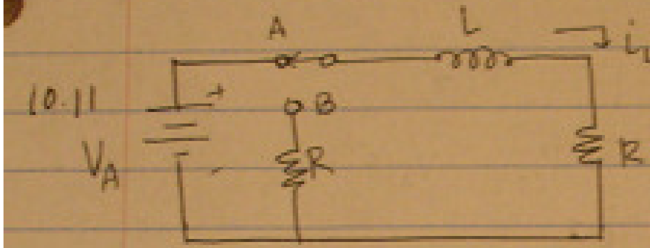
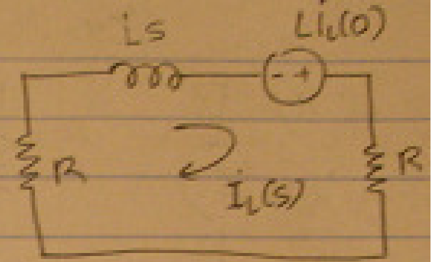



$\xrightarrow{\mathcal{L}}$




$$i_L(0) = \frac{V_A}{R}$$

KVL: $R I_L(s) + Ls\, I_L(s) - L i_L(0) + R I_L(s) = 0$

$$2R I_L(s) + Ls\, I_L(s) - \frac{L V_A}{R} = 0$$

$$I_L(s)(2R + Ls) = \frac{L V_A}{R}$$

$$I_L(s) = \frac{L V_A}{R(sL + 2R)}$$

$$I_L(s) = \frac{V_A}{R\left(s + \frac{2R}{L}\right)}$$

$$\boxed{i_L(t) = \frac{V_A}{R} e^{-t(2R/L)} u(t)}$$

## 10.13

First solve for the initial condition of the capacitor. As the battery is connected for an extended period of time, the circuit characteristic of the capacitor tends to an open circuit. With this simplification and by straightforward voltage division, we have

$$v_C(0) = \frac{V_A}{2}. \tag{1}$$

Now we proceed to describe the circuit after the switch is flipped, i.e. for $t \geq 0$. Note that we mathematically represent the action of flipping the switch by describing the second battery as a step function $V_A u(t)$ rather than as a constant over time. KCL at the top node yields

$$\frac{V_C(s) - (-V_A/s)}{R} + \frac{V_C(s)}{R} + I_C(s) = 0. \tag{2}$$

Making the substitution,

$$V_C(s) = \frac{I_C(s)}{sC} + \frac{v_C(0)}{s},$$

Equation (2) (note that an equation with its number is a proper noun) becomes, after some algebraic wrangling,

$$\left(\frac{I_C(s)}{sC} + \frac{v_C(0)}{s}\right)\left(\frac{2}{R}\right) = -\frac{V_A}{sR} - I_C(s),$$

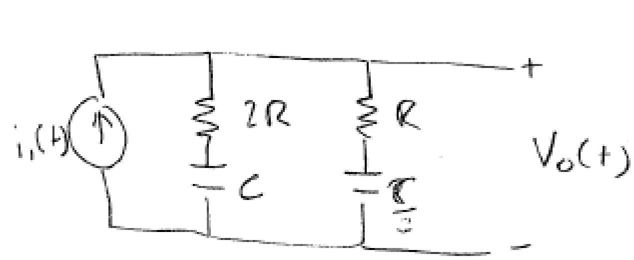
$$\vdots$$

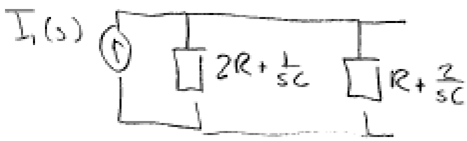
$$I_C(s) = -\frac{2V_A}{R} \times \frac{1}{s + 2(RC)^{-1}}.$$

Hence, taking the inverse Laplace transform,

$$i_C(t) = -\frac{2V_A}{R} e^{-2t(RC)^{-1}}.$$

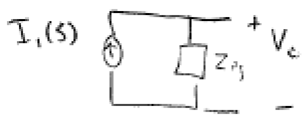
## 10.20




$$Z_{eq} = \frac{\left(2R+\frac{1}{sC}\right)\left(R+\frac{2}{sC}\right)}{\left(3R+\frac{3}{sC}\right)} = \frac{2R^2+\frac{5R}{sC}+\frac{2}{s^2C^2}}{3\left(R+\frac{1}{sC}\right)} \cdot \frac{s^2C^2}{s^2C^2}$$

