Hw #7

9-2 $f(t) = A[(3-\alpha t)e^{-\alpha t}]u(t)$

$$F(s) = \frac{3A}{s+\alpha} - \frac{\alpha A}{(s+\alpha)^2} = A\frac{3(s+\alpha)-\alpha}{(s+\alpha)^2} = \frac{A(3s+2\alpha)}{(s+\alpha)^2}\frac{\frac{1}{3}}{\frac{1}{3}}$$

$$= \boxed{\frac{A}{3}\frac{(s+2/3\alpha)}{(s+\alpha)^2}}$$

Zero: $s = -\frac{2}{3}\alpha$

Poles: $s = -\alpha$

9-4

$f(t) = A(\cos(\beta t) - \sin(\beta t))u(t)$

$$F(s) = A\left(\frac{s}{s^2+\beta^2} - \frac{\beta}{s^2+\beta^2}\right) = \boxed{A\left(\frac{s-\beta}{s^2+\beta^2}\right)}$$

Zero: $s = -\beta$

Poles: $s = \pm\beta j$

9.8 a) $f_1(t) = 4\delta(t) + [20e^{-20t} + 40e^{-40t}]u(t)$

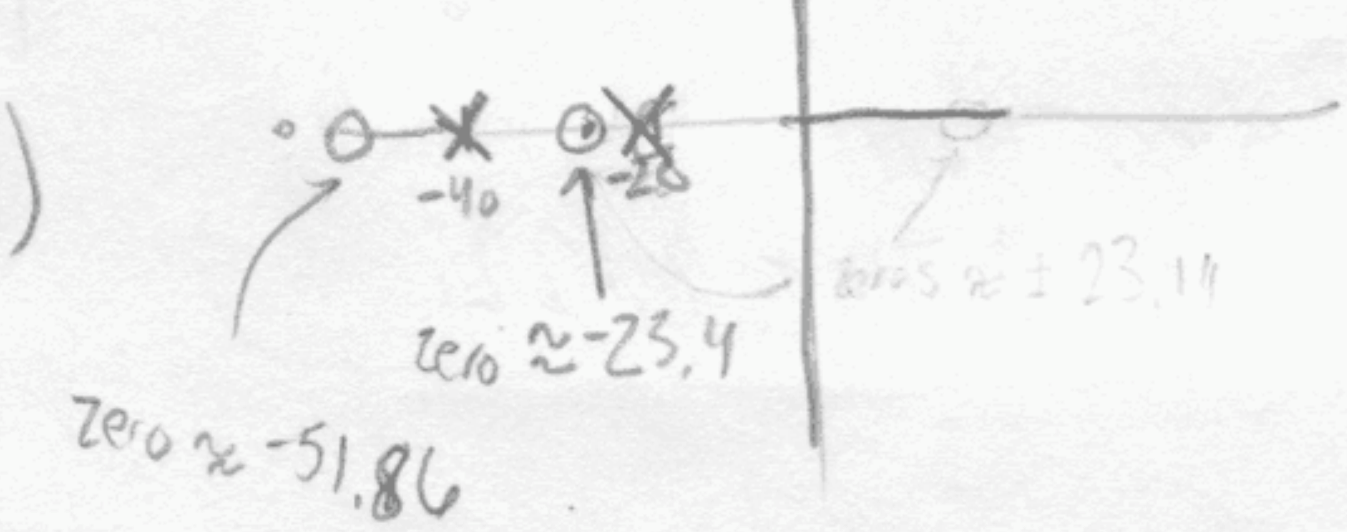
$$F_1(s) = 4 + \frac{20}{s+20} + \frac{40}{s+40}$$

$$= \frac{4(s+20)(s+40) + 20(s+40) + 40(s+20)}{(s+20)(s+40)} = \frac{1\cdot(s^2+75s+1200)}{4(s+20)(s+40)}$$

