

MAE140 - Linear Circuits - Fall 14
Midterm, November 6

Instructions

- (i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities
- (ii) You have 80 minutes
- (iii) Do not forget to write your **name** and **student number**

Good luck!

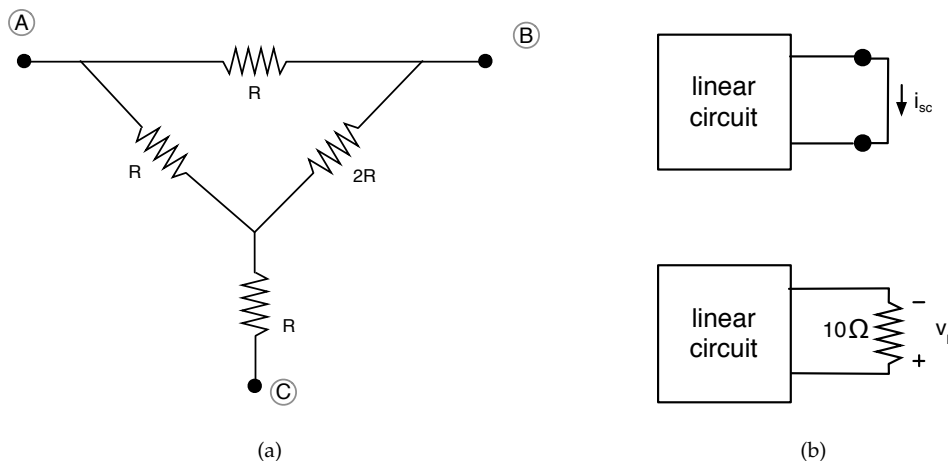


Figure 1: Circuits for question 1.

1. Equivalent circuits

Part I: [5 points] For the circuit in Figure 1(a), find the equivalence resistances as seen from terminals (A) and (B), (A) and (C), and (B) and (C).

Part II: [5 points] A TA performed two experiments on a linear circuit composed only of independent sources and resistors. As indicated in Figure 1(b), the short-circuit current $i_{sc} = 10mA$ was measured and, after connecting a 10Ω resistor load, the voltage drop $v_R = -75mV$ was observed. Use this information to construct the Norton and Thévenin equivalent circuits of the linear circuit.

Solution:

Part I: From (A) and (B), the resistor at the bottom plays no role (+ 1 point)
 and the equivalence resistance is

$$R \parallel (R + 2R) = R \parallel 3R = \frac{3}{4}R \quad (+ 1 \text{ point})$$

From (A) and (C), the equivalence resistance is

$$R + R \parallel (R + 2R) = R + R \parallel 3R = \frac{7}{4}R \quad (+ 1.5 \text{ point})$$

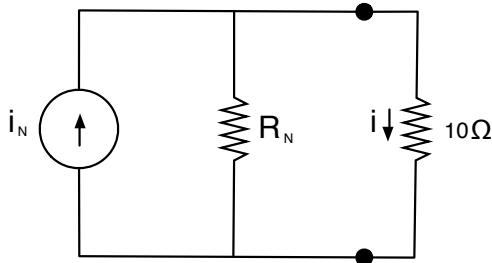
From (B) and (C), the equivalence resistance is

$$R + 2R \parallel (R + R) = R + 2R \parallel 2R = R + R = 2R \quad (+ 1.5 \text{ point})$$

Part II: According to the statement, the circuit acting as source is composed of linear components, so we are ok using Norton and Thévenin equivalence. We know the Norton current source i_N has value equal to the short-circuit current, therefore

$$i_N = 10mA \quad (+ 1 \text{ point})$$

On the other hand, if R_N is the value of the Norton resistance, when we connect the resistor load of 10Ω ,



the latter gets the current

$$i = \frac{1/10}{1/10 + 1/R_N} i_N \quad (+ 1 \text{ point})$$

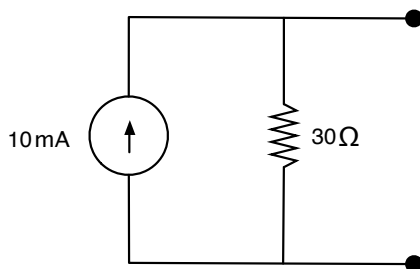
and therefore,

$$v_R = -10i = -10 \frac{1/10}{1/10 + 1/R_N} i_N \quad (+ 1 \text{ point})$$

Using $v_R = -75 \cdot 10^{-3}V$ and $i_N = 10^{-2}A$, we get $R_N = 30\Omega$.

