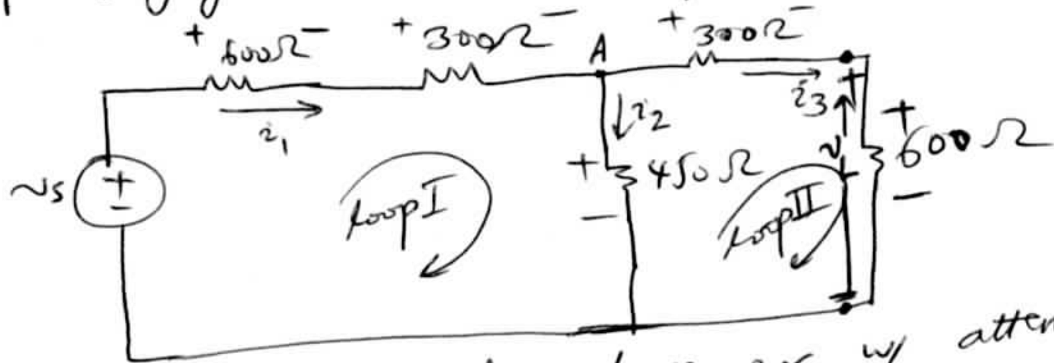
P 3-72:

Topology of the circuit w/ proper labeling:



1st) Compute the delivered power w/ attenuator:

- KCL at node A:

$$i_1 = i_2 + i_3 \quad (1)$$

- KVL loop II:

$$300\, i_3 + 600\, i_3 = 450\, i_2 \quad (2.1) \Rightarrow 900\, i_3 = 450\, i_2 \Rightarrow 2 i_3 = i_2 \quad (2.2)$$

- KVL loop I:

$$v_s = 600\, i_1 + 300\, i_1 + 450\, i_2 \quad (3)$$

Equations (1) & (2.2) yeild:

$$i_1 = i_2 + i_3 \xrightarrow{(2.2)} i_1 = i_3 + 2 i_3 \Rightarrow i_1 = 3 i_3 \quad (4)$$

Derive i_3 in terms of v_s based on the equations (2.2), (4) & (3):

$$v_s = 600\,(3 i_3) + 300\,(3 i_3) + 450\,(2 i_3) \Rightarrow v_s = 3600\, i_3$$

$$\Downarrow$$

$$\boxed{i_3 = \frac{1}{3600} v_s} \quad (5)$$

Then, derive v_L i.e. the voltage drop across the load based on equation (5):

$$v_L = 600 \cdot i_3 = 600 \cdot \frac{1}{3600} v_s \Rightarrow \boxed{v_L = \frac{1}{6} v_s} \quad (6)$$

Hence, the power delivered at the load side:

$$P_L = v_L\, i_L = \frac{1}{6} v_s \cdot \frac{1}{3600} v_s \Rightarrow \boxed{P_L = \frac{1}{21600} v_s^2} \quad (7)$$

2^{nd}) Compute the delivered power w/o attenuator:
The considered topology will set thus:

It is easy to check the following:

$$v_s = 600 i_1 + 600 i_1 \Rightarrow v_s = 1200\, i_1 \Rightarrow \boxed{i_1 = \frac{1}{1200} v_s} \quad (8)$$

$$\therefore\ v_L = v_s - 600\, i_1 = v_s - \frac{600}{1200} v_s \Rightarrow \boxed{v_L = \frac{v_s}{2}} \quad (9)$$

Denote $P_L' =$ the power delivered at the load side w/o attenuator:

$$\text{Equations (8), (9)} \Rightarrow P_L' = \frac{1}{1200} v_s \cdot \frac{v_s}{2} \Rightarrow \boxed{P_L' = \frac{1}{2400} v_s^2} \quad (10)$$

3^{rd}) Having developed equations (7) & (10), we compute the fraction of delivered power in these cases as follows:

(Let f_p be the desired fraction)

$$f_p = \frac{P_L}{P_L'} = \frac{1/21600\ v_s^2}{1/2400\ v_s^2} = 11.11 \times 10^{-2} = \frac{1}{9}$$

Therefore:

$$f_{p,dB} = 10 \log_{10} f_p = 10 \log_{10} (1.11 \times 10^{-1}) \Rightarrow \boxed{f_{p,dB} = -9.54\,dB}$$

- Requirements: $\begin{cases} v_{oc} = 36\,v \\ R_T \leq 10\,\Omega \end{cases}$

- 1^{st} type of cells: $\begin{cases} v_{oc1} = 9\,v \\ R_{T1} = 4\,\Omega \\ m_1 = 40\,g \end{cases}$ 2^{nd} type of cells: $\begin{cases} v_{oc2} = 4\,v \\ R_{T2} = 0.5\,\Omega \\ m_2 = 15\,g \end{cases}$