poles: $s = -20$, $s = -40$

zeros: $s = \frac{5}{2}(\sqrt{33}-15)$

$s = \frac{5}{2}(-15-\sqrt{33})$

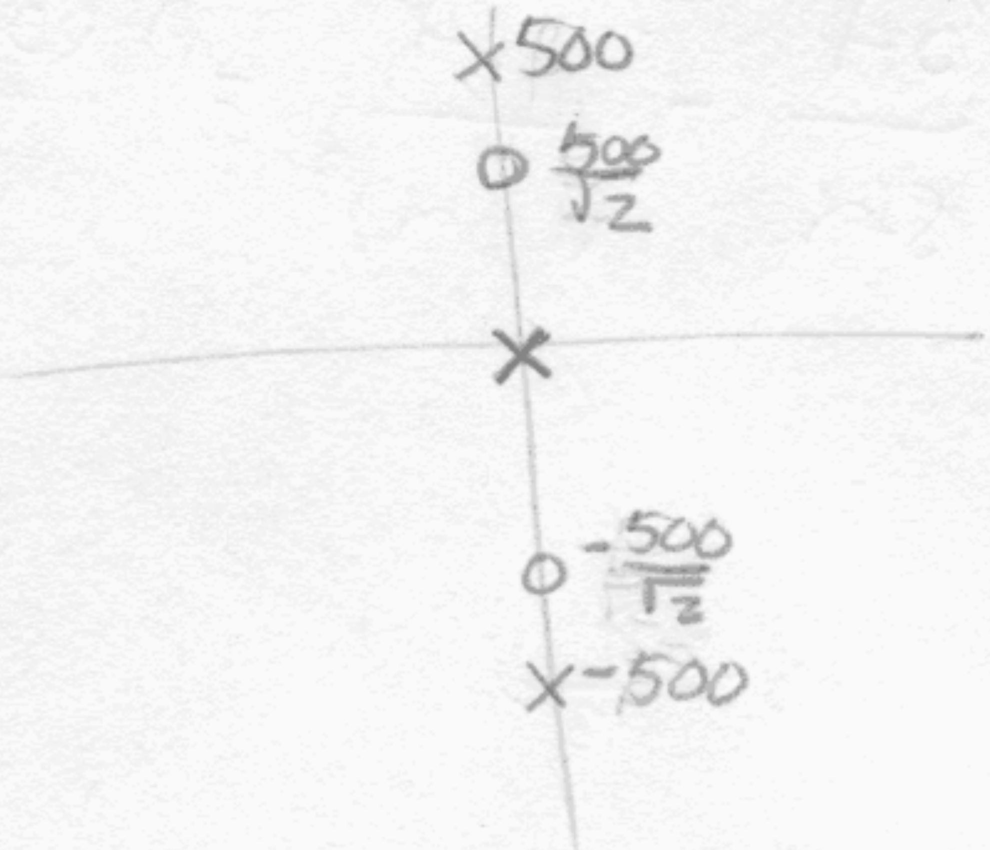


9-8b) $f_2(t) = (15 + 15\cos(500t))u(t)$

$$F_2(s) = 15\left(\frac{1}{s} + \frac{s}{s^2+500^2}\right) = 15\left(\frac{(s^2+500^2) + s^2}{s(s^2+500^2)}\right) = \boxed{30\left(\frac{s^2+\frac{500^2}{2}}{s(s^2+500^2)}\right)}$$

zeros: $\pm\frac{500}{\sqrt{2}}j$

poles: $0, \pm 500j$



9-9 a) $f_1(t) = \delta(t) + (625te^{-25t})u(t)$

$$F_1(s) = 1 + \frac{625}{(s+25)^2} = \frac{(s+25)^2+625}{(s+25)^2} = \boxed{\frac{s^2+50s+1250}{(s+25)^2}}$$

poles: $s=-25, s=-25$

zeros: $s = -25 \pm 25j$

b) $f_2(t) = [10 + e^{-10t} + 5\cos(10t) + 5\sin(10t)]u(t)$

$$F_s(s) = \frac{10}{s} + \frac{1}{s+10} + \frac{5s}{s^2+100} + \frac{5(10)}{s^2+100}$$

