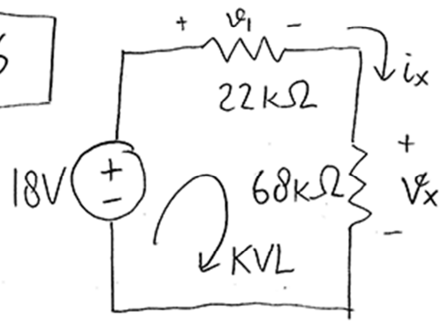
MAE 140 HW 2 Solution

2-26



To find i_x and v_x, use the KVL to get the equation

$$-18V + v_1 + v_x = 0$$

$$-18V + i_x(22k\Omega) + i_x(68k\Omega) = 0$$

$$(22k\Omega + 68k\Omega)\, i_x = 18V$$

$$\boxed{i_x = \frac{18V}{90k\Omega} = 20mA}$$

$$\boxed{v_x = 20mA \cdot 68k\Omega = 13.6V}$$

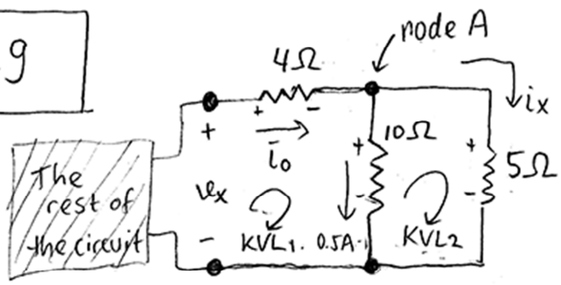
Can also use the voltage divider equation to get v_x

$$\hookrightarrow v_x = \left(\frac{68k\Omega}{22k\Omega + 68k\Omega}\right) 18V = 13.6V$$

Can also use resistor in series equation to get i_x

$$\hookrightarrow i_x = \frac{18V}{R_{equivalent}} = \frac{18V}{22k\Omega + 68k\Omega} = 20mA$$

2-29



To find the v_x and i_x, use 2 KVL loops and one KCL at node A to get 3 equations:

$$KVL_1 \rightarrow -v_x + i_o(4\Omega) + (0.5A)(10\Omega) = 0$$

$$KVL_2 \rightarrow (-0.5A)(10\Omega) + i_x(5\Omega) = 0$$

$$KCL_A \rightarrow i_o - 0.5A - i_x = 0$$

$\rightarrow$ 3 Equations, 3 unknowns: i_o, i_x, v_x

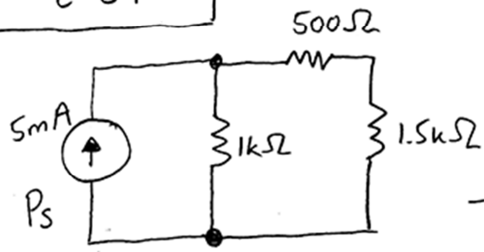
From $KVL_2 \rightarrow \boxed{i_x = \frac{(0.5A)(10\Omega)}{5\Omega} = 1A}$

From KVL_1 & $KCL_2 \rightarrow -v_x + (0.5A + i_x)(4\Omega) + 5V = 0$

$$\boxed{v_x = 5V + (1.5A)(4\Omega) = 11V}$$

2-31

Find the power provided by the source (P_s)!



Use the series and parallel equivalent resistance to find the total resistance of the circuit.