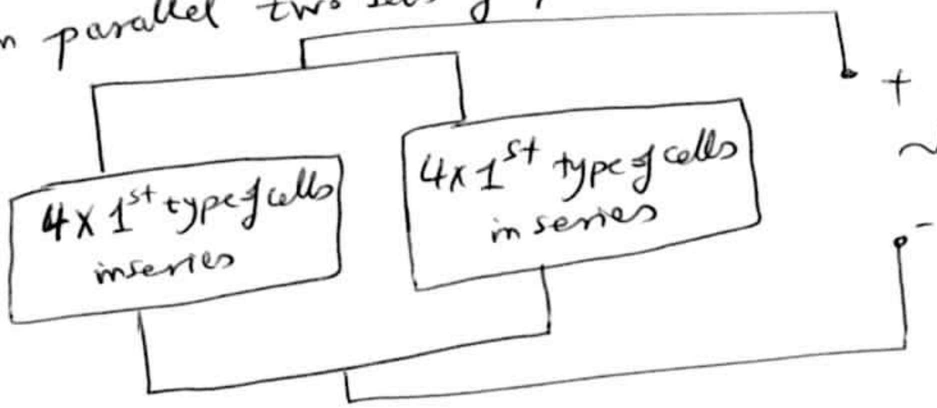
- we are allowed to use EITHER 1^{st} type OR 2^{nd} type:

– using 1^{st} type of cells:

Design I: to put 4 of these cells in series $\Rightarrow \begin{cases} v_{oc} = 4 \times 9 = 36\,v \checkmark \\ R_T = 4 \times R_{T1} = 16 \geq 10\,\Omega \times \end{cases}$

this design is NOT desirable, because $R_T = 16 \geq 10\,\Omega$

Design II: to put in parallel two sets of 4 series cells of this type:



$$v_T = 4 \times 9 = 36\,v \checkmark$$

$$R_T = (4 \times R_{T1}) \| (4 \times R_{T1}) = \frac{16 \times 16}{16 + 16} \Rightarrow R_T = 8\,\Omega \leq 10\,\Omega \checkmark$$

this design is, so far, desirable, w/ $m_{tot} = 8 \times 40 = 320\,g.$

– using 2^{nd} type of cells:

Design III: to put in series 9 of these cells in series $\Rightarrow \begin{cases} v_{oc} = 9 \times 4 = 36\,v \checkmark \\ R_T = 9 \times R_{T2} = 4.5 \leq 10 \checkmark \end{cases}$

this design is, so far, desirable, w/ $m_{tot} = 9 \times 15 = 135\,g.$

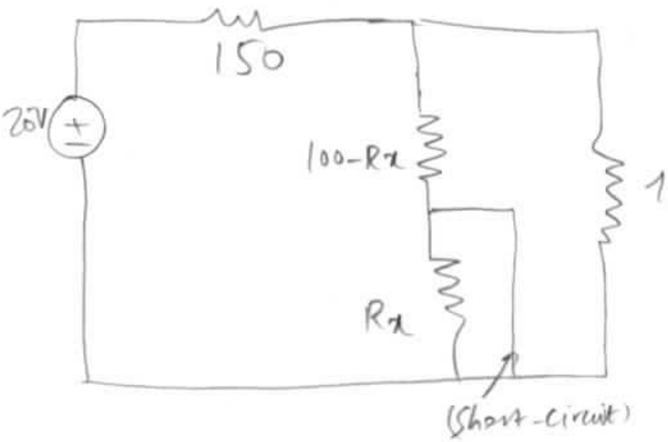
– The best design:

Comparing Design II & Design III $\Rightarrow$ as $m_{tot}\big|_{\text{Design II}} > m_{tot}\big|_{\text{Design III}}$,

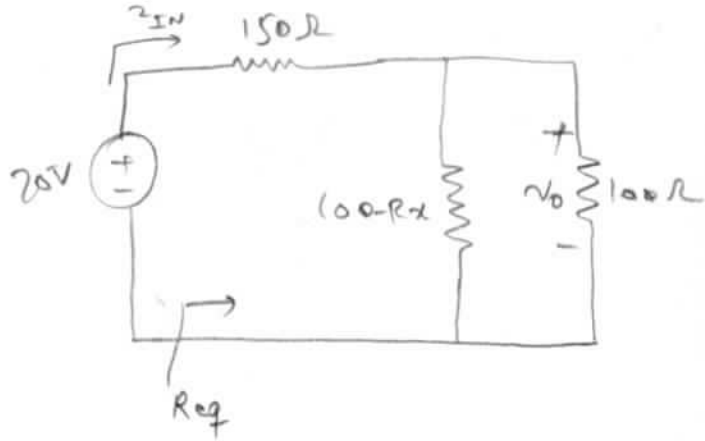
So, the best design is Design III.