$$= \frac{10(s+10)(s^2+100) + s(s^2+100) + (5s+50)(s)(s+10)}{s(s+10)(s^2+100)}$$

$$= \boxed{\frac{8(2s^3+25s^2+200s+1250)}{s(s+10)(s^2+100)}}$$

poles: $s=0, s=-10, s=\pm 10j$

zeros: $s \approx -9.071$

$s \approx -1.714 \pm 8.122j$

9.11 $f(t) = (10 + 2e^{-10t})u(t) + 5\cos(100(t-.05))\,u(t-.05)$

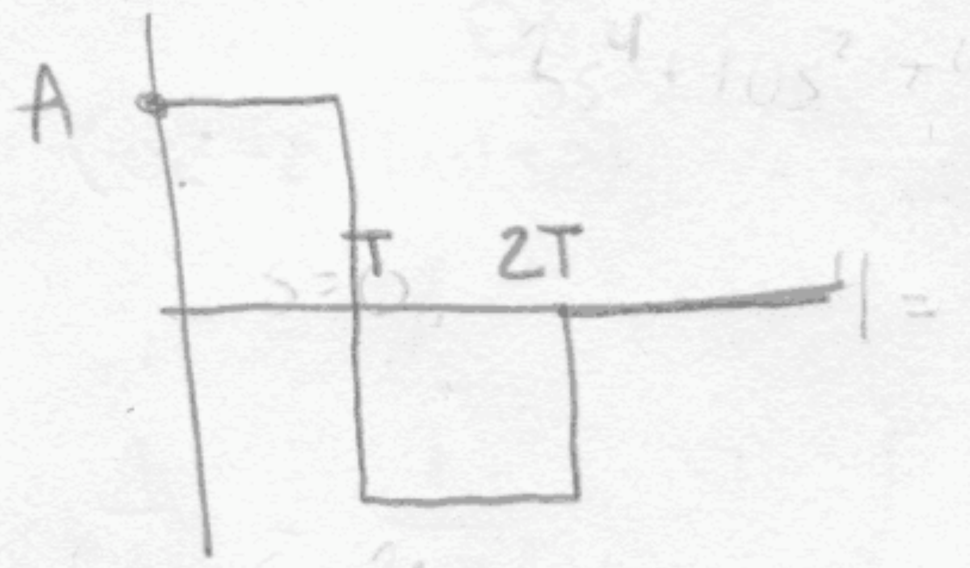
$$\boxed{F(s) = \frac{10}{s} + \frac{2}{s+10} + \frac{5s}{s^2+(100)^2} \circ e^{-.05s}}$$

Here, we used the t-domain translation property that

$$f(t-a)u(t-a) \Longrightarrow e^{-as}F(s)$$

9.14

a)



at $t=0$, there is a step to A

at $t=T$, there is a step of $-2A$

at $t=2T$, there is a step of A

so $\boxed{f(t) = Au(t) - 2A\,u(t-T) + A\,u(t-2T)}$

b) using time domain translation, $u(t-a) \Rightarrow \frac{e^{-as}}{s}$

so $\boxed{F(s) = \frac{A}{s} - \frac{2A}{s}e^{-Ts} + \frac{A}{s}e^{-2Ts}}$

c) $F(s) = \int_{0^-}^{\infty}(Au(t) - 2Au(t-T) + Au(t-2T))e^{-st}dt = A\int_{0^-}^{\infty} e^{-st}dt + \int_{T}^{\infty} e^{-st}dt + \int_{2T}^{\infty} e^{-sT}dt$

because $u(t-T) = 0$, for $t<T$

$\int e^{-st}dt = \frac{-e^{-st}}{s}$

$$F(s) = -\frac{Ae^{-st}}{s}\Big|_{0^-}^{\infty} + \frac{2Ae^{-st}}{s}\Big|_{T}^{\infty} - \frac{Ae^{-st}}{s}\Big|_{2T}^{\infty}$$

for $s = \sigma + j\omega$ with $\sigma > 0$, the upper limits go to zero as $t \to \infty$ $\left(\frac{1}{e^{\sigma t}} \xrightarrow[t\to\infty]{} 0,\ \sigma > 0\right)$

$$F(s) = 0 + \frac{Ae^{-s(0)}}{s} + 0 - \frac{2Ae^{-sT}}{s} + 0 + \frac{Ae^{-s(2T)}}{s}$$

$$\boxed{F(s) = \frac{A}{s} - \frac{2Ae^{-sT}}{s} + \frac{Ae^{-s(2T)}}{s}}$$

This was shown for s with $\sigma > 0$ but can be extended to all $s \neq 0$.