→ 500Ω and 1.5kΩ are in series

↳ $R_1 = 500\Omega + 1.5k\Omega = \underline{2k\Omega}$

→ 1kΩ and R_1 are in parallel

↳ $R_{total} = 1k\Omega \| R_1 = \left(\frac{1}{1k\Omega} + \frac{1}{2k\Omega}\right)^{-1}$

$= \left(\frac{3k\Omega}{2k\Omega^2}\right)^{-1} = \underline{\frac{2}{3}k\Omega}$

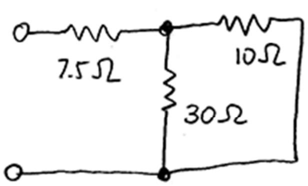
Then the total power is =

$P_s = (5mA)^2(R_{total}) = (5mA)^2\left(\frac{2}{3}k\Omega\right) \rightarrow \boxed{P_s = -0.01667\ W}$

(providing power)

2-35

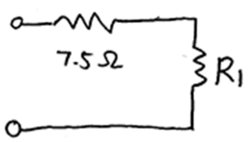
Find R_{eq} for the circuit below!



→ 30Ω and 10Ω are in parallel

↳ $R_1 = \left(\frac{1}{10\Omega} + \frac{1}{30\Omega}\right)^{-1} = \left(\frac{40\Omega}{300\Omega^2}\right)^{-1}$

$\underline{R_1 = 7.5\Omega}$



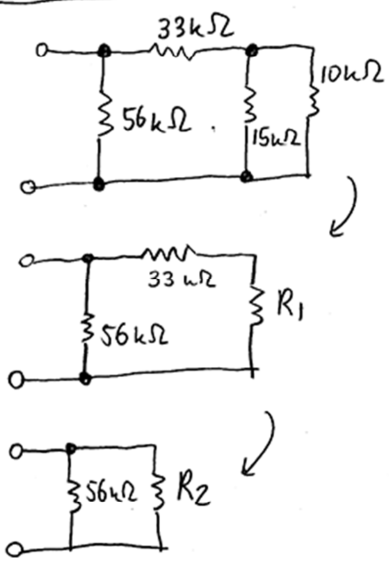
→ R_1 and 7.5Ω are in series

↳ $R_{eq} = R_1 + 7.5\Omega = 7.5\Omega + 7.5\Omega$

$\boxed{R_{eq} = 15\Omega}$

2-37

Find the equivalent resistance R_{eq}!



→ 10 kΩ and 15 kΩ are in parallel

↳ $R_1 = \left(\frac{1}{10k\Omega} + \frac{1}{15k\Omega}\right)^{-1} = \left(\frac{25k\Omega}{150\ k\Omega^2}\right)^{-1} = \underline{6\ k\Omega}$

→ 33kΩ and R_1 are in series

↳ $R_2 = 33k\Omega + 6k\Omega = 39\ k\Omega$

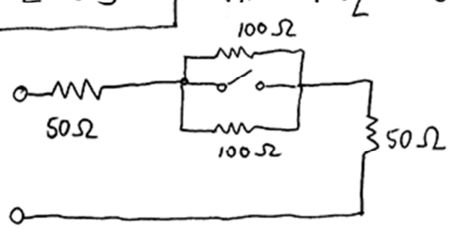
→ 56 kΩ and R_2 are in parallel

↳ $R_{eq} = \left(\frac{1}{56k\Omega} + \frac{1}{39k\Omega}\right)^{-1} = \left(\frac{95\ k\Omega}{2184\ k\Omega^2}\right)^{-1}$

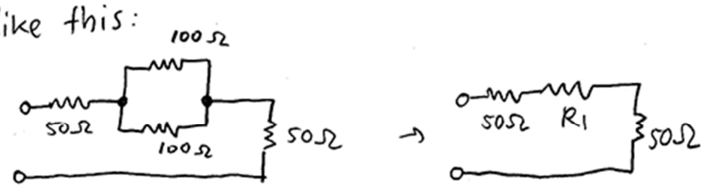
$$\boxed{R_{eq} = 22.99\ k\Omega}$$

2-39

Find R_{eq} when the switch is open and repeat for when switch is closed!



