Instructions

(i) The exam is open book. You may use your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 180 minutes

(iii) Do not forget to write your **name** and **student number**

(iv) On the questions for which the answers are given, please provide detailed derivations

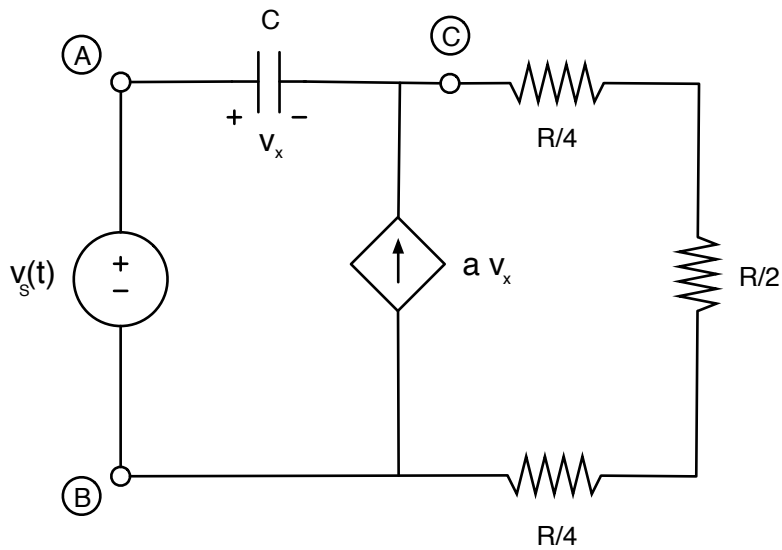(v) The exam has 5 questions for a total of 50 points and 3 bonus points

Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

   Part I: [2 points] Assuming zero initial conditions, transform the circuit in Figure 1 into the s-domain.

   Part II: [4 points] Find the impedance equivalent in the circuit obtained in Part I as seen from terminals Ⓐ and Ⓑ. The answer should be given as a ratio of two polynomials.

   Part III: [4 points] Find the s-domain Thévenin equivalent of the circuit obtained in Part I as seen from terminals Ⓒ and Ⓑ.
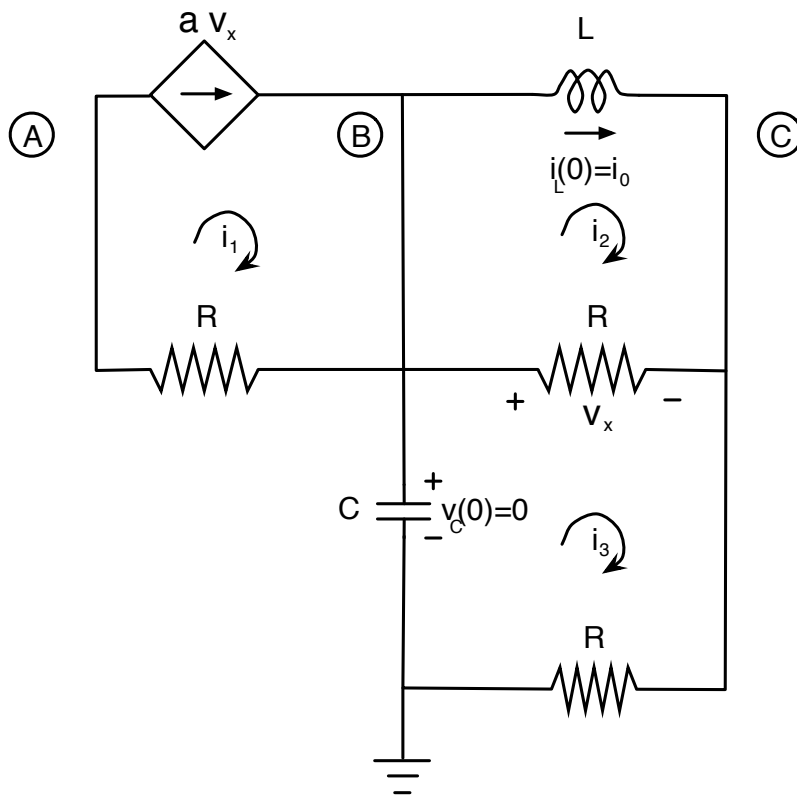
Figure 2: Nodal and Mesh Analysis Circuit

2. **Nodal and Mesh Analysis**

   Part I: [5 points] Formulate node-voltage equations in the s-domain for the circuit in Figure 2. Use the reference node and other labels as shown in the figure. Use the initial conditions indicated in the figure and transform them into current sources. Explain how you deal with the presence of the dependent source. Make sure your final answer has the same number of independent equations as unknown variables. No need to solve any equations!