P3-75:

Circuit 1:



$\Longrightarrow$



where $0 \leq Rx \leq 100$ & to maintain $\begin{cases} 0 \leq v_0 \leq 5 \\ P_{IN} \leq 2.5W \quad [P_{IN} = 20 \cdot i_{IN}] \end{cases}$

$$Req = 150 + (100 - Rx) \| (100\,\Omega) = 150 + \frac{(100-Rx)(100)}{100 - Rx + 100} = 150 + \frac{10^4 - 10^2 Rx}{200 - Rx}$$

$$\Rightarrow v_0 = 20 - 150 \times \underbrace{\frac{20}{Req}}_{i_{IN}} = 8 + \frac{480}{Rx - 160}$$

So, to check: $0 \leq v_0 \leq 5 \Rightarrow 0 \leq 8 + \frac{480}{Rx-160} \leq 5$

$\Downarrow$ some simplifications

$0 \leq Rx \leq 100$ ✓

Then, to check: $P_{IN} \leq 2.5 \Rightarrow 20 \cdot i_{IN} \leq 2.5 \Rightarrow 20\left(\frac{20}{Req}\right) \leq 2.5$

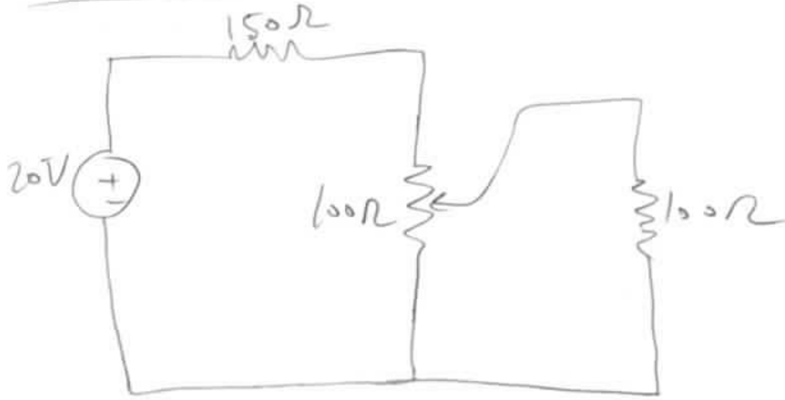
$$\Rightarrow Req \geq 160 \quad \Rightarrow 150 + \frac{10^4 - 10^2 Rx}{200 - Rx} \geq 160$$

$\Downarrow$ some simplifications

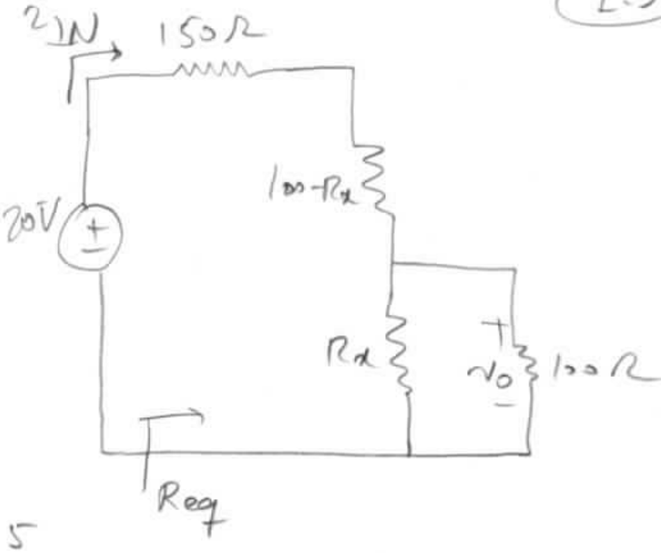
$Rx \leq 88.89$ ✗

it means that for $88.89 < Rx \leq 100$, $P_{IN} \leq 2.5$ is NOT satisfied, so it is not acceptable.

Circuit 2:



$\Longrightarrow$



where $0 \leq R_x \leq 100$ & to maintain $\begin{cases} 0 \leq v_o \leq 5 \\ P_{IN} \leq 2.5W \quad [P_{IN} = 20 \cdot i_{IN}] \end{cases}$

$$R_{eq} = (150 + 100 - R_x) + R_x \,//\, 100 = 250 - R_x + \frac{100 R_x}{100 + R_x}$$

$$v_o = 20 - (150 + (100 - R_x)) \underbrace{\frac{20}{R_{eq}}}_{i_{IN}} \xrightarrow[\text{simplification}]{\text{some}} v_o = -\frac{2000 R_x}{R_x^2 - 250 R_x - 25000}$$

So, to check: $0 \leq v_o \leq 5 \Longrightarrow 0 \leq -\dfrac{2000 R_x}{R_x^2 - 250 R_x - 25000} \leq 5$

$\Downarrow$ some simplifications

$0 \leq R_x \leq 100$ ✓

Then, to check: $P_{IN} \leq 2.5 \Rightarrow 20 \cdot i_{IN} \leq 2.5 \Rightarrow 20 \left(\dfrac{20}{R_{eq}}\right) \leq 2.5$

