac{-1}{2RC}$

10.6

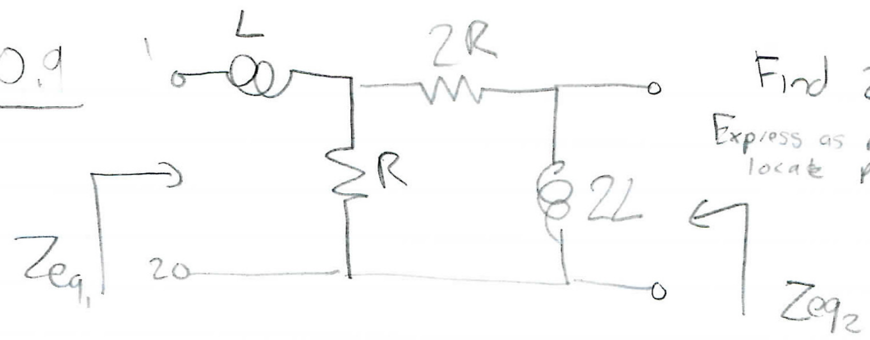
The equivalent impedance follows from transforming the circuit into the s-domain and assuming the initial conditions are zero (without loss of generality due to superposition). We have

$$\begin{aligned} Z_{eq} &= sL + \left(R || \frac{1}{sC}\right), \\ &= sL + \frac{R \times \frac{1}{sC}}{R + \frac{1}{sC}}, \\ &= \frac{RLCs^2 + Ls + r}{RCs + 1}. \end{aligned}$$

Substituting the values for R, L, and C, we find a pole at $s = -1/(RC) = -2000$.

To find the zeros, we use MATLAB's root-finding function on the numerator polynomial (after substituting parameters). The two zeros are both at $s = -1000$.

10.9



Find Z_{eq_1} & Z_{eq_2}
Express as rational function
locate poles & zeros

$$Z_{eq_1} = L + \left(R \parallel (2R + 2L)\right)$$

$$2R + 2L = 2R + 2Ls$$

$$Z_{eq_1} = \frac{R(2R + 2Ls)}{3R + 2Ls} + \frac{Ls(3R + 2Ls)}{3R + 2Ls}$$

$$Z_{eq_1} = \frac{2R^2 + 2LsR + 3LRs + 2L^2s^2}{3R + 2Ls} = \boxed{\frac{2L^2s^2 + 5LRs + 2R^2}{3R + 2Ls}}$$

$$\boxed{\text{Pole @ } s = \frac{-3R}{2L}}, \quad \frac{-5LR \pm \sqrt{25L^2R^2 - 4 \cdot 2L^2 \cdot 2R^2}}{4L^2} = \frac{-5LR \pm 3LR}{4L^2}$$

$$\boxed{\text{Zeros @ } s = \frac{-R}{2L}, \; s = \frac{-2R}{L}}$$

$$Z_{eq_2} = 3R \parallel 2L$$

$$= \boxed{\frac{6LsR}{3R + 2Ls}}$$

$$\boxed{\begin{array}{l} \text{pole @ } \frac{-3R}{2L} \\ \text{zero @ } 0 \end{array}}$$