(+ 1 point)

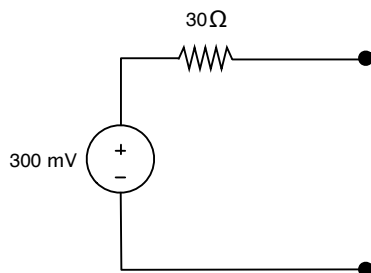
The Norton equivalent circuit looks like



Regarding Thévenin, we get

$$R_T = R_N = 30\Omega \quad V_T = R_T i_{SC} = 30 \cdot 10^{-2} = 300mV \quad (+ 1 \text{ point})$$

and the equivalent circuit looks like



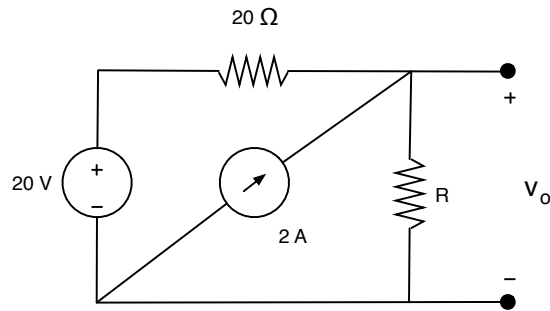


Figure 2: Circuit for question 2.

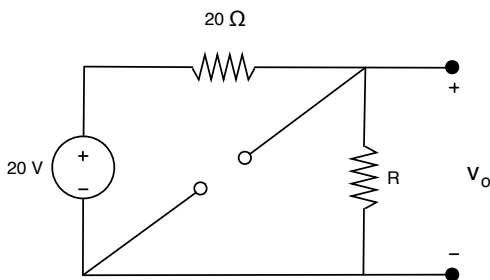
2. (10 points) **Superposition**

For the circuit in Figure 2, use only superposition, voltage division, and current division to determine the value of the resistance R if the measured output voltage is $v_o = 10V$.

Solution: By superposition, we know that the total output voltage is the sum of the output voltages when (i) we turn off the current source and consider only the voltage source, and (ii) we turn off the voltage source and consider only the current source,

$$v_o = v_{O,(i)} + v_{O,(ii)} \quad (+ 3 \text{ points})$$

In case (i), turning off the current source, we substitute it by an open circuit, and this yields,

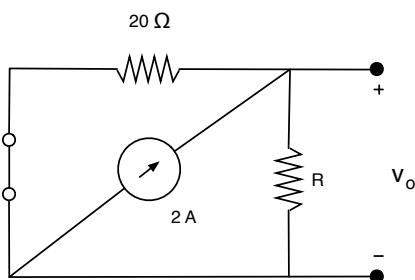


(+ 1 point)

Therefore, using voltage division, we deduce

$$v_{O,(i)} = \frac{R}{R + 20} 20 \quad (+ 1 \text{ point})$$

In case (ii), turning off the voltage source, we substitute it by a closed circuit, and this yields,



(+ 1 point)

Therefore, using current division, we deduce

$$v_{O,(ii)} = R \frac{1/R}{1/R + 1/20} 2 \quad (+ 2 \text{ points: } 1 \text{ point for correct current, } 1 \text{ point for correct voltage})$$

Consequently, we have

$$10 = v_O = v_{O,(i)} + v_{O,(ii)} = \frac{20R}{R + 20} + \frac{2}{1/R + 1/20} = \frac{20R}{R + 20} + \frac{40R}{R + 20},$$

from which we deduce $R = 4\Omega$.

(+ 2 points)

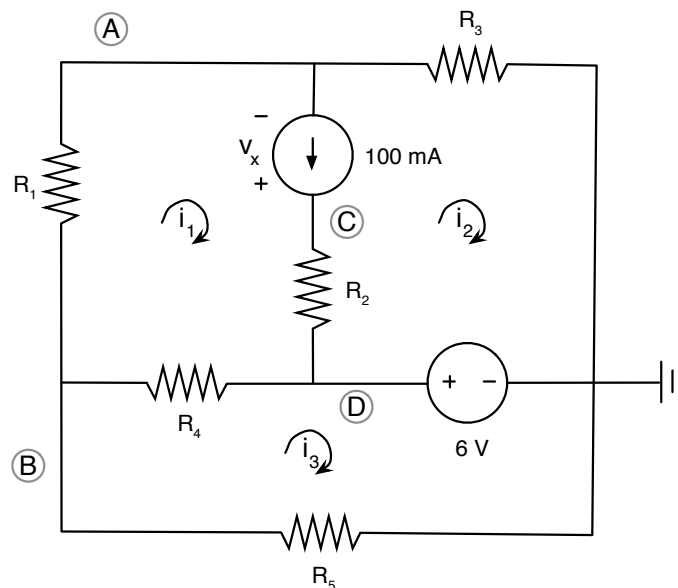


Figure 3: Circuit for question 3.