$\Rightarrow R_{eq} \geq 160 \Longrightarrow 250 - R_x + \dfrac{100 R_x}{100 + R_x} \geq 160$

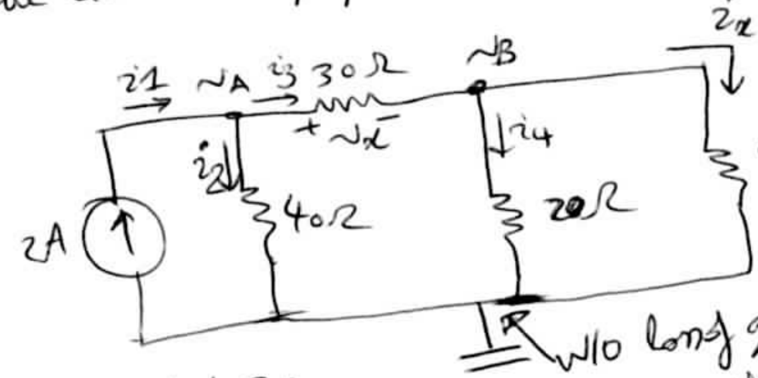
$\Downarrow$ some simplifications

$R_x \leq 100$ ✓

Hence, both requirements are met for $0 \leq R_x \leq 100$, so this design is desirable.

P3-1:

The topology of the circuit w/ proper labeling is drawn as follows:



w/o loss generality, we can assume this node to be the reference node.

(I) KCL at node A & B:

$$i_1 = i_2 + i_3$$

$$i_3 = i_4 + i_x$$

(II) Element eq'ns:

$$i_1 = 2A \quad \text{(current source)}$$

$$i_2 = \frac{v_A}{40}$$

$$i_3 = \frac{v_A - v_B}{30}$$, [where we also note that: $\underbrace{v_x = v_A - v_B}_{(*)}$]

$$i_4 = \frac{v_B}{20}$$

$$i_x = \frac{v_B}{80} \quad (**)$$

(III) plugging back the obtained eq'ns in step(II), back into step(I):

$$2 = \frac{v_A}{40} + \frac{v_A - v_B}{30} \Rightarrow \begin{cases} 7v_A - 4v_B = 240 \\ -4v_A + 11.5v_B = 0 \end{cases}$$

$$\frac{v_A - v_B}{30} = \frac{v_B}{20} + \frac{v_B}{80} \Rightarrow$$

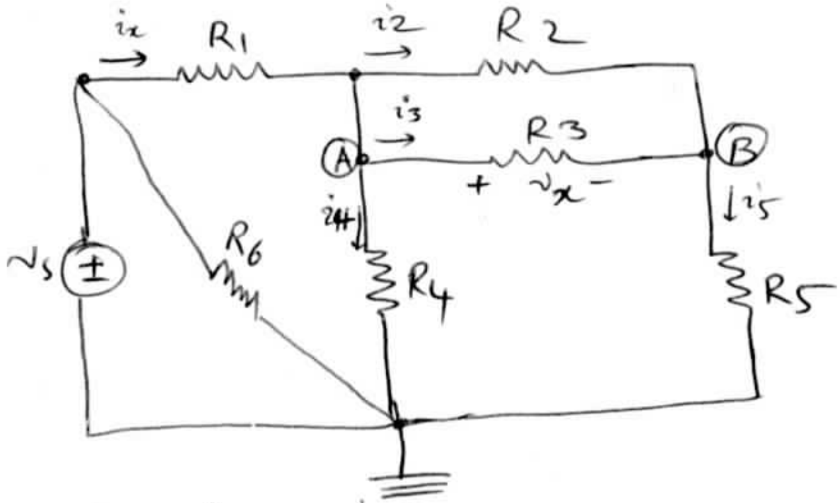
(IV) Solving for the set of linear eq'ns obtained in step(III):

$$\begin{bmatrix} 7 & -4 \\ -4 & 11.5 \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 240 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} \boxed{v_A = 42.79\,V} \\ \boxed{v_B = 14.88\,V} \end{cases}$$

Therefore, from (*): $v_x = v_A - v_B \Rightarrow \boxed{v_x = 27.91\,V}$

from (**): $i_x = v_B/80 \Rightarrow \boxed{i_x = 0.1860\,A}$

P 3-6:



(a) node-voltage eq'ns:

In analogous way to the sol'n of P 3-1, we obtain:

node (A): $i_x = i_2 + i_3 + i_4 \Leftrightarrow \dfrac{v_s - v_A}{R_1} = \dfrac{v_A - v_B}{R_2} + \dfrac{v_A - v_B}{R_3} + \dfrac{v_A}{R_4}$

$$\Leftrightarrow \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) v_A - \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_B = \left(\frac{1}{R_1}\right) v_s$$

node (B): $i_2 + i_3 = i_5 \Leftrightarrow \dfrac{v_A - v_B}{R_2} + \dfrac{v_A - v_B}{R_3} = \dfrac{v_B}{R_5}$

$$\Leftrightarrow -\left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_A + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) v_B = 0$$

and so, in matrix form:

