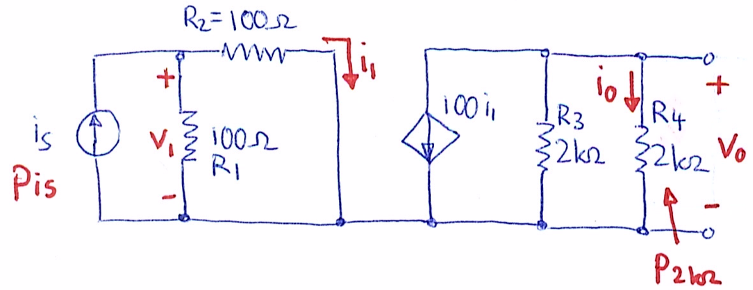
MAE 140 fall 2014
Homework 5 solution

4.2 4.7 4.23 4.27 4.30 4.31 4.35 4.36 4.37 4.43

4.2 Find $\frac{V_o}{V_1}$ and $\frac{i_o}{i_s}$. For $i_s = 5mA$, find the powers P_{is} and $P_{2k\Omega}$



Using current division we find $i_o = \frac{\frac{1}{R_4}}{\frac{1}{R_3}+\frac{1}{R_4}}(-100 i_1) = -50 i_1$ (1)

for the circuit on the RHS and $i_1 = \frac{\frac{1}{R_2}}{\frac{1}{R_1}+\frac{1}{R_2}} * i_s = \frac{1}{2} i_s$ (2)

for the circuit on the LHS.

Substituting (2) in (1) we obtain $i_o = -25 i_s \rightarrow \boxed{\frac{i_o}{i_s} = -25}$

Using Ohm's law we know that $V_o = i_o R_4 = -25 i_s \cdot 2k\Omega$ (3)

Since R_1 and R_2 are in parallel, the voltages across R_1 and R_2 are equal and thus

$V_1 = V_2 = i_1 R_2$ (4)

Substituting (2) in (4) $V_1 = \frac{1}{2} R_2 i_s \rightarrow i_s = \frac{2V_1}{R_2}$ (5)

Substituting (5) in (3) $V_o = -25 \cdot 2k\Omega \frac{2V_1}{100\Omega} = -1000 V_1$

$\rightarrow \boxed{\frac{V_o}{V_1} = -1000}$

The power supplied by i_s is

$P_{is} = V \cdot i_s = i_s^2 R_{EQ} = i_s^2 \frac{100 \cdot 100}{100+100}$

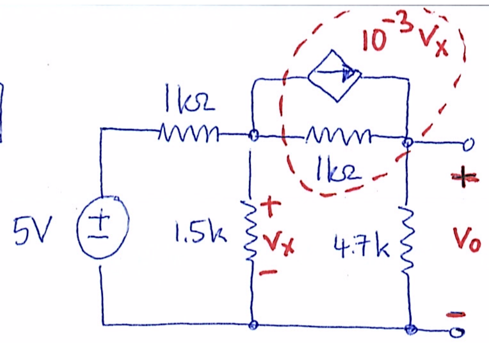
$P_{is} = i_s^2 \cdot 50\Omega = \boxed{1.3 \text{ mW}}$

The power delivered to the $2k\Omega$ resistor is

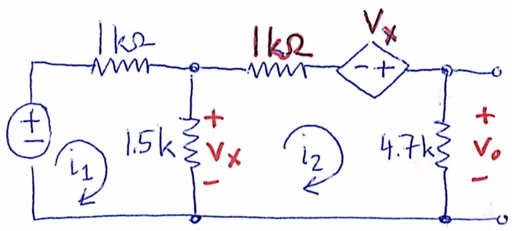
$P_{2k\Omega} = i_o^2 R_4 = (-25 i_s)^2 2k\Omega$