9.16

a) $\frac{s+100}{s(s+50)} = \frac{A}{s} + \frac{B}{s+50}$, $A(s+50) + B(s) = s+100$

(4)

residue:

$\frac{s+100}{s+50} = A + \frac{Bs}{s+50}$, let $s=0$, $A=2$

$\frac{s+100}{s} = \frac{A}{s}(s+50) + B$, let $s=-50$ $B=-1$

$\frac{s+100}{s(s+50)} = \frac{2}{s} - \frac{1}{s+50}$

$$\boxed{f_1(t) = (2-e^{-50t})u(t)}$$

9.16b) $F_2(s) = \dfrac{s+10}{s(s+50)(s+100)} = \dfrac{A}{s} + \dfrac{B}{s+50} + \dfrac{C}{s+100}$

$s+10 = A(s+50)(s+100) + B(s)(s+100) + C(s+50)s$

if $s=0$, $10 = A(50)(100)$, $A = \frac{1}{500}$

if $s=-50$, $+40 = B(+50)(+50)$, $B = \frac{+4}{250} = \frac{+2}{125}$

if $s=-100$, $-90 = C(-50)(-100)$, $C = \frac{-9}{500}$

$$F_2(s) = \frac{\frac{1}{500}}{s} + \frac{\frac{2}{125}}{s+50} + \frac{-\frac{9}{500}}{s+100}$$

$$\boxed{f_2(t) = \left(\frac{1}{500} + \frac{2}{125}e^{-50t} - \frac{9}{500}e^{-100t}\right)u(t)}$$

9.19a) $F_1(s) = \frac{\beta(s+\beta)}{s(s^2+\beta^2)} = \frac{\beta(s+\beta)}{s(s+\beta j)(s-\beta j)} = \frac{A}{s} + \frac{B}{s+\beta j} + \frac{B^*}{s-\beta j}$ ⑥

B and B^* are complex conjugates.

residue method: $\frac{\beta(s+\beta)}{(s+\beta j)(s-\beta j)} = A + \frac{B(s)}{s+\beta j} + \frac{B^*(s)}{s-\beta j}$

set $s=0$ $\frac{\beta^2}{(\beta j)(-\beta j)} = A = 1$

$\frac{\beta(s+\beta)}{s(s-\beta j)} = \frac{A(s+\beta j)}{s} + B + \frac{B^*(s+\beta j)}{(s-\beta j)}$

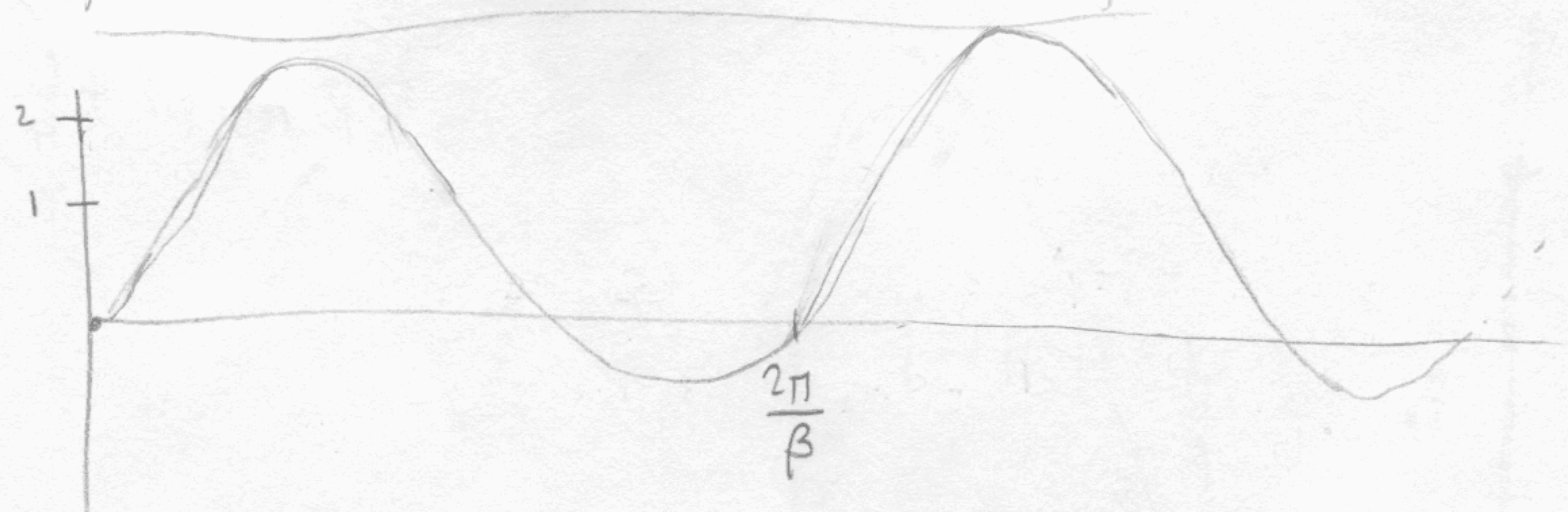
set $s=-\beta j$ $\frac{\cancel{\beta^2}(-j+1)}{-\cancel{\beta^2} j(-2j)} = B = -\frac{1}{2} + \frac{1}{2}j$ $\quad \frac{-\frac{1}{2}+\frac{1}{2}j}{s+\beta j} + \frac{-\frac{1}{2}-\frac{1}{2}j}{s-\beta j} = \frac{-s+\beta}{s^2+\beta^2}$

