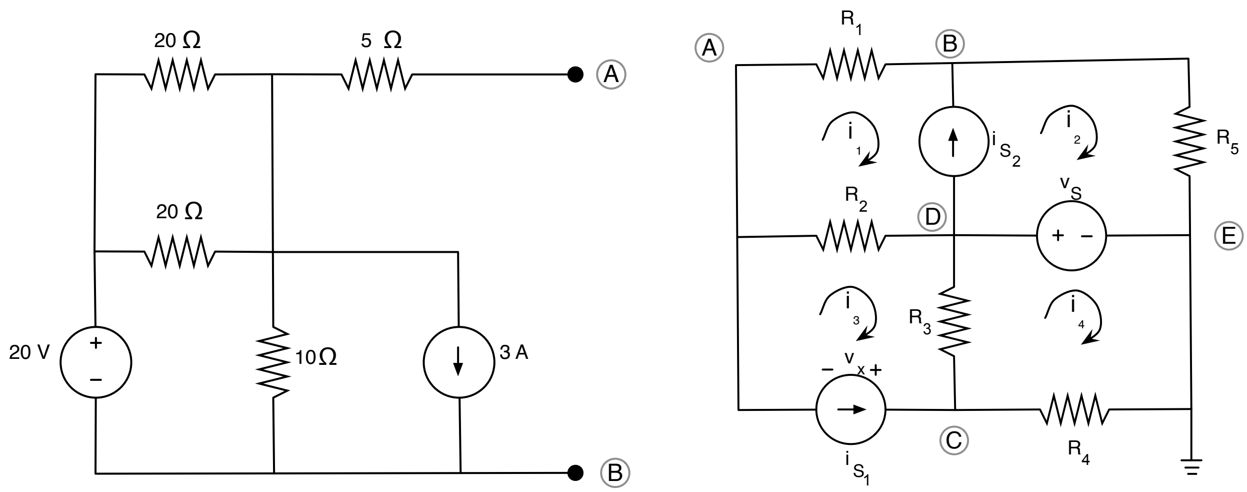
MAE140 - Linear Circuits - Fall 13
Midterm, October 31

Instructions

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 70 minutes

(iii) Do not forget to write your **name**, **student number**, and **instructor**



(a) Question 1.

(b) Questions 2-4.

Figure 1: Circuits for questions 1-4

1. **Equivalent circuits**

Part I: [2 points] Turn off all the sources in the circuit of Figure 1(a) and find the equivalent resistance as seen from terminals A and B.

Part II: [3 points] Find the Thévenin equivalent as seen from terminals A and B.
Hint: If you want, you can use the result obtained in Part I. When you are deciding what source transformation to use, be careful not to lose track of terminals A and B!

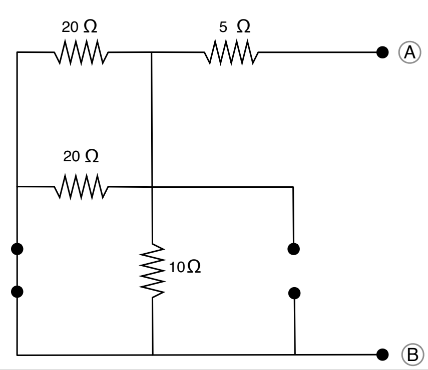
Part III: [1 point] Show that the power absorbed by a $40\,\Omega$ resistor that is connected to terminals A and B is $0.4W$.

Solution:

Part I: We start by switching off the sources.

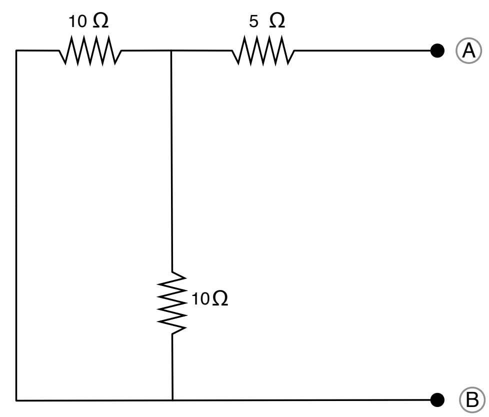
We substitute the voltage source by a short circuit, and the current source by an open circuit. Then, we get the circuit on the right

