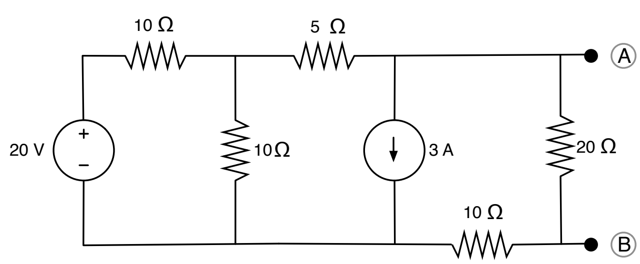
MAE140 - Linear Circuits - Fall 11
Midterm, October 27

Instructions

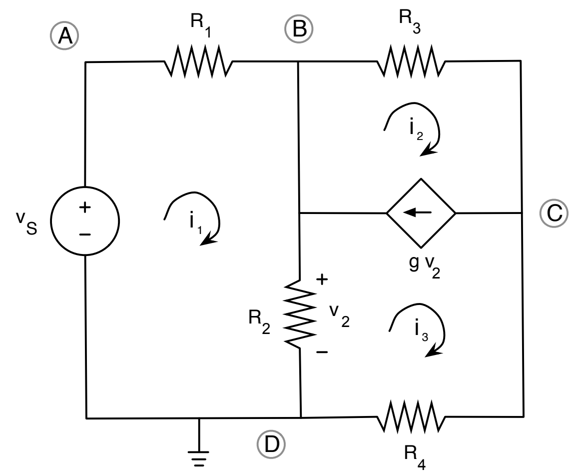
(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 70 minutes

(iii) Do not forget to write your **name**, **student number**, and **instructor**



(a) Question 1.



(b) Questions 2-3.

Figure 1: Circuits for questions 1-3

1. **Equivalent circuits**

Part I: [2 points] Turn off all the sources in the circuit of Figure 1(a) and find the equivalent resistance as seen from terminals A and B.

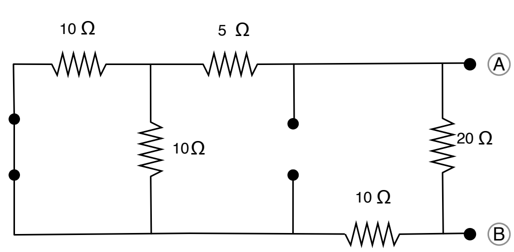
Part II: [3 points] Find the Thévenin equivalent as seen from terminals A and B.
Hint: If you want, you can use the result obtained in Part I

Part III: [1 point] Find the power absorbed by a $40\,\Omega$ resistor that is connected to terminals A and B.

Solution:

Part I: We start by switching off the sources.

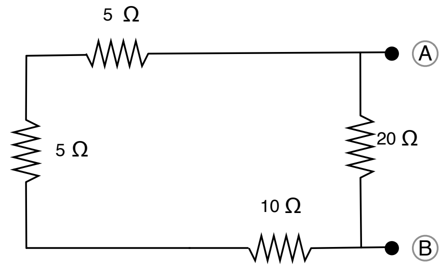
We substitute the voltage source by a short circuit, and the current source by an open circuit. Then, we get the circuit on the right



[.5 point]

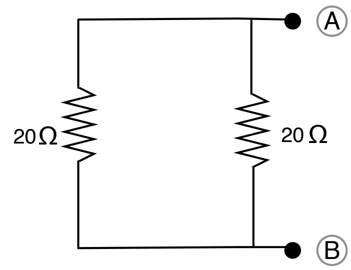
Now we combine the two resistances in parallel on the left to get the circuit

[.5 point]



Next, we sum the three resistances in series to reduce the circuit to:

[.5 point]



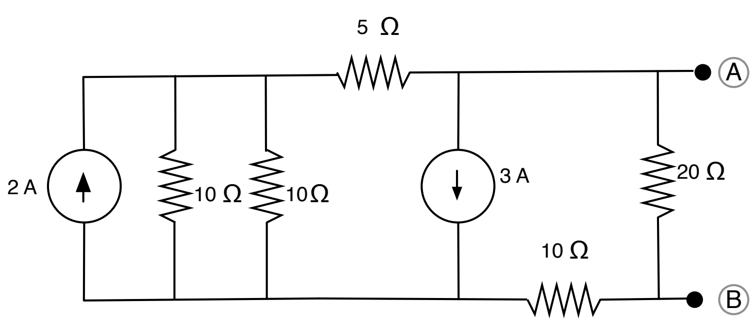
The final equivalent resistance is given by

$$R_{\text{final}} = \frac{20 \cdot 20}{20 + 20} = 10\Omega$$ **(.5 point)**

Part II: From Part I, we can say that $R_T = 10\Omega$. The Thévenin voltage v_T is equal to the voltage v_{oc} across the 20Ω resistor.

