MAE140 - Linear Circuits - Fall 11
Final, December 7

Instructions

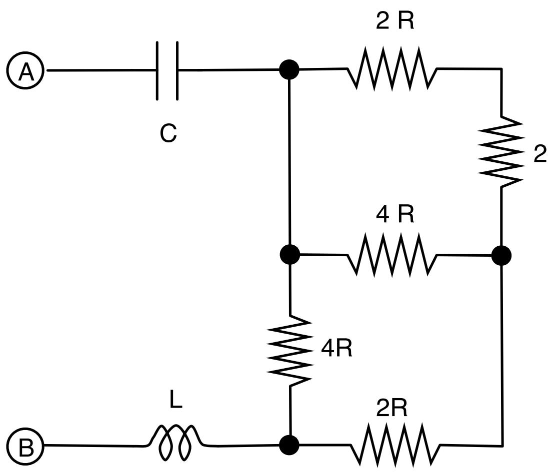
(i) The exam is open book. You may use your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 180 minutes

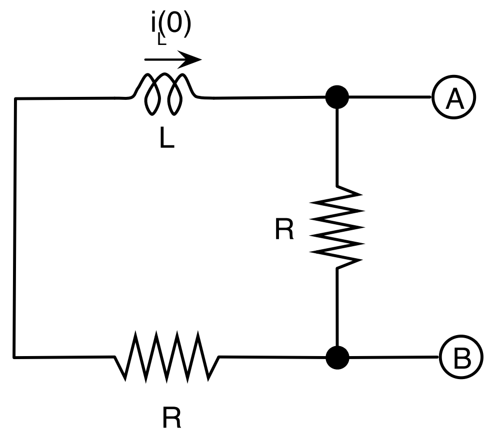
(iii) Do not forget to write your name, student number, and instructor

(iv) On the questions for which we have given the answers, please provide detailed derivations.

(v) The exam has 6 questions for a total of 60 points and 5 bonus points



(a) Question 1, Part I



(b) Question 1, Part II

Figure 1: Circuits for Question 1.

1. **Equivalent Circuits**

Part I: [5 points] Assuming zero initial conditions, transform the circuit into the s-domain and find the impedance equivalent to the circuit in Figure 1(a) as seen from terminals A and B. The answer should be given as a ratio of two polynomials.

Part II: [5 points] Assuming that the initial condition of the inductor is as indicated in the diagram, redraw the circuit shown in Figure 1(b) in the s-domain. Then use source transformation to find the s-domain Norton equivalent of this circuit as seen from terminals A and B.

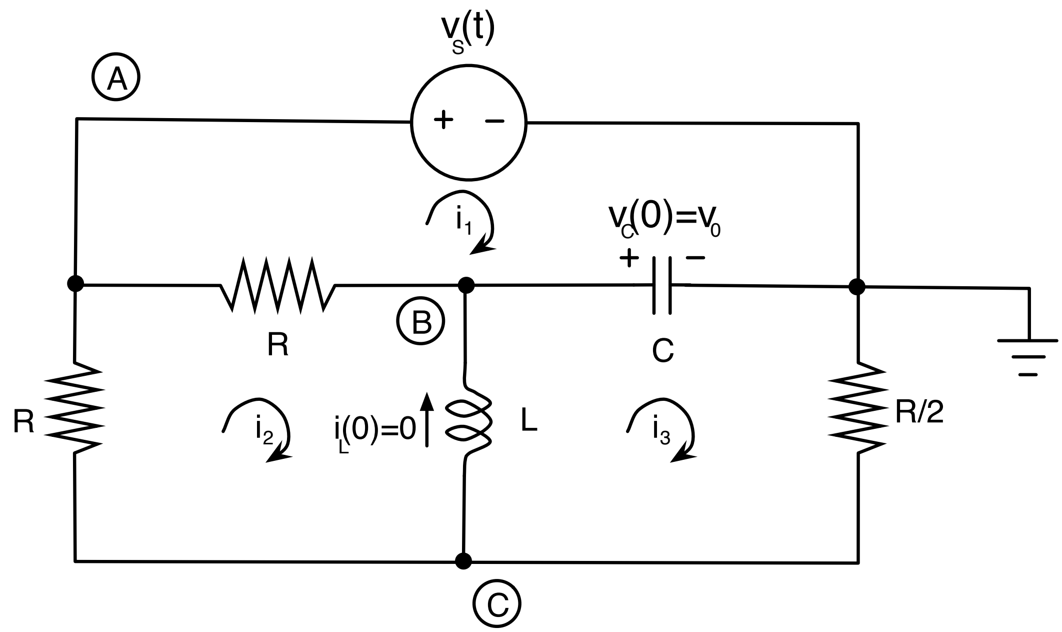


Figure 2: Nodal and Mesh Analysis Circuit

2. **Nodal and Mesh Analysis**

Part I: [5 points] Formulate node-voltage equations in the s-domain for the circuit in Figure 2. Use the reference node and other labels as shown in the figure. Use the initial conditions indicated in the figure! Transform initial conditions on the capacitor and on the inductor into current sources. No need to solve any equations!