If the switch is open, then the circuit will behave like this:

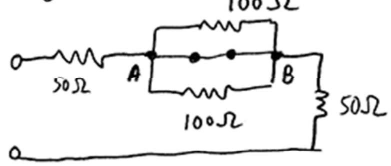
↳ 

→ Both 100Ω are in parallel

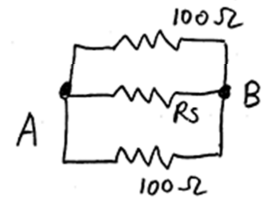
$R_1 = \left(\frac{1}{100\Omega} + \frac{1}{100\Omega}\right)^{-1} = \left(\frac{200\ \Omega}{10000\ \Omega^2}\right)^{-1} = \underline{50\Omega}$

→ R_1 and both 50Ω are in series → $\boxed{R_{eq} = 50\Omega + 50\Omega + 50\Omega = 150\Omega}$

If the switch is closed, then you will have a short circuit at the switch's junction.



So, we have a parallel junction from A to B



Where $R_s = 0\Omega$

Then we have the equivalent resistance across A-B

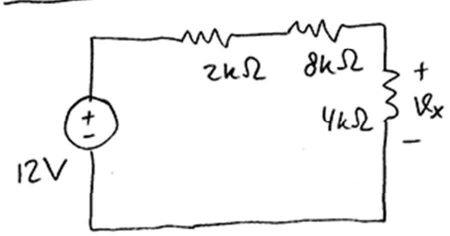
$$\hookrightarrow R_1 = \left(\frac{1}{100\Omega} + \frac{1}{100\Omega} + \frac{1}{R_s}\right)^{-1} = \left(\frac{200\Omega + R_s}{10000\Omega\, R_s}\right)^{-1} = \frac{10000\Omega\, R_s}{200\Omega + R_s}$$

with $R_s = 0$, we have. $R_1 = \frac{0}{200\Omega} = 0\Omega$.

Then the equivalent resistance is $R_{eq} = 50\Omega + 0\Omega + 50\Omega \rightarrow \boxed{R_{eq} = 100\Omega}$

2-54

Use Voltage division to find v_x!

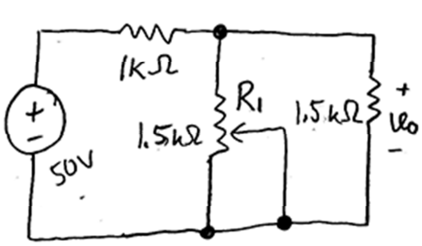


$$v_x = \left(\frac{4k\Omega}{2k\Omega + 8k\Omega + 4k\Omega}\right) 12V = \frac{4}{14} \cdot 12V$$

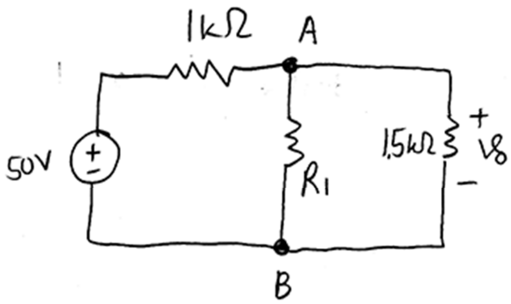
$$\boxed{v_x = 3.43\ V}$$

2-61

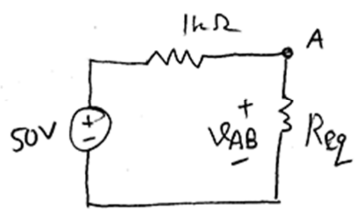
Find the range of values of v_o!



The resistor in the middle is called a potentiometer. It has a variable resistance with a range from 0Ω to $1.5k\Omega$. $\rightarrow R_1 \in [0, 1.5k\Omega]$



v_0 is the voltage across node AB, so we can find v_0 by using the equivalent resistance across AB.