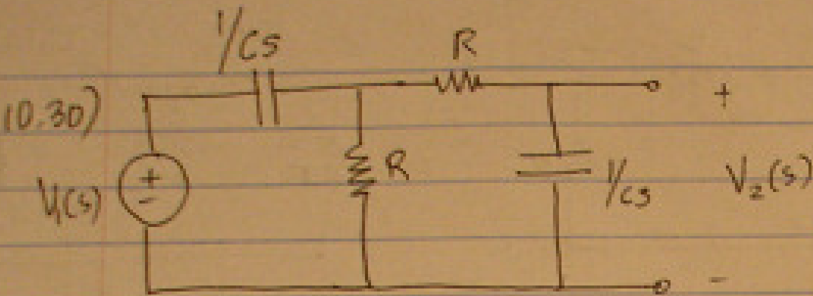
$$\frac{2R^2s^2C^2+5RsC+2}{3(Rs^2C^2+sC)} = \frac{2R^2s^2C^2+3RCs+2}{3sC(RsC+1)} = \frac{(2RCs+1)(RCs+2)}{3sC(RsC+1)}$$

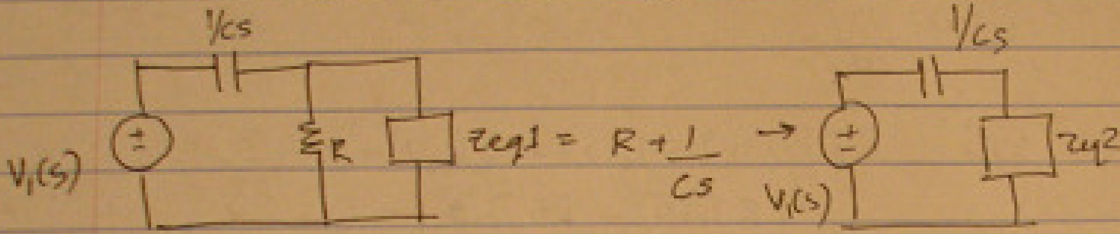



$V_o(s) = I_1(s)\,Z_{eq}(s)$

$$\boxed{V_o = \frac{(2RCs+1)(RCs+2)}{3sC(RsC+1)}\,I_1(s)}$$

10.30)




$$Z_{eq2} = \frac{R\left(R+\frac{1}{Cs}\right)}{2R+\frac{1}{Cs}}$$

$$V_{zeq2} = \frac{Z_{eq2}}{Z_{eq2}+\frac{1}{Cs}} V_1(s)$$

$$V_2(s) = \frac{1/Cs}{\frac{1}{Cs}+R} V_{zeq2}$$

$$= \frac{1}{1+RCs}\left(\frac{\frac{R(RCs+1)}{2RCs+1}}{\frac{R(RCs+1)}{2RCs+1}+\frac{1}{Cs}}\right)V_1(s)$$

$$= \frac{1}{1+RCs}\left(\frac{\frac{R\cancel{(RCs+1)}}{\cancel{2RCs+1}}}{\frac{R(RCs+1)Cs+2RCs+1}{(\cancel{2RCs+1})Cs}}\right)V_1(s)$$

$$V_2(s) = \frac{RCs}{RCs(RCs+1) + 2RCs + 1} V_1(s)$$

$$= \frac{RCs}{R^2C^2s^2 + 3RCs + 1}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{s/RC}{s^2 + \frac{3s}{RC} + 1/R^2C^2}$$

poles: $\dfrac{-3}{2RC} \pm \dfrac{1}{2}\dfrac{\sqrt{5}}{RC}$

Solving system of linear eqns.

$$\frac{-3}{2RC} + \cancel{\frac{\sqrt{5}}{2RC}} = -2618$$

$$\frac{-3}{2RC} - \cancel{\frac{\sqrt{5}}{2RC}} = -382$$

$$\frac{-6}{2RC} = -3000$$

$$RC = \frac{1}{1000} = 10^{-3}$$

$$\boxed{\begin{array}{l} R = 1k \\ C = 1\mu F \end{array}}$$

## 10.31

*Part (a)* Denote the left mesh by a mesh current $I_A(s)$, and the right mesh by a mesh current $I_B(s)$. Because the initial conditions are zero, the mesh equations follow immediately as

$$\left[\begin{array}{cc} sL+R_1 & -R_1 \\ -R_1 & R_1+R_2+\frac{1}{sC} \end{array}\right]\left[\begin{array}{c} I_A(s) \\ I_B(s) \end{array}\right]\left[\begin{array}{c} V_1(s) \\ 0 \end{array}\right].$$

*Part (b)* Note that $I_2(s) = I_B(s)$. The recommended method to solve this matrix equation to use MATLAB's symbolic solver package. Note that Wolfram Alpha also has a free symbolic solver capable of evaluating these expressions. From either package, or cranking through the algebra by hand, we have that

$$I_2(s) = \frac{sR_1CV_1(s)}{s^2LC(R_1+R_2)+s(R_1R_2C+L)+R_1}.$$

*Part (c)* The transfer function from $V_1(s)$ to current $I_2(s)$ has a zero at $s=0$. When $v_1(t)$ is a step function, there are no zeroes. This is somewhat ambiguous.