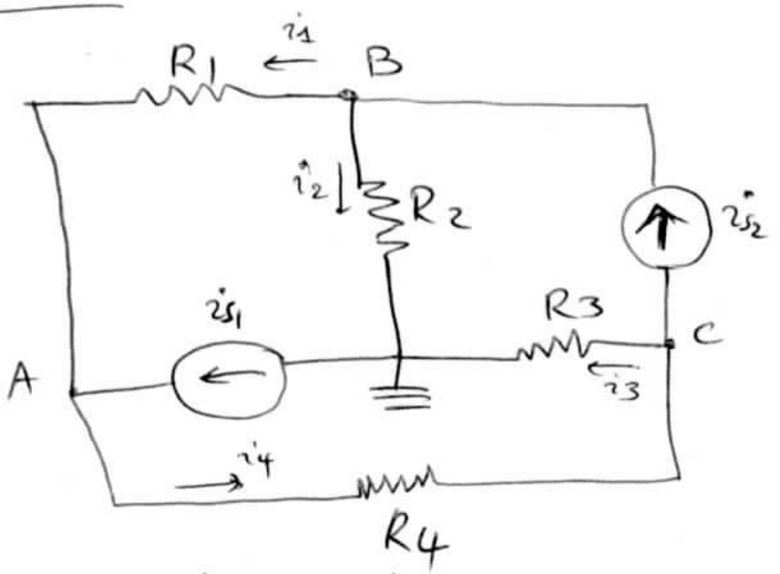
$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) & -\left(\frac{1}{R_2} + \frac{1}{R_3}\right) \\ -\left(\frac{1}{R_2} + \frac{1}{R_3}\right) & \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} \\ 0 \end{bmatrix} v_s$$

(b) Given: $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 10\,k\Omega$ & $v_s = 24\,V$:

$$\begin{bmatrix} 0.4\times10^{-3} & -0.2\times10^{-3} \\ -0.2\times10^{-3} & 0.3\times10^{-3} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 0.1\times10^{-3} \\ 0 \end{bmatrix} 24 \Rightarrow \begin{cases} v_A = 9\,V \\ v_B = 6\,V \end{cases}$$

where we have: $\begin{cases} v_x = v_A - v_B \Rightarrow \boxed{v_x = 3\,V} \\ i_x = \dfrac{v_s - v_A}{} \Rightarrow \boxed{i_x = 15\times10^{-4}\,A} \end{cases}$

P 3-7:



(a) node-voltage eq'ns:

In analogous way to P 3-1:

node (A): $i_{s_1} + i_1 = i_4 \Leftrightarrow \frac{v_A - v_C}{R_4} - \frac{v_B - v_A}{R_1} = i_{s_1} \Leftrightarrow \left(\frac{1}{R_1} + \frac{1}{R_4}\right) v_A - \frac{1}{R_4} v_C - \frac{1}{R_1} v_B = i_{s_1}$

node (B): $i_{s_2} = i_1 + i_2 \Leftrightarrow \frac{v_B - v_A}{R_1} + \frac{v_B}{R_2} = i_{s_2} \Leftrightarrow \left(-\frac{1}{R_1}\right) v_A + \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_B = i_{s_2}$

node (C): $i_{s_2} + i_3 = i_4 \Leftrightarrow \frac{v_A - v_C}{R_4} - \frac{v_C}{R_3} = i_{s_2} \Leftrightarrow -\left(\frac{1}{R_4}\right) v_A + \left(\frac{1}{R_4} + \frac{1}{R_3}\right) v_C = -i_{s_2}$

and so, in matrix form:

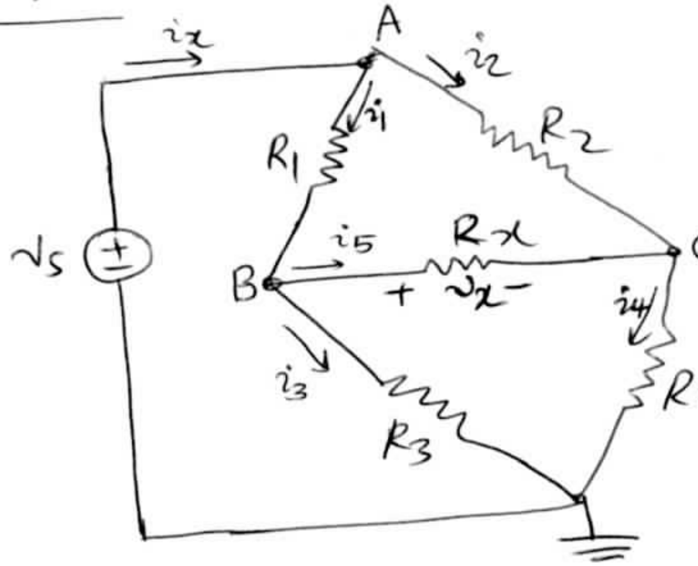
$$\begin{bmatrix} \left(\frac{1}{R_1} + \frac{1}{R_4}\right) & -\frac{1}{R_1} & -\frac{1}{R_4} \\ -\frac{1}{R_1} & \left(\frac{1}{R_1} + \frac{1}{R_2}\right) & 0 \\ -\frac{1}{R_4} & 0 & +\left(\frac{1}{R_4} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} i_{s_1} \\ i_{s_2} \\ -i_{s_2} \end{bmatrix}$$