Part II: [5 points] Formulate mesh-current equations in the s-domain for the circuit in Figure 2. Use the currents shown in the figure. Use the initial conditions indicated in the figure! Transform initial conditions on the capacitor and on the inductor into voltage sources. No need to solve any equations!

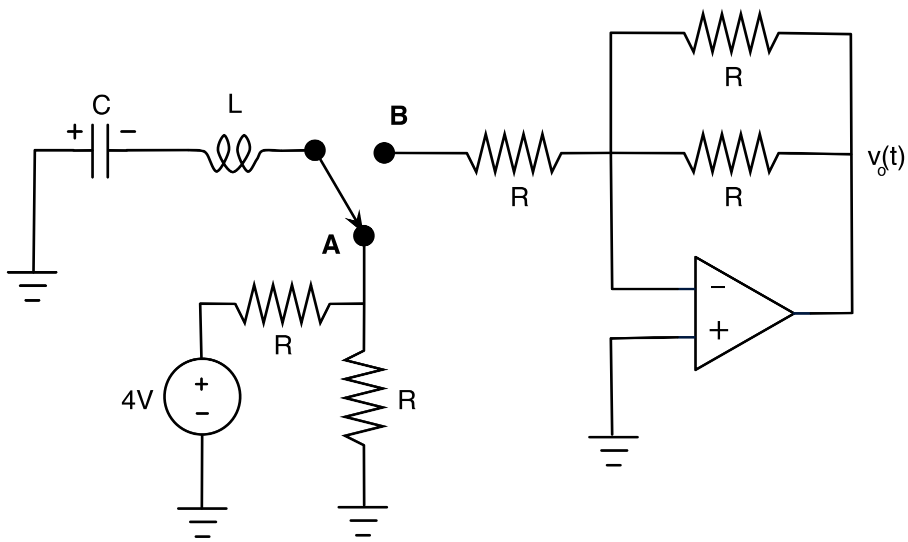


Figure 3: RCL circuit for Laplace Analysis

3. **Laplace Domain Circuit Analysis**

Part I: [3 points] Consider the circuit depicted in Figure 3. The voltage source is constant. The switch is kept in position **A** for a very long time. At $t = 0$ it is moved to position **B**. Show that the initial capacitor voltage and inductor currents are given by

$$v_C(0^-) = -2V, \quad i_L(0^-) = 0A.$$

[Show your work]

Part II: [2 points] Use these initial conditions to transform the circuit into the s-domain for $t \geq 0$. Use equivalent models for the capacitor and the inductor in which the initial conditions appear as voltage sources.
[Show your work]

Part III: [5 points] Use domain circuit analysis and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $C = \frac{1}{2}F$, $L = 6H$, and $R = 8\Omega$ is

$$v_o(t) = (2e^{-t} - 2e^{-t/3})u(t).$$

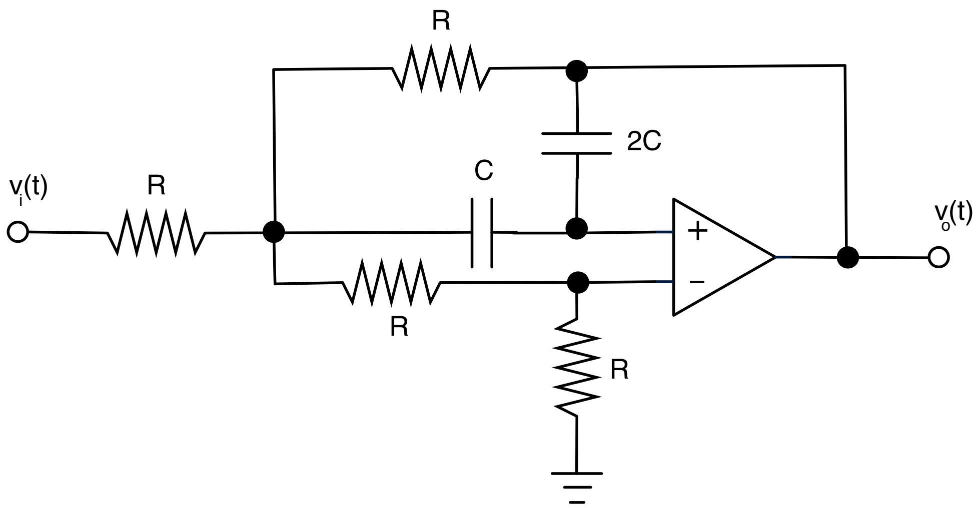


Figure 4: Frequency Response Analysis.

4. **Frequency Response Analysis**

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 4 into the s-domain.