3. Node voltage and mesh current analysis

Part I: [5 points] Formulate node-voltage equations for the circuit in Figure 3. Use the node labels (A) through (D) provided in the figure and clearly indicate how you handle the presence of a voltage source. The final equations must depend only on unknown node voltages and the resistor values R_1 through R_5 . **Do not modify the circuit or the labels.** No need to solve any equations!

Part II: [5 points] Formulate mesh-current equations for the circuit in Figure 3. Use the mesh currents shown in the figure and clearly indicate how you handle the presence of the current source. The final equations should only depend on the unknown mesh currents and the resistor values R_1 through R_5 . **Do not modify the circuit or the labels.** No need to solve any equations!

Part III: [2 bonus points] Would removing the resistor R_2 have any effect on the currents of the circuit? Would it have any effect on the voltage drop from node D to node A? How about on the voltage drop v_x seen by the independent current source? Justify your answers.

Solution:

Part I: For nodal analysis, the presence of a voltage source is a problem that we need to deal with. Fortunately, ground has been chosen properly, and we can simply say that

$$v_D = 6 \quad (+ 1 \text{ point})$$

Therefore, we only need to write 3 KCL equations, for nodes **(A)**, **(B)**, and **(C)**. We do this by inspection as follows,

$$\begin{pmatrix} G_1 + G_3 & -G_1 & 0 & 0 \\ -G_1 & G_1 + G_4 + G_5 & 0 & -G_4 \\ 0 & 0 & G_2 & -G_2 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} -0.1 \\ 0 \\ 0.1 \end{pmatrix} \quad (+ 3 \text{ points})$$

For convenience, we have used the short-hand notation $G_i = 1/R_i$. These are three equations in three unknowns v_A, v_B, v_C (since we already know $v_D = 6$). **(+ 1 point)**

NOTE TO GRADERS: Students have to clearly indicate their node-voltage equations and the unknown voltages they need to solve for to get the final point. Any reasonable set of equations is fine.

Part II: For mesh analysis, the presence of a current source is a problem that we need to deal with. The current source is not in parallel with any resistor (so we cannot use source transformation) and it belongs to two meshes, so we definitely need to combine meshes 1 and 2 into a supermesh. **(+ 1 point)**

The equation defining the supermesh is then

$$i_1 - i_2 = 0.1 \quad (+ 1 \text{ point})$$

KVL for the supermesh is written as

$$R_3 i_2 - 6 + R_4(i_1 - i_3) + R_1 i_1 = 0 \quad (+ 1 \text{ point})$$

Finally, KVL for mesh 3 is written as

$$6 + R_5 i_3 + R_4(i_3 - i_1) = 0 \quad (+ 1 \text{ point})$$

These are three equations in three unknowns i_1, i_2, i_3 , so we're good. **(+ 1 point)**

In matrix form, this would look like

$$\begin{pmatrix} 1 & -1 & 0 \\ R_1 + R_4 & R_3 & -R_4 \\ -R_4 & 0 & R_4 + R_5 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 0.1 \\ 6 \\ -6 \end{pmatrix}$$

NOTE TO GRADERS: Students have to clearly indicate their mesh-current equations and the unknown currents they need to solve for to get the final point. Any reasonable set of equations is fine.

Part III: Removing the resistor R_2 from the circuit would have no effect in the currents (in fact, one can easily see this by realizing that R_2 does not appear in the mesh current equations in Part II). **(+ 1 bonus point)**

NOTE TO GRADERS: Alternative explanations to get this point and the next are also valid, like for instance the fact that we have discussed in class how a current source in series with a resistor is equivalent, for the rest of the circuit, to just the current source.

Removing the resistor would not have any effect either in the voltage drop from node D to node A. This can be seen by noticing that

$$v_D - v_A = R_4(i_1 - i_3) + R_1 i_1, \quad (+ 1 \text{ bonus point})$$

and this expression does not change with or without the presence of R_2 .

Solution:

However, removing the resistor has an effect in the voltage drop seen by the current source. With R_2 present, the voltage drop that the current source sees is

$$v_x = v_C - v_A = R_2(i_1 - i_2) + R_4(i_1 - i_3) + R_1 i_1 \quad (+ .5 \text{ bonus point})$$

Without R_2 present, the voltage drop that the current source sees is

$$v_x = v_D - v_A = R_4(i_1 - i_3) + R_1 i_1 \quad (+ .5 \text{ bonus point})$$