$B^* = -\frac{1}{2} - \frac{1}{2}j$

$F_1(s) = \frac{1}{s} + \frac{-s+\beta}{s^2+\beta^2} = \frac{1}{s} + \frac{(-1)s}{s^2+\beta^2} + \frac{\beta}{s^2+\beta^2}$

$$\boxed{f_1(t) = \left(1 - \cos(\beta t) + \sin(\beta t)\right)u(t)}$$

b)



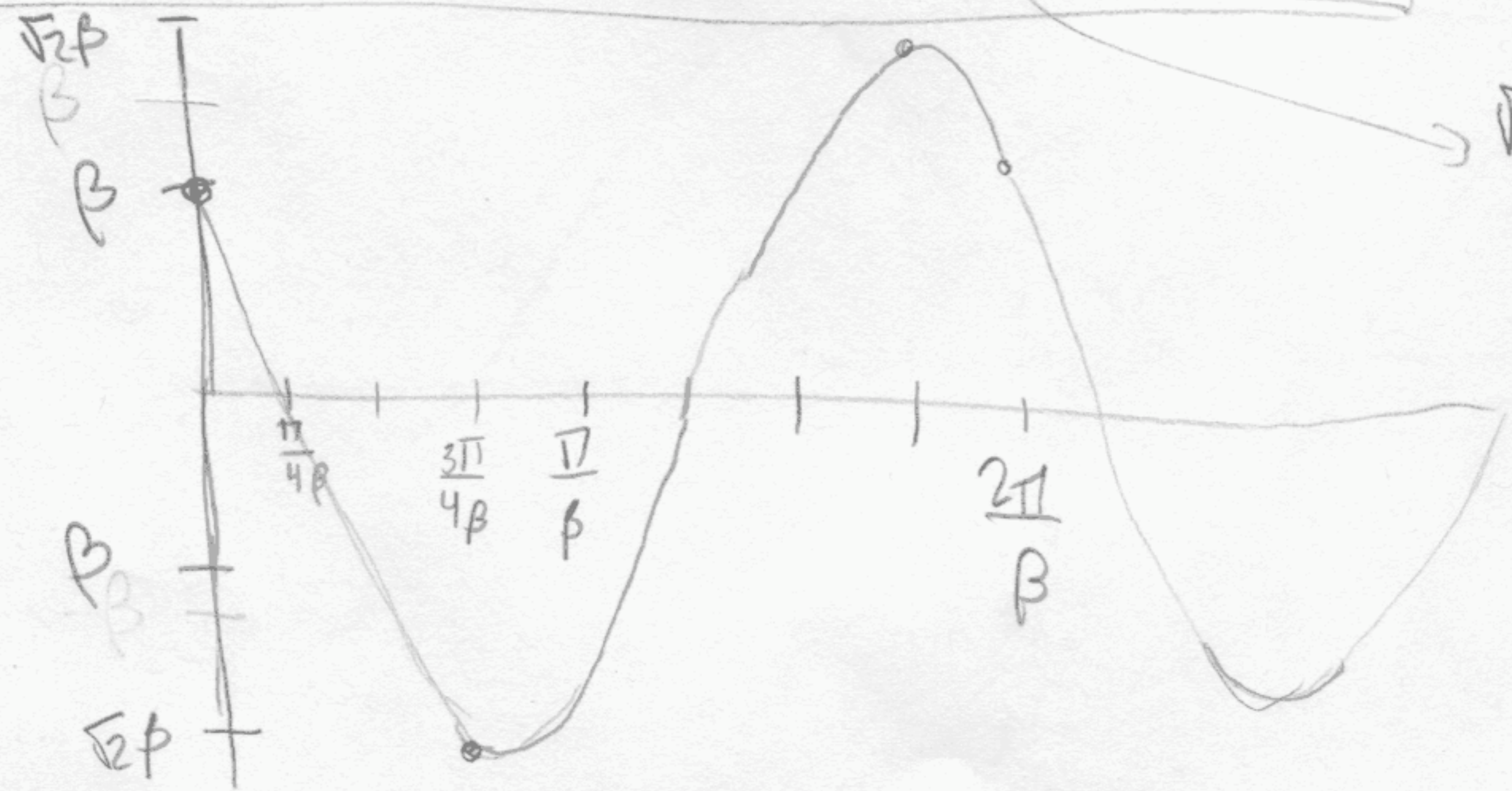
b) $F_2(s) = \dfrac{s(s+\beta)}{s^2+\beta^2}$