Part II: [4 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ is given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{9 + 2RCs}.$$

[Show your work]

Hint: use node voltage analysis

Part III [3 points] Let $R = 10$ KΩ, $C = 100$ μF. Compute the gain and phase functions of $T(s)$. What are the DC gain and the ∞-freq gain? What is the cut-off frequency ω_c? Use these values to sketch the magnitude of the frequency response of the circuit. Is this circuit a low-pass, high-pass, or band-pass filter?
[Explain your answer]

Part IV [2 points] Using what you know about frequency response, compute the steady state response $v_o^{SS}(t)$ of this circuit when $v_i(t) = 2\cos(\frac{9}{2}t + \frac{\pi}{2})$ using the same values of R and C as in Part III.

5. **Active Filter Design**

Consider the transfer function

$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{2\lambda^2}{s^2 + 3\lambda s + 2\lambda^2}$$

where the parameter $\lambda \geq 0$ is to be specified by the user. In this question, always assume zero initial conditions.

Part I: [3 points] Show that the transfer function $T(s)$ can be realized as a product of two first-order low-pass filters of the form

$$T_1(s) = \frac{\pm\omega_1}{s + \omega_1}, \qquad T_2(s) = \frac{\pm\omega_2}{s + \omega_2}$$

that is, $T(s) = T_1(s) \times T_2(s)$. What is the cut-off frequency and gain of $T_1(s)$ and $T_2(s)$ in terms of λ?

Part II: [4 points] Design a circuit that implements $T(s)$ as the product of the two filters $T_1(s)$ and $T_2(s)$ using no more than 2 OpAmps.

Part III: [3 points] Find values of the components in your design so that $\lambda = 2000\pi$ rad/s.

Part IV: [3 bonus points] Design a circuit that implements $T(s)$ as the product of the two filters $T_1(s)$ and $T_2(s)$ using only 1 OpAmp.

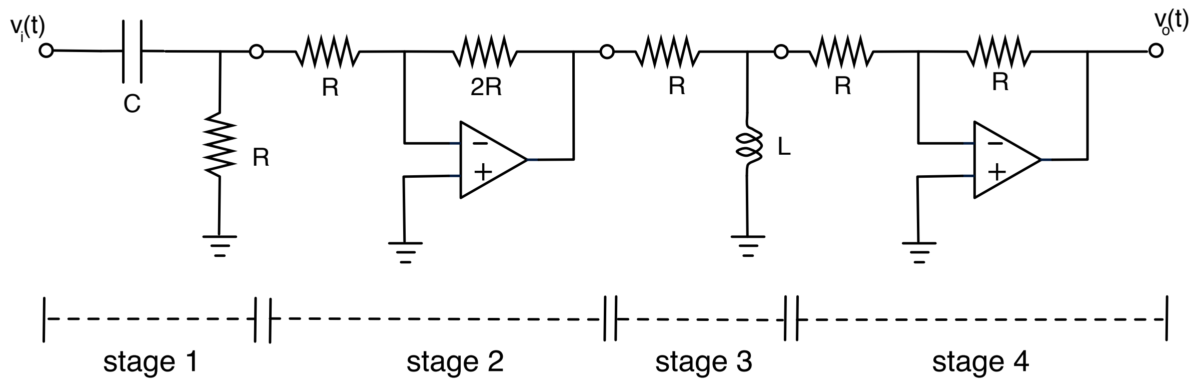


Figure 5: Circuit for Question 6.

6. **Chain rule and circuit design**

   Consider the circuit in Figure 5. You can assume zero initial conditions.

   Part I: [3 points] Redraw the circuit in Figure 5 in the s-domain and compute the transfer functions $T_1(s)$, $T_2(s)$, $T_3(s)$, $T_4(s)$ of each one of the stages.

   Part II: [2 points] Somebody with a rusty recollection of linear circuits analyzed the circuit in Figure 5 and concluded that the transfer function $T(s)$ from $V_i(s)$ to $V_o(s)$ is equal to the product of the transfer functions

   $$\widetilde{T}(s) = T_1(s) \times T_2(s) \times T_3(s) \times T_4(s) = \frac{2RCLs^2}{RCLs^2 + (R^2C + L)s + R}$$

   of the 4 stages identified in the plot. Identify two problems that invalidate this conclusion.

   Part III: [2 points] Modify Figure 5, keeping all 4 stages, so that the resulting circuit does have transfer function $\widetilde{T}(s)$ by adding at most 2 OpAmps.
   [Justify your answer]

   Part IV: [3 points] Use stage 1, stage 3 and a noninverting OpAmp to design yet another circuit with transfer function $\widetilde{T}(s)$.
   [Provide reasons that justify how you arrived at your design]

   Part V: [2 bonus points] What design would you recommend to realize the transfer function $\widetilde{T}(s)$, the answer in Part III or the answer in Part IV? Why?
   Hint: you can provide at least 2 different reasons