$P_{2k\Omega} = \boxed{31.25 \text{ W}}$

$\boxed{4.7}$



Using source transformation we obtain



Mesh-current analysis yields

$$\begin{bmatrix} (1+1.5)\,k\Omega & -1.5\,k\Omega \\ -1.5\,k\Omega & (1.5+1+4.7)\,k\Omega \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5V \\ V_x \end{bmatrix}$$

$$V_x = Ri = 1.5\,k\Omega\,(i_1 - i_2)$$

Rearranging

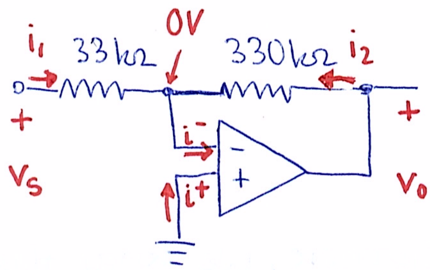
$$\begin{bmatrix} (1+1.5)\,k\Omega & -1.5\,k\Omega \\ -(1.5+1.5)\,k\Omega & (1.5+1+4.7+1.5)\,k\Omega \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5V \\ 0 \end{bmatrix}$$

$$i_1 = 2.5217 \text{ mA}$$
$$i_2 = 0.8696 \text{ mA}$$

V_o can be computed using Ohm's law

$$V_o = i_2\, 4.7\,k\Omega = \boxed{4.087\text{ V}}$$

$\boxed{4.23}$ Find the voltage gain for both circuits



From the properties of operational amplifiers learned in class we know that

$$i^- = 0 \text{ and } i^+ = 0$$

Thus, and since the '+' terminal is connected to ground, we know that the voltage potential at the '−' terminal is also 0 V.

Consequently, we have

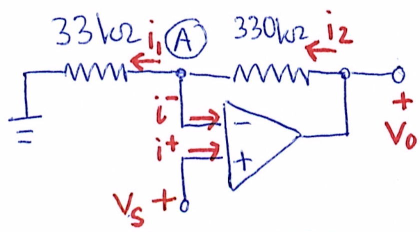
$$i_1 = -i_2 \quad \text{with } i_1 = \frac{V_s}{33k\Omega}$$
$$i_2 = \frac{V_o}{330k\Omega}$$

Substituting yields

$$\frac{V_s}{33k\Omega} = -\frac{V_o}{330\,k\Omega}$$

$$\frac{V_o}{V_s} = -\frac{330k\Omega}{33k\Omega} = \boxed{-10}$$

This is an inverting op-amp!
$\rightarrow \frac{V_o}{V_s} = -\frac{R_f}{R_{in}} = -\frac{330\,k\Omega}{33\,k\Omega}$



With i^- and i^+ being equal to zero, the voltage at node A is equal to V_s.
Thus, we can write

$$i_1 = i_2 \quad \text{with } i_1 = \frac{V_s}{33k\Omega}$$
$$i_2 = \frac{V_o - V_s}{330\,k\Omega}$$

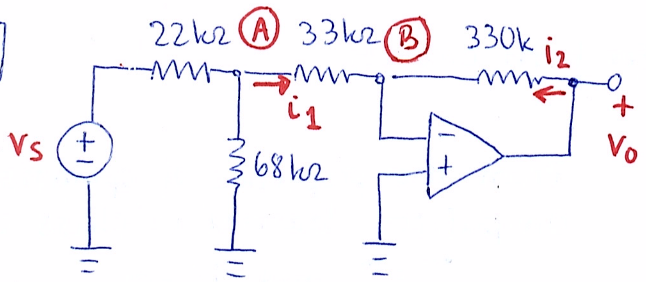
Substituting yields

$$\frac{V_s}{33k\Omega} = \frac{V_o - V_s}{330k\Omega}$$

$$10 = \frac{V_o}{V_s} - 1 \quad \text{so} \quad \boxed{\frac{V_o}{V_s} = 11}$$

This is a non-inverting op-amp
$\rightarrow \frac{V_o}{V_s} = 1 + \frac{R_f}{R_{in}} = 1 + \frac{330k\Omega}{33k\Omega}$