$$\begin{array}{r} 1 \\ s^2+\beta^2 \overline{\big)\, s^2 + s\beta} \\ -\ \underline{s^2 + \beta^2} \\ 0 + s\beta - \beta^2 \end{array}$$

$$= 1 + \frac{\beta(s)}{s^2+\beta^2} - \frac{\beta^2}{s^2+\beta^2}$$

$$\boxed{F_2(t) = \delta(t) + \beta\big(\cos(\beta t) - \sin(\beta t)\big)u(t)}$$

$\sqrt{2}\beta \cos\left(\beta t + \frac{\pi}{4}\right)$



9-23 a) $F(s) = \dfrac{(s+4000)(s+8000)}{s(s+2000)(s+6000)} = \dfrac{A}{s} + \dfrac{B}{s+2000} + \dfrac{C}{s+6000}$

$(s+4000)(s+8000) = A(s+2000)(s+6000) + B(s)(s+6000) + C(s)(s+2000)$

$s=0, \quad (4000)(8000) = A(2000)(6000)$

$A = \frac{8}{3}$

$s=-2000, \quad (2000)(6000) = B(-2000)(4000)$

$B = -\frac{3}{2}$

$s=-6000, \quad (-2000)(2000) = C(-6000)(-4000)$

$C = \frac{-1}{6}$

$$\boxed{f(t) = \left(\frac{8}{3} - \frac{3}{2}e^{-2000t} - \frac{1}{6}e^{-6000t}\right)u(t)}$$

b) $F(s) = \dfrac{3s^4 + 10s^2 + 4}{s(s^2+1)(s^2+4)} = \dfrac{A}{s} + \dfrac{Bs+C}{s^2+1} + \dfrac{Ds+E}{s^2+4}$

$3s^4 + 10s^2 + 4 = A(s^2+1)(s^2+4) + (Bs+C)(s)(s^2+4) + (Ds+E)(s)(s^2+1)$

$s=0, \quad 4 = A(1)(4) \qquad A = 1$

$s=i, \quad 3 + 10(-1) + 4 = (Bi + C)(i)(3)$

$-3 = (-B + Ci)(3) \qquad B = 1, \; C = 0$

$s=2i, \quad 3(16) - 10(4) + 4 = (D(2i) + E)(2i)(-3)$

$12 = 12D - 6Ei, \quad D = 1, \; E = 0$

$F(s) = \frac{1}{s} + \frac{s}{s^2+1} + \frac{s}{s^2+4}$

$$\boxed{F(t) = u(t)\left(1 + \cos(t) + \cos(2t)\right)}$$

9-28 a)

$$F_1(s) = \frac{e^{-3s}\cdot(s+20)}{(s+10)(s+30)}$$

by partial fractions

$$= e^{-3s}\left(\frac{\frac{1}{2}}{s+30} + \frac{\frac{1}{2}}{s+10}\right)$$

using t-domain translation
$e^{-as}F(s) \Rightarrow f(t-a)u(t-a)$

$$\boxed{f_1(t) = \frac{1}{2}\left(e^{-30(t-3)} + e^{-10(t-3)}\right)u(t-3)}$$

b)

$$F_2(s) = \frac{se^{-3s}+20}{(s+10)(s+30)} = \frac{e^{-3s}(s)}{(s+10)(s+30)} + \frac{20}{(s+10)(s+30)}$$

For this one, we use t-domain translation.

$$e^{-3s}\left[\frac{3/2}{s+30} - \frac{1/2}{s+10}\right] \qquad \frac{1}{s+10} - \frac{1}{s+30}$$

$$\boxed{f_2(t) = \left(e^{-10t} - e^{-30t}\right)u(t) + \left(\frac{3}{2}e^{-30(t-3)} - \frac{1}{2}e^{-10(t-3)}\right)u(t-3)}$$