It is also possible to use "writing node equations by inspection" method, instead.

(b) Given $R_1 = 1.5\,k\Omega$, $R_2 = 2.2\,k\Omega$, $R_3 = 3.3\,k\Omega$, $R_4 = 4.7\,k\Omega$, $i_{s_1} = i_{s_2} = 2\,mA$:

$$\begin{bmatrix} 9\times10^{-4} & -7\times10^{-4} & -2\times10^{-4} \\ -7\times10^{-4} & 1.1\times10^{-3} & 0 \\ -2\times10^{-4} & 0 & +5\times10^{-4} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \times 2\times10^{-3} \Rightarrow \begin{cases} \boxed{v_A = 5.98\,V} \\ \boxed{v_B = 5.34\,V} \\ \boxed{v_C = -1.41\,V} \end{cases}$$

P-3-8:



(a) node-voltage eq'ns:

In analogous way to P 3-1:

node Ⓑ: $i_1 = i_5 + i_3 \Longleftrightarrow \dfrac{v_S - v_B}{R_1} = \dfrac{v_B - v_C}{R_x} + \dfrac{v_B}{R_3} \Leftrightarrow \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_x}\right)v_B - \frac{1}{R_x}v_C = \frac{v_s}{R_1}$

node Ⓒ: $i_2 + i_5 = i_4 \Leftrightarrow \dfrac{v_S - v_C}{R_2} + \dfrac{v_B - v_C}{R_x} = \dfrac{v_C}{R_4} \Leftrightarrow \left(-\frac{1}{R_x}\right)v_B + \left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_x}\right)v_C = \frac{v_s}{R_2}$

and so, in matrix form:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_x} & -\frac{1}{R_x} \\ -\frac{1}{R_x} & \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_x} \end{bmatrix} \begin{bmatrix} v_B \\ v_C \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} \\ \frac{1}{R_2} \end{bmatrix} v_S$$

(b) Given: $R_1 = R_4 = 1\,k\Omega$, $R_2 = R_3 = 250\,\Omega$, $R_x = 500\,\Omega$, $v_s = 15V$:

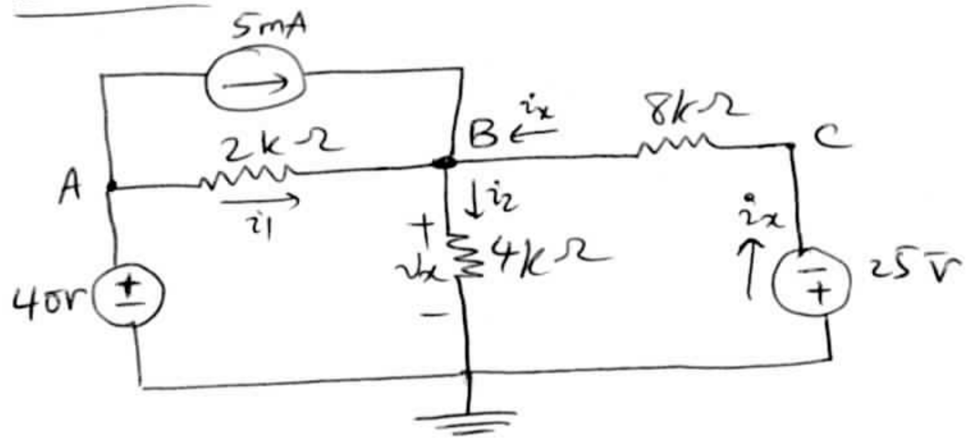
$$\begin{bmatrix} 7\times10^{-3} & -2\times10^{-3} \\ -2\times10^{-3} & 7\times10^{-3} \end{bmatrix} \begin{bmatrix} v_B \\ v_C \end{bmatrix} = \begin{bmatrix} 1.5\times10^{-2} \\ 6\times10^{-2} \end{bmatrix} \Rightarrow \begin{cases} \boxed{v_B = 5V} \\ \boxed{v_C = 10V} \end{cases}$$

Referring to circuit:

$v_x = v_B - v_C \Rightarrow \boxed{v_x = -5\,V}$

kcl at node Ⓐ: $i_x = i_1 + i_2 = \dfrac{v_s - v_B}{R_1} + \dfrac{v_s - v_C}{R_2} \Rightarrow \boxed{i_x = 0.03\,A}$

P 3-15:



(b) node-voltage eq'n:

node (B): $i_1 + 5\times10^{-3} = i_2 - i_x \Leftrightarrow \dfrac{40 - v_B}{2\times10^3} + 5\times10^{-3} = \dfrac{v_B}{4\times10^3} - \dfrac{-25 - v_B}{8\times10^3}$

$\Leftrightarrow v_B\left(\frac{1}{4} + \frac{1}{8} + \frac{1}{2}\right) = (5 - 3.125 + 20)$

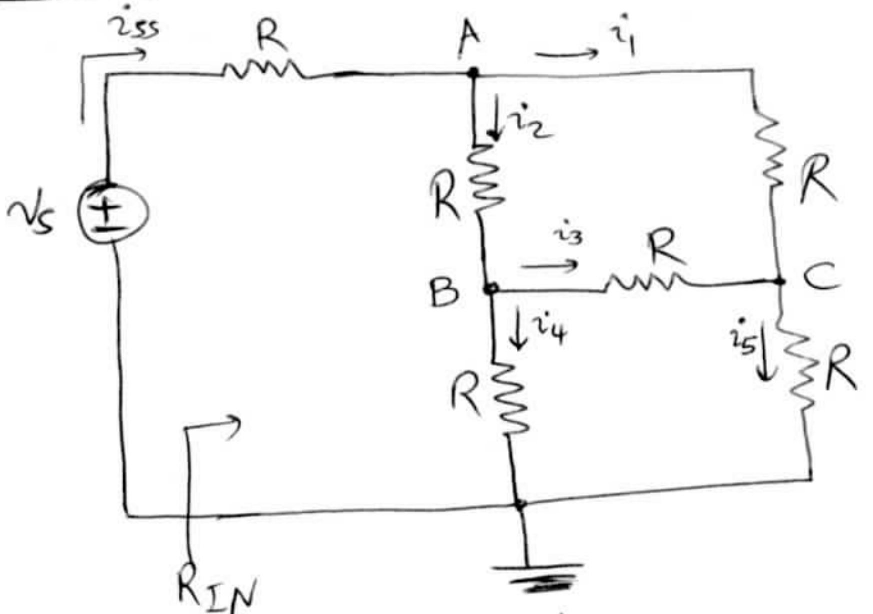
$\Leftrightarrow \boxed{v_B = 25\,V}$

(d) Referring to the circuit:

$i_x = \dfrac{-25 - v_B}{8\times10^3} \Rightarrow i_x = \dfrac{-25-25}{8\times10^3} \Rightarrow \boxed{i_x = -6.25\,mA}$

$v_x = v_B - v_{Ground} = v_B - 0 \Rightarrow \boxed{v_x = 25\,V}$

P3-16:



GOAL: find $R_{IN} = \dfrac{v_s}{i_{ss}}$.

node (A): $i_{ss} = i_1 + i_2$

[we note that: $v_s = R i_{ss} + v_A \Leftrightarrow i_{ss} = \dfrac{v_s - v_A}{R}$]

$\Updownarrow$

$$\frac{v_s - v_A}{R} = \frac{v_A - v_C}{R} + \frac{v_A - v_B}{R} \Leftrightarrow \left(\frac{3}{R}\right) v_A - \frac{1}{R} v_B - \frac{1}{R} v_C = \frac{v_s}{R}$$

$$\Leftrightarrow 3v_A - v_B - v_C = v_s$$

node (B): $i_2 = i_3 + i_4 \Leftrightarrow \dfrac{v_A - v_B}{R} = \dfrac{v_B - v_C}{R} + \dfrac{v_B}{R} \Leftrightarrow \left(\dfrac{3}{R}\right) v_B - \dfrac{1}{R} v_A - \dfrac{1}{R} v_C = 0$

$$\Leftrightarrow 3v_B - v_A - v_C = 0$$

node (C): $i_3 + i_1 = i_5 \Leftrightarrow \dfrac{v_B - v_C}{R} + \dfrac{v_A - v_C}{R} = \dfrac{v_C}{R} \Leftrightarrow \left(\dfrac{3}{R}\right) v_C - \dfrac{1}{R} v_B - \dfrac{1}{R} v_A = 0$