   Part II: [5 points] Formulate mesh-current equations in the s-domain for the circuit in Figure 2. Use the currents shown in the figure. Use the initial conditions indicated in the figure and transform them into voltage sources. Explain how you deal with the presence of the dependent source. Make sure your final answer has the same number of independent equations as unknown variables. No need to solve any equations!

   Part III: [1 bonus point] Express the transform $I_L(s)$ of the inductor current in terms of your unknown variables of Part I and also in terms of your unknown variables of Part II.
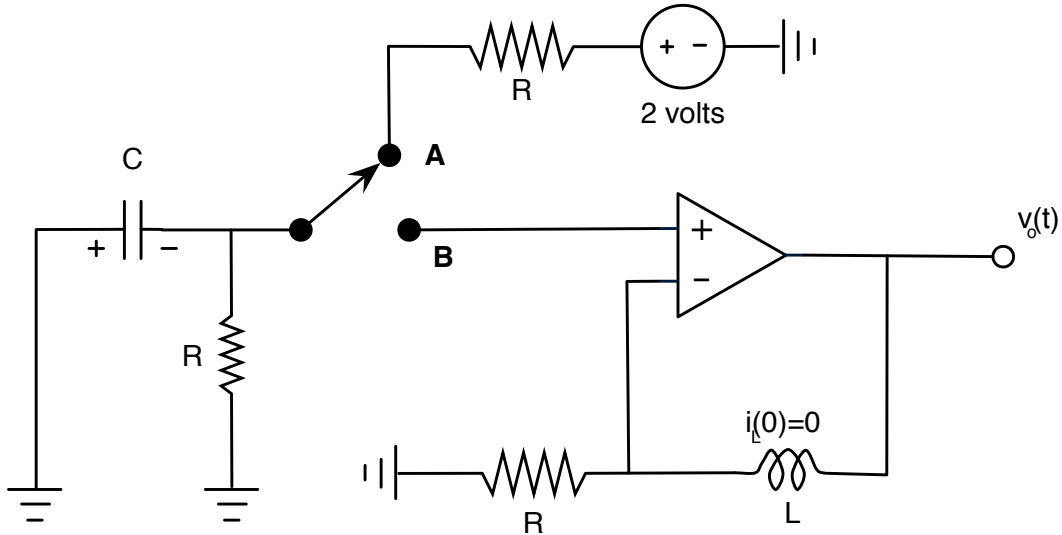
Figure 3: RCL circuit for Laplace Analysis

3. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 3. The voltage source is constant. The switch is kept in position **A** for a very long time. At $t = 0$ it is moved to position **B**. Show that the initial capacitor voltage is given by

$$v_C(0^-) = -1V.$$

[Show your work]

Part II: [4 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the capacitor in which the initial condition appears as a voltage source. Find the transfer function of the circuit.

[Show your work]

Part III: [4 points] Use domain circuit analysis and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $C = 1000\,\mu F$, $L = 1000\,\mu H$, and $R = 1\,k\Omega$ is
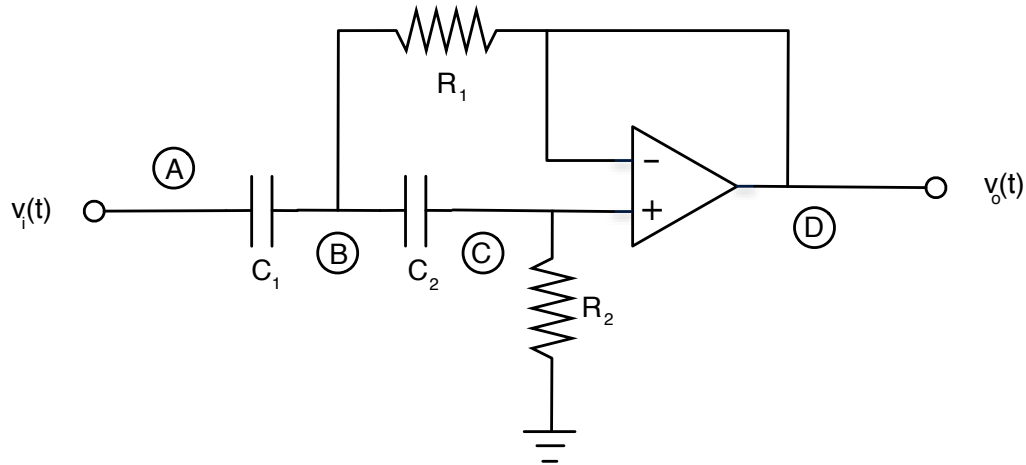
$$v_o(t) = 10^{-6}\delta(t) + (1 - 10^{-6})e^{-t}u(t).$$

Figure 4: Frequency Response Analysis.