(+ .5 point)



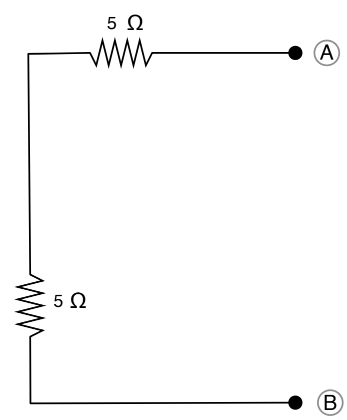
Next, we combine the two resistances in parallel in the upper left corner to get the circuit

(+ .5 point)



Next, we combine the two remaining resistances that are still in parallel to get the circuit

(+ .5 point)



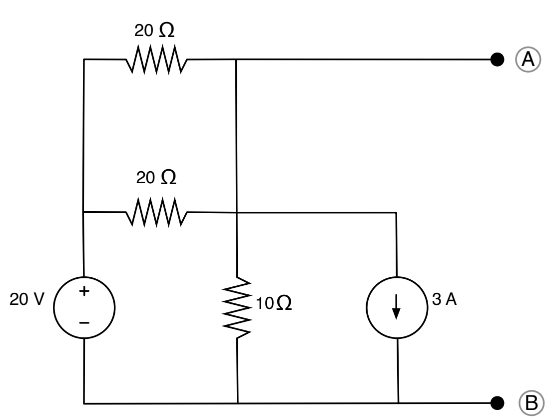
The final equivalent resistance is given by summing the two resistances in series

$$R_{\text{final}} = 5 + 5 = 10\Omega$$ **(+ .5 point)**

Part II: From Part I, we can say that $R_T = 10\Omega$. The Thévenin voltage v_T is equal to the open circuit voltage seen from terminals A-B.

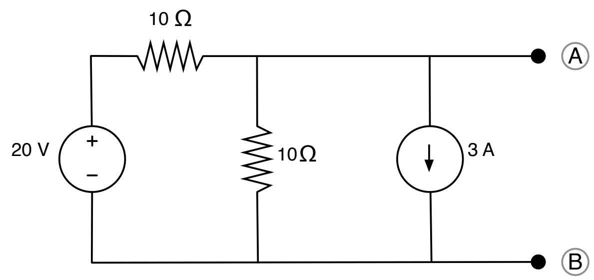
(+ .5 point)

To find the open-circuit voltage, we first note that, since the 5Ω resistor does not see any current, its voltage drop is zero, and hence it can be eliminated.



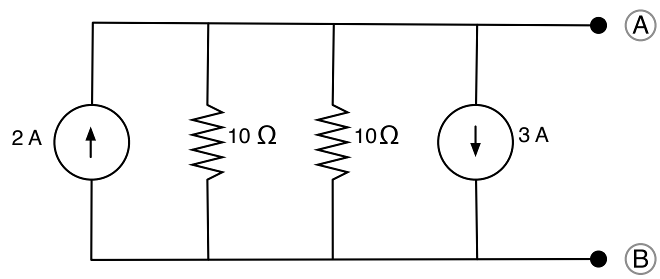
(+ .5 point)

Combining the two resistors in parallel, we obtain the equivalent circuit:



(+ .5 point)

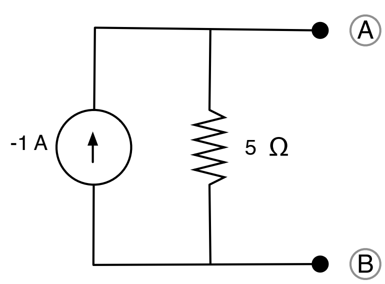
Now we transform the voltage source in series with the resistor to obtain



(+ .5 point)

(Note that, if one instead transforms the current source in parallel with a resistor into a voltage source of $-30V$ in series with a resistor, then one loses track of the terminals A-B. This would most likely lead you to think that the voltage drop is $-30V$, which is incorrect – because the nodes connected to this voltage source are not A,B anymore.)

Combining all the elements, we arrive at



(+ .5 point)

The voltage drop that the 5Ω resistor sees is then $-5V$, which is precisely the open-circuit voltage from terminals A-B, i.e., $v_T = -5V$.