4.27



since $i^- = i^+ = 0$, $V_B = 0V$. Using KCL at node B we obtain

$$i_1 = -i_2 \quad \text{with} \quad i_1 = \frac{V_A - V_B}{33k\Omega} = \frac{V_A}{33k\Omega} \qquad (1)$$

$$i_2 = \frac{V_0 - V_B}{330k\Omega} = \frac{V_0}{330k\Omega}$$

To find V_A we use voltage division. Note that the 33kΩ and 68kΩ resistors are in parallel, so the equivalent resistance is

$$R_{EQ} = \frac{33k\Omega \cdot 68k\Omega}{33k\Omega + 68k\Omega} = 22.2178\ k\Omega$$

Using voltage division we obtain

$$V_A = \frac{R_{EQ}}{R_{EQ} + 22k\Omega} V_S = 0.5025\ V_S \qquad (2)$$

Substituting (2) in (1) yields

$$\frac{V_A}{33k\Omega} = -\frac{V_0}{330k\Omega}$$

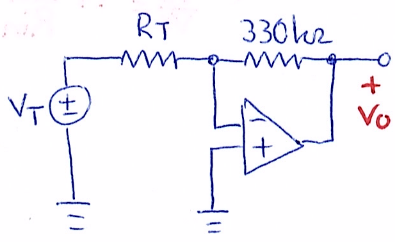
$$\frac{0.5025 V_S}{33k\Omega} = -\frac{V_0}{330k\Omega} \rightarrow \boxed{\frac{V_0}{V_S} = -5.025}$$

! Alternative way of solving this problem:
Find Thevenin equivalent circuit (of everything to the left of B)

→ open circuit yields Thevenin voltage

$$V_T = V_B = V_A = \frac{68k\Omega}{22k\Omega + 68k\Omega} V_S = \underline{0.7556 V_S}$$

$$R_T = 33k\Omega + 22k\Omega \| 68k\Omega = \underline{49.62k\Omega}$$

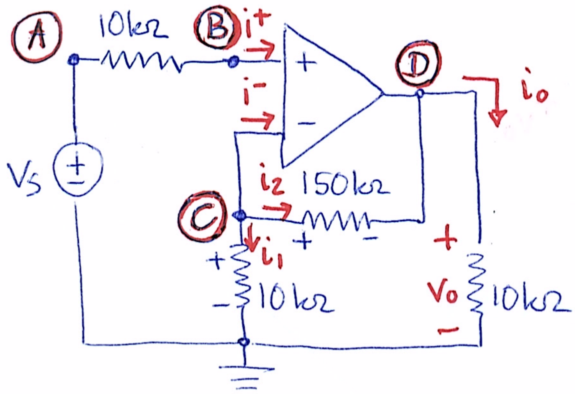


This is an inverting op-amp!

$$V_0 = -\frac{330k\Omega}{R_T} V_T = -\frac{330k\Omega}{49.62k\Omega} \cdot 0.7556 V_S$$

$$\boxed{\frac{V_0}{V_S} = -5.025}$$

4.30



$V_{cc} = \pm 24V$

(a) Find V_o in terms of V_S

Remember that $i^+ = i^- = 0$ and thus, $V_B = V_C$.
Also, since $i^+ = 0$ we have $V_B = V_A = V_S$.
Using KCL we can write for node C

$$i_1 = -i_2$$

$$\frac{V_C}{10k\Omega} = -\frac{(V_C - V_D)}{150k\Omega} = \frac{V_D - V_C}{150k\Omega}$$