The poles may be evaluated using the quadratic formula as

$$\begin{aligned} s &= -\frac{-R_1R_2C - L \pm \sqrt{(R_1R_2C+L)^2 - 4R_1LC(R_1+R_2)}}{2LC(R_1+R_2)}, \\ &= -\left(\frac{1}{2}\right)\frac{L+R_1R_2C \pm \sqrt{(R_1R_2C)^2 - 2R_1R_2LC + L^2 - 4R_1^2LC}}{LC(R_1+R_2)}. \end{aligned}$$

Unfortunately, there is no easy way to do this.

*Part (d)* It is important to one's sanity to evaluate by computer as much of the following as possible. Substituting the given values and noting that $V_1(s) = 50/s$, we have

$$\begin{aligned} I_2(s) &= \frac{.025}{.003s^2+3s+1000}, \\ &\approx \frac{c_1}{s+500+288.7j} + \frac{c_1^*}{s+500-288.7j}, \end{aligned}$$

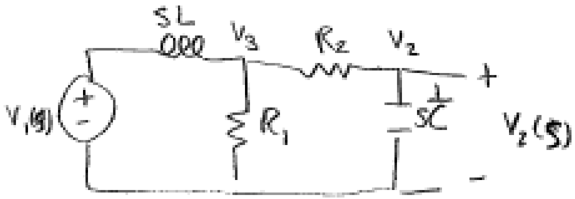
where $c_1^*$ denotes the conjugate of $c_1$, and $c_1$ is given by evaluating

$$\begin{aligned} c_1 &= \lim_{s\to -500-288.7j}(s+500+288.7j)I_2(s), \\ &= .0144j. \end{aligned}$$

Performing the inverse Laplace transform on this expression yields,

$$\begin{aligned} i_2(t) &= .0144e^{-500t}j\left(e^{-288.7jt} - e^{288.7jt}\right), \\ &= 29e^{-500t}\sin(288.7t)u(t) \text{ mA}. \end{aligned}$$

# 10.32




a)

$$\begin{bmatrix} \frac{1}{R_2}+sC & \frac{-1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{sL} \end{bmatrix}\begin{bmatrix} V_2 \\ V_3 \end{bmatrix}=\begin{bmatrix} 0 \\ \frac{V_1}{sL} \end{bmatrix}$$

b) symbolically solving with matlab

$$V_2(s)=\frac{R_1}{s^2LC(R_1+R_2)+s(R_1R_2C+L)+R_1}V_1(s)$$

c) The forced poles depend on $V_1(s)$, which isn't known. They're the poles of $V_1(s)$.
The natural poles are the roots of the denominator of $V_2(s)$

$$\boxed{S_{1,2}=\left(-\frac{1}{2}\right)\frac{L+R_1R_2C \pm\sqrt{(R_1R_2C)^2+2R_1R_2LC+L^2-4R_1^2LC}}{LC(R_1+R_2)}}$$

d)

$$V_2(s)=\frac{500\cdot 10^3\cdot 15}{s(s+500+500i)(s+500-500i)}=\frac{C_1}{s}+\frac{C_2}{s+500+500i}+\frac{C_2^*}{s+500-500i}$$

$$500\cdot 10^3\cdot 15(1)=C_1(s+500+500i)(s+500-500i)+C_2(s+500-500i)(s)+C_2^*(s+500+500i)s$$

let $s=0$, $500\cdot 15(1)=(500^2+500^2)C_1$

$C_1=15$

let $s=-500-500i$

$1000\cdot 500\ 15=0+C_2(-1000i)(-500-500i)$

$C_2=-7.5-7.5i$

$$V_2(t)=u(t)\left[15-7.5e^{-500t}\left(e^{500it}+e^{-500it}\right)+7.5ie^{-500t}\left(e^{-500it}-e^{500it}\right)\right]$$

$$V_2(t)=15u(t)\left[1-e^{-500t}\cos(500t)+e^{-500t}\sin(500t)\right]$$