(+ .5 point)

Part III: We compute the voltage across the 40Ω resistor with the Thévenin equivalent as:

$$v = \frac{40}{40 + R_T} v_T = \frac{40}{50}(-5) = -4V$$ **(+ .5 point)**

Then, the power absorbed by a 40Ω resistor is $p = \frac{v^2}{40} = \frac{16}{40} = 0.4W$. **(+ .5 point)**

2. **Node voltage analysis**

[6 points] Formulate node-voltage equations for the circuit in Figure 1(b). Use the node labels A through E provided in the figure and clearly indicate how you handle the presence of a voltage source. The final equations must depend only on unknown node voltages and the source values v_S, i_{S_1}, and i_{S_2}. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: There are five nodes in this circuit and the ground node (E) is directly connected to the voltage source. Therefore, this helps us taking care of it by setting $v_D = v_S$ (this is method 2).

(+ 1.5 point)

We need to derive equations for the other three unknown node voltages v_A, v_B, and v_C. We do this using KCL and write equations by inspection. The matrix is 3×4 and the independent vector has 3 components.

(+ .5 point),

We write,

$$\begin{pmatrix} G_1 + G_2 & -G_1 & 0 & -G_2 \\ -G_1 & G_1 + G_5 & 0 & 0 \\ 0 & 0 & G_3 + G_4 & -G_3 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} -i_{S_1} \\ i_{S_2} \\ i_{S_1} \end{pmatrix} \quad \textbf{(+ 3 points)}$$

where, as we do usually, $G_i = 1/R_i$. Since we know that $v_D = v_S$, we can rewrite the equations above as

$$\begin{pmatrix} G_1 + G_2 & -G_1 & 0 \\ -G_1 & G_1 + G_5 & 0 \\ 0 & 0 & G_3 + G_4 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} -i_{S_1} + G_2 v_S \\ i_{S_2} \\ i_{S_1} + G_3 v_S \end{pmatrix} \quad \textbf{(+ 1 point)}$$

3. **Mesh current analysis**

[6 points] Formulate mesh-current equations for the circuit in Figure 1(b). Use the mesh currents shown in the figure and clearly indicate how you handle the presence of each of the current sources. The final equations should only depend on the unknown mesh currents and the source values v_S, i_{S_1}, and i_{S_2}. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: There are four meshes in this circuit. We can easily take care of the current source i_{S_1} by realizing that it only belongs to one mesh, and therefore $i_3 = -i_{S_1}$.

(+ 1 point)

The other current source, i_{S_2}, belongs to two meshes and is not in parallel with any resistor, so we need to use a supermesh (combining meshes 1 and 2) to deal with it.

(+ 1 point)

The current source imposes the constraint

$$i_2 - i_1 = i_{S_2}. \quad \textbf{(+ 1 point)}$$

KVL for the supermesh reads like

$$R_1 i_1 + R_5 i_2 - v_S + R_2(i_1 - i_3) = 0, \quad \textbf{(+ 1 point)}$$

The remaining equation comes from KVL for mesh 4,

$$v_S + R_4 i_4 + R_3(i_4 - i_3) = 0 \quad \textbf{(+ 1 point)}$$

Finally, using that $i_3 = -i_{S_1}$, the equations can be rewritten as:

$$\begin{aligned} i_2 - i_1 &= i_{S_2} \\ (R_1 + R_2)i_1 + R_5 i_2 &= v_S - R_2 i_{S_1} \\ (R_3 + R_4)i_4 &= -v_S - R_3 i_{S_1} \end{aligned}$$

(+ 1 point)

4. **Bonus question**

[1 point] Express v_x in the circuit of Figure 1(b) in terms of the node voltage variables of Question 2 and also in terms of the mesh current variables of Question 3.

Solution: In terms of the node voltage variables, we have

$$v_x = v_C - v_A$$

(+ .5 point)

and in terms of the mesh current variables, we realize that the voltage drop that the current source sees is equal to the sum of the voltage drops of the resistors R_3 and R_2,

$$v_x = R_3(i_4 - i_3) + R_2(i_1 - i_3)$$

(+ .5 point)