4. **Frequency Response Analysis**

   Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 4 into the s-domain.

   Part II: [3 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ is given by

   $$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{R_1 R_2 C_1 C_2 s^2}{R_1 R_2 C_1 C_2 s^2 + R_1(C_1 + C_2)s + 1}.$$

   [Show your work]
   Hint: use node voltage analysis

   Part III [4 points] Let $R_1 = R_2 = 10$ kΩ, $C_1 = C_2 = 220$ nF. Compute the gain and phase functions of $T(s)$. What are the DC gain and the ∞-freq gain? What is the cut-off frequency ω_c? Use these values to sketch the magnitude of the frequency response of the circuit. Is this circuit a low-pass, high-pass, or band-pass filter?
   [Explain your answer]

   Part IV [2 points] Using what you know about frequency response, compute the steady state response $v_o^{SS}(t)$ of this circuit when $v_i(t) = 2\cos(706.26t - \frac{\pi}{2})$ using the same values of R_1, R_2, C_1, and C_2 as in Part III.
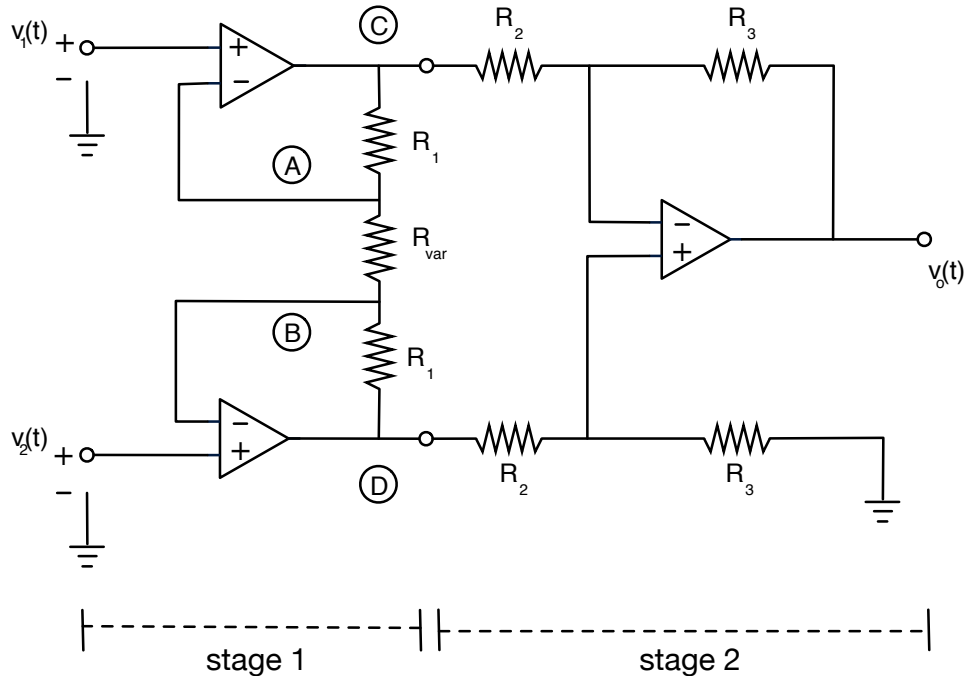
Figure 5: Circuit for Question 5.

5. **Op-Amps and Loading**

Consider the circuit in Figure 5

Part I: [3 points] Considering stage 1 and stage 2 separately, use nodal analysis to obtain the output voltages v_C and v_D of the Op-amps in stage 1.

Part II: [2 points] Do you recognize stage 2 as any of the basic op-amp circuits? If so, specify which one and provide an explicit expression for the output voltage v_o.

Part III: [2 points] Consider now the connection of stages 1 and 2 depicted in Figure 5. Is there loading? Justify your answer.

Part IV: [1 point] For the circuit in Figure 5, show that the output voltage v_o as a function of the input voltages v_1 and v_2 is

$$v_o(t) = \frac{R_3}{R_2} \left(1 + \frac{2R_1}{R_{var}} \right) (v_2(t) - v_1(t))$$

[Justify your work]

[Hint: use your answer to Part III]

Part V: [2 points] Consider the case when R_{var} is removed and substituted by an open circuit. How would you describe stage 1 then? What would the expression for the output voltage v_o in this case?

Part VI: [2 bonus points] The circuit is called *instrumentation amplifier*. To compute the difference between the voltages produced by two different instruments, it would seem that simply using stage 2 would do. Do you agree with this? What do you think the role of stage 1 is? Finally, can you explain what the role of R_{var} is in this design?