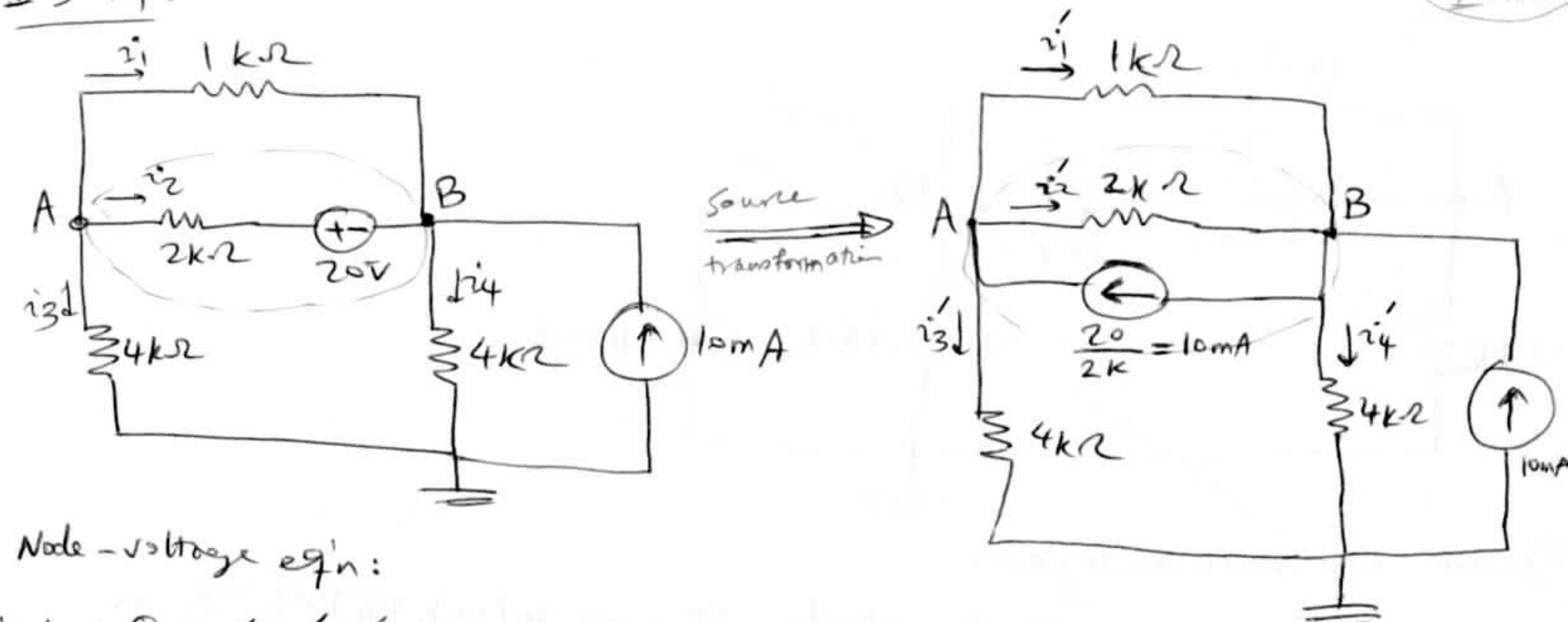
$$\Leftrightarrow 3v_C - v_B - v_A = 0$$

and so, in matrix form:

$$\begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} v_A = v_s/2 \\ v_B = v_s/4 \\ v_C = v_s/4 \end{cases}$$

$$\Rightarrow i_{ss} = \frac{v_s - v_A}{R} = \frac{v_s}{2R} \Rightarrow R_{IN} = \frac{v_s}{i_{ss}} = \frac{v_s}{v_s/2R} \Rightarrow \boxed{R_{IN} = 2R}$$

P 3-19:



Node-voltage eq'n:

kcl at (A): $i_1' + i_2' + i_3' = 10 \Leftrightarrow \frac{v_A - v_B}{1} + \frac{v_A - v_B}{2} + \frac{v_A}{4} = 10 \Leftrightarrow v_A(1+1/2+1/4) - v_B(1+1/2) = 10$

kcl at (B): $i_1' + i_2' = i_4' \Leftrightarrow \frac{v_A - v_B}{1} + \frac{v_A - v_B}{2} = \frac{v_B}{4} \Rightarrow -v_A(1+1/2) + v_B(1+1/2+1/4) = 0$

So, in matrix form:

$$\begin{bmatrix} 1+1/2+1/4 & -(1+1/2) \\ -(1+1/2) & 1+1/2+1/4 \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix} \Rightarrow \begin{cases} \boxed{v_A = 21.54V} \\ \boxed{v_B = 18.46V} \end{cases}$$

Then, we can find the currents:

$$i_1 = \frac{v_A - v_B}{1k} \Rightarrow \boxed{i_1 = 3.08mA}$$

$$i_2 = \frac{v_A - 20}{2k} \Rightarrow \boxed{i_2 = 0.77mA}$$

$$i_3 = \frac{v_A}{4k} \Rightarrow \boxed{i_3 = 5.385mA}$$

$$i_4 = \frac{v_B}{4k} \Rightarrow \boxed{i_4 = 4.6150mA}$$