$$15 = \frac{V_D}{V_C} - 1 \rightarrow V_D = 16\,V_C$$

where $V_D = V_O$, $V_C = V_S$

$$\boxed{V_o = 16\,V_S}$$

(b) Find i_o for $V_S = 1V$ and $V_S = 3V$

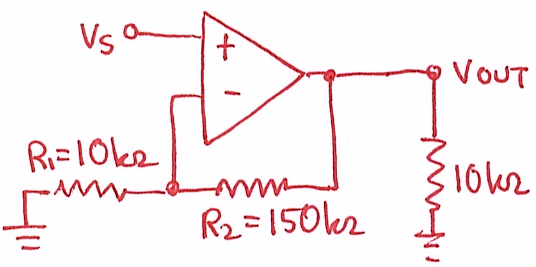
$$i_o = \frac{V_o}{10k\Omega}$$

$V_o(V_S = 1V) = 16V \rightarrow \boxed{i_o = 1.6mA}$

$V_o(V_S = 3V) = 48V > V_{cc}$ op-amp becomes saturated

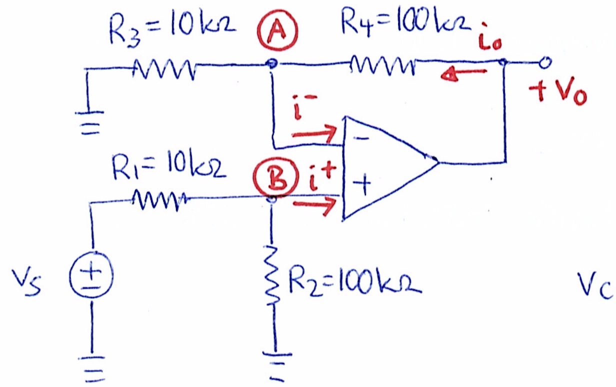
and thus $V_o = 24V \rightarrow \boxed{i_o(V_S = 3V) = 2.4mA}$

Note: This is a non-inverting op-amp! We know from class that



$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_S = 16\,V_S$$

4.31



$V_{cc} = \pm 18V$

a) Find v_o in terms of v_s

For an ideal op-amp we have $i^- = i^+ = 0$ and $v_A = V_B$.
Using voltage division we can write

$$V_B = \frac{R_2}{R_1+R_2} V_s = \frac{10}{11} V_s \qquad (1)$$

$$V_A = \frac{R_3}{R_3+R_4} V_o = \frac{1}{11} V_o \qquad (2)$$

Substituting (1) in (2) using $V_A = V_B$ yields

$$\boxed{V_o = 10 V_s}$$

b) Find i_o for $V_s = 0.5V$ and $V_s = 2V$

$$i_o = \frac{V_o - V_B}{100k\Omega} = \frac{V_o - \frac{10}{11} V_s}{100\,k\Omega}$$

- $V_o(V_s = 0.5V) = 10 \cdot 0.5V = 5V$ $\rightarrow$ $\boxed{i_o(V_s = 0.5V) = 45.45\,\mu A}$

- $V_o(V_s = 2V) = 20\,V$

20V > V_{cc} so the op-amp becomes saturated so $V_o(V_s = 2V) = \underline{18\,V}$

$\rightarrow$ $\boxed{i_o(V_s = 2V) = \frac{(18 - \frac{20}{11})V}{100\,000 \frac{V}{A}} = 161.8\,\mu A}$

Note that this is a voltage divider connected to a non-inverting op-amp. Thus we know that

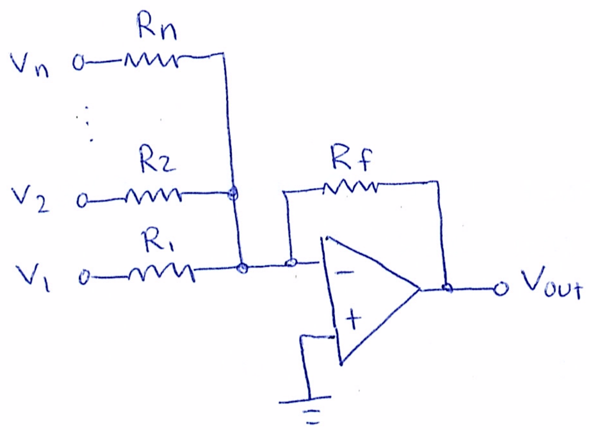
$$V_o = \left(1 + \frac{R_4}{R_3}\right) V_B \quad \text{where} \quad V_B = \frac{R_2}{R_1+R_2} V_s = \frac{10}{11} V_s$$