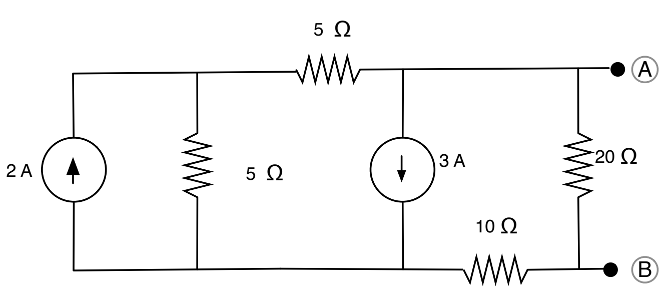
(+ .5 point)

To find the open-circuit voltage, a possible series of source transformations is shown here. We first transform the voltage source



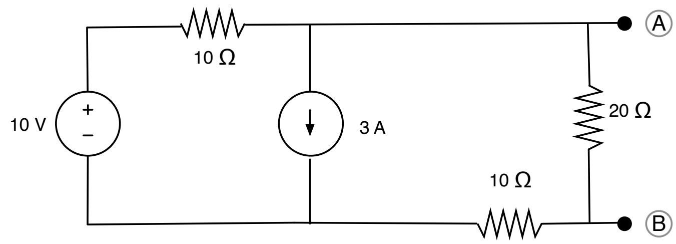
(+ .5 point)

This is equivalent to:



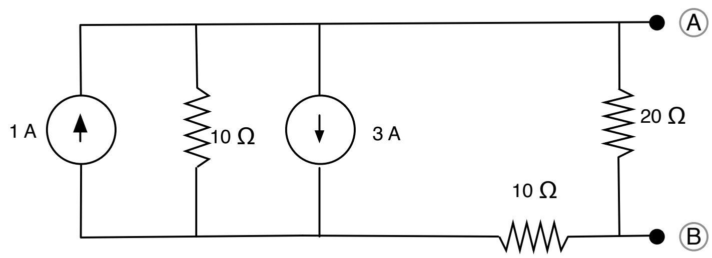
(+ .25 point)

We transform the current source back to a voltage source, then sum the resistors of 5Ω that become in series:



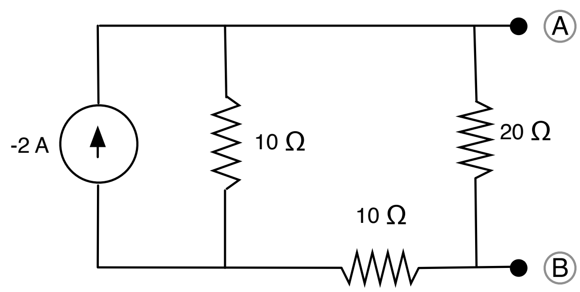
(+ .5 point)

The voltage source can be transformed now into a current source:



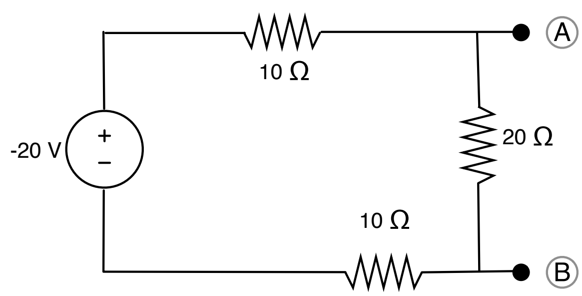
(+ .25 point)

Summing the two current sources in parallel results into:



(+ .25 point)

Applying a source transformation, reduces the circuit to:



(+ .25 point)