→ R_1 and 15kΩ are parallel

$$\hookrightarrow R_2 = \left(\frac{1}{R_1} + \frac{1}{1.5k\Omega}\right)^{-1} = \left(\frac{R_1 + 1.5k\Omega}{R_1 \cdot 1.5k\Omega}\right)^{-1}$$

$$R_2 = \frac{R_1 \cdot 1.5k\Omega}{R_1 + 1.5k\Omega}$$

So, using voltage dividing equation, we can find $v_{AB} = v_0$

$$v_0 = \left(\frac{R_{eq}}{R_{eq} + 1k\Omega}\right) 50V = \left(\frac{\frac{R_1 \cdot 1.5k\Omega}{R_1 + 1.5k\Omega}}{\frac{R_1 \cdot 1.5k\Omega}{R_1 + 1.5k\Omega} + 1k\Omega}\right) 50V = \left(\frac{R_1 \cdot 1.5k\Omega}{R_1 \cdot 1.5k\Omega + (R_1 + 1.5k\Omega)(1k\Omega)}\right) 50V$$

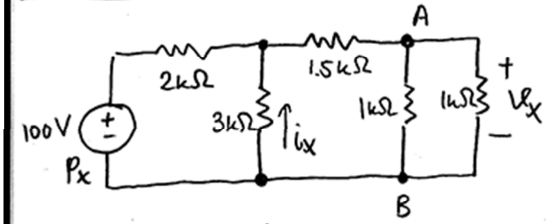
$$\boxed{v_0 = \left(\frac{R_1 \cdot 1.5\,k\Omega}{R_1\, 2.5k\Omega + 1.5\,k\Omega^2}\right) 50V}$$

At $R_1 = 0 \rightarrow v_0 = 0$

At $R_1 = 15k\Omega \rightarrow v_0 = \left(\frac{1.5k\Omega \cdot 1.5k\Omega}{1.5k\Omega \cdot 2.5k\Omega + 1.5\,k\Omega^2}\right) 50V \rightarrow \boxed{v_0 = 21.43V}$

You can also use KCL and KVL, but it will be more complicated.

2-71 Use circuit reduction to find v_x, i_x, P_x!



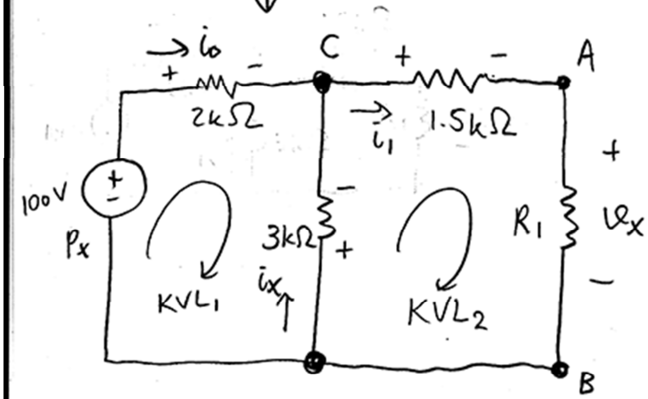
v_x is the voltage difference across AB, so we should reduce the connection between AB.

$\longrightarrow$ Both 1kΩ are parallel

$\hookrightarrow R_1 = \left(\frac{1}{1k\Omega} + \frac{1}{1k\Omega}\right)^{-1} = \left(\frac{2k\Omega}{1k\Omega^2}\right)^{-1}$

$R_1 = 0.5\ k\Omega$

↓



$KVL_1 \rightarrow -100V + i_0\,(2k\Omega) + (-i_x)(3k\Omega) = 0$

$KVL_2 \rightarrow i_x\,(3k\Omega) + i_1\,(1.5k\Omega) + i_1 R_1 = 0$

$KCL_C \rightarrow i_0 + i_x - i_1 = 0$

$i_0 = i_1 - i_x$

So, we have:

$$KVL_2 \rightarrow i_1 = -i_x\left(\frac{3k\Omega}{1.5k\Omega + 0.5k\Omega}\right) = -\frac{3}{2}i_x$$

$$KVL_1 \text{ \& } KCL_C \rightarrow -100V + (i_1 - i_x)(2k\Omega) + (-i_x)(3k\Omega) = 0$$

$$i_1\,(2k\Omega) - i_x\,(5k\Omega) = 100V$$

$$-\frac{3}{2}i_x\,(2k\Omega) - i_x\,(5k\Omega) = 100\ V$$

$$\hookrightarrow \boxed{i_x = \frac{-100\ V}{\left(\frac{3}{2}\cdot 2k\Omega + 5k\Omega\right)} = -12.5\ mA}$$