Now we can compute v_{oc} using a voltage division rule as:

$$v_T = v_{\text{oc}} = \frac{20}{10+10+20}(-20V) = -10V$$ **(+ .5 point)**

Part III: We compute the voltage across the 40Ω resistor with the Thévenin equivalent as:

$$v = \frac{40}{40+R_T} v_T = \frac{40}{50}(-10) = \frac{-40}{5} = -8V$$ **(+ .5 point)**

Then, the power absorbed by a 40Ω resistor is $p = \frac{v^2}{40} = \frac{64}{40} = 1.6W$. **(+ .5 point)**

2. **Node voltage analysis**

[6 points] Formulate node-voltage equations for the circuit in Figure 1(b). Use the node labels A, B, C provided in the figure and clearly indicate how you handle the presence of a voltage source. The final equations must depend only on unknown node voltages or the value v_S. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: There are four nodes in this circuit and the ground node (D) is directly connected to the voltage source. Therefore, this helps us taking care of it by setting $v_A = v_S$.

(+ 1.5 point)

We need to derive equations for the other two unknown node voltages v_B and v_C.

KCL for node B is:

$$\frac{v_B - v_S}{R_1} + \frac{v_B}{R_2} + \frac{v_B - v_C}{R_3} = gv_2$$

(+ 1 point)

KCL for node C is:

$$\frac{v_C - v_B}{R_3} + \frac{v_C}{R_4} = -gv_2$$

(+ 1 point)

The current source gv_2 depends on the voltage v_2 across R_2. Using the nodal variables, we see that

$$v_2 = v_B$$

(+ 1.5 point)

Thus, we can rewrite the equations above as:

$$\frac{v_B - v_S}{R_1} + \frac{v_B}{R_2} + \frac{v_B - v_C}{R_3} - gv_B = 0$$
$$\frac{v_C - v_B}{R_3} + \frac{v_C}{R_4} + gv_B = 0,$$

or, equivalently,

$$v_B\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} - g\right) - \frac{v_C}{R_3} = \frac{v_S}{R_1}$$
$$v_C\left(\frac{1}{R_3} + \frac{1}{R_4}\right) + v_B\left(-\frac{1}{R_3} + g\right) = 0.$$

(+ 1 point)

3. **Mesh current analysis**

[6 points] Formulate mesh-current equations for the circuit in Figure 1(b). Use the mesh currents shown in the figure and clearly indicate how you handle the presence of a dependent current source. The final equations should only depend on the unknown mesh currents and the source value v_S. **Do not modify the circuit or the labels**. **Do not use any source transformation**. No need to solve any equations!

Hint: Use a supermesh

Solution: KVL for mesh 1 becomes:

$$i_1 R_1 + (i_1 - i_3)R_2 = v_S$$

(+ 1 point)

Mesh 2 and Mesh 3 share a current source. We can deal with it defining a supermesh consisting of Mesh 2 and Mesh 3.

(+ 1 point)

KVL for the supermesh reads like

$$i_2R_3 + i_3R_4 + R_2(i_3 - i_1) = 0, \qquad \textbf{(+ 1 point)}$$

and the current source imposes the constraint:

$$i_2 - i_3 = gv_2. \qquad \textbf{(+ 1 point)}$$

The value of the dependent source can be expressed in terms of mesh currents as

$$gv_2 = gR_2(i_1 - i_3). \qquad \textbf{(+ 1 point)}$$

Finally, the equations can be rewritten as:

$$\begin{aligned} &i_1(R_1 + R_2) - i_3R_2 = v_S, \\ &- R_2i_1 + i_2R_3 + i_3(R_2 + R_4) = 0, \\ &- gR_2i_1 + i_2 + i_3(gR_2 - 1) = 0. \end{aligned} \qquad \textbf{(+ 1 point)}$$

4. **Bonus question**

   [1 point] If you were allowed to use source transformations in the circuit of Figure 1(b) and node C was the ground (instead of D), describe what would you do in Question 2 to take care of the voltage source using node voltage analysis. **Do not write or solve any equations**!

Solution: The voltage source is connected in series with resistor R_1. Applying an equivalent source transformation, we can replace the series connection of v_S and R_1 with an independent current source $\frac{v_S}{R_1}$ connected in parallel with R_1. The current source can now be handled using nodal analysis. This transformation has the added advantage of eliminating node A.