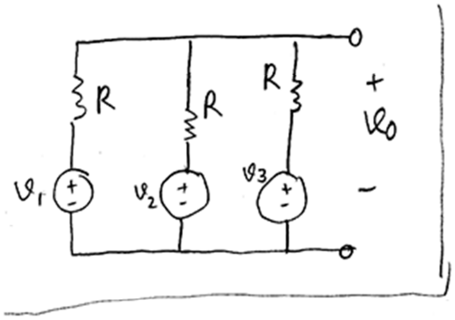
Find $i_1 \rightarrow$ $i_1 = -\frac{3}{2}i_x = -\frac{3}{2}(-12.5\ mA) = +18.75\ mA$

Find $v_x \rightarrow$ $v_x = i_1 R_1 = (18.75mA)(0.5k\Omega) \rightarrow \boxed{v_x = 9.375\ V}$

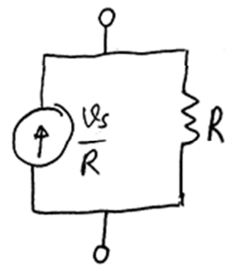
Find $i_0 \rightarrow$ $i_0 = i_1 - i_x = 18.75mA - (-12.5\ mA) = 31.25mA$

$\hookrightarrow$ $P_x = i_0(-100V) = (31.25mA)(-100V) \rightarrow \boxed{P_x = -3.125\ W}$ ↙ providing power

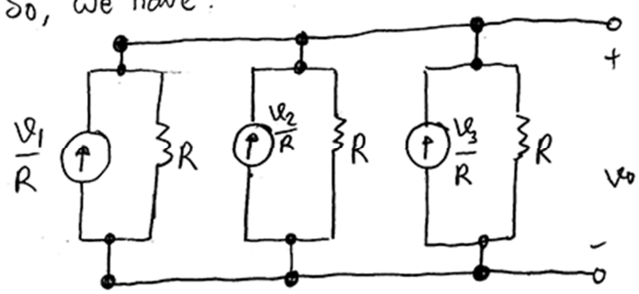
2-75 Use source transformation to relate v_o to v_1, v_2, v_3!



is equivalent to:



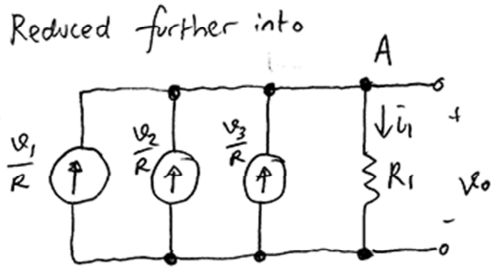
So, we have:



All R are parallel, so we can reduce the circuit further.

$$R_1 = \left(\frac{1}{R}+\frac{1}{R}+\frac{1}{R}\right)^{-1} = \left(\frac{3}{R}\right)^{-1} = \frac{R}{3}$$

Reduced further into



Using KCL at node A, we have

$$\hookrightarrow \frac{v_1}{R} + \frac{v_2}{R} + \frac{v_3}{R} - i_1 = 0$$

$$i_1 = \frac{1}{R}\left(v_1 + v_2 + v_3\right)$$

Then, $v_{AB} = v_o = i_1 R_1 = \frac{1}{\cancel{R}}\left(v_1+v_2+v_3\right)\frac{\cancel{R}}{3}$

$$\hookrightarrow \boxed{v_o = \frac{1}{3}\left(v_1+v_2+v_3\right)}$$