$$\rightarrow V_o = 11 \cdot \frac{10}{11} V_s = 10\,V_s$$

4.35 Three-input inverting summer with

$$V_o = -\left[v_1 + 10v_2 + 100v_3\right]$$

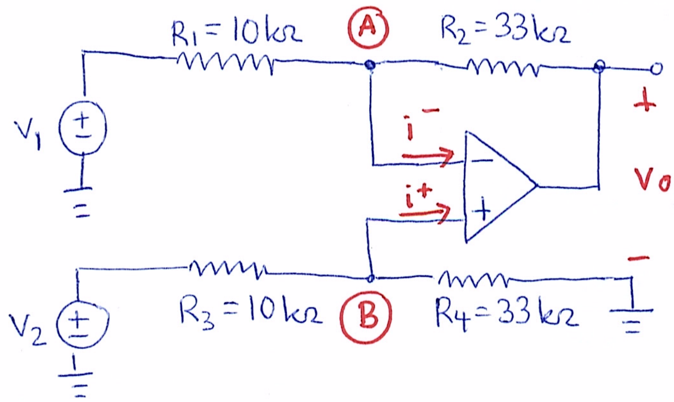
The resistance of the feedback resistor is $100\,k\Omega$.
What is R_1, R_2, R_3?



$$V_{out} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \dots + \frac{V_n}{R_n}\right)$$

so $\boxed{\begin{aligned} R_1 &= 100\,k\Omega \\ R_2 &= 10\,k\Omega \\ R_3 &= 1\,k\Omega \end{aligned}}$

4.36



For an ideal op-amp we have $i^- = i^+ = 0$

$$V_A = V_B$$

To find v_o in terms of v_1 and v_2 we use the superposition principle.

(1) Turn off v_2: Since both R_3 and R_4 are connected to ground we have $V_A = V_B = 0$ and the circuit acts like an inverting amplifier. Thus

$$V_{o,1} = -\frac{R_f}{R_{in}} V_1 = -\frac{R_2}{R_1} V_1 = -3.3\,V_1$$

(2) Turn off v_1: The circuit acts like a non-inverting amplifier with gain

$$V_{o,2} = \left(1 + \frac{R_2}{R_1}\right) V_B = 4.3\,V_B$$

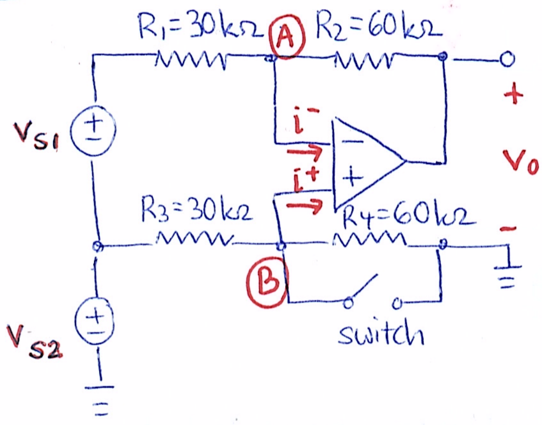
V_B can be obtained using voltage division

$$V_B = \frac{R_4}{R_3 + R_4} V_2 = \frac{33}{43} V_2$$

$$\Rightarrow V_o = V_{o,1} + V_{o,2} = -3.3V_1 + 4.3\,\frac{3.3}{4.3} V_2 = \boxed{3.3(V_2 - V_1) = V_o}$$

← This is a differential amplifier

4.37 Find v_o in terms of v_{s1} and v_{s2} for case 1: switch open
case 2: switch closed



For an ideal op-amp we have

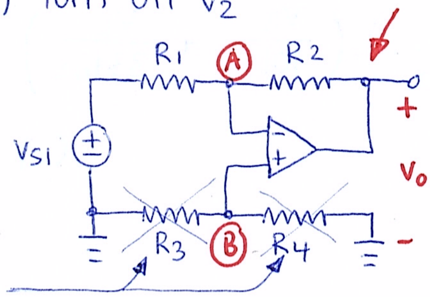
$i^- = i^+ = 0$

$V_A = V_B$

We use the superposition principle to find v_o in terms of v_{s1} and v_{s2}.

Case 1 (switch open)

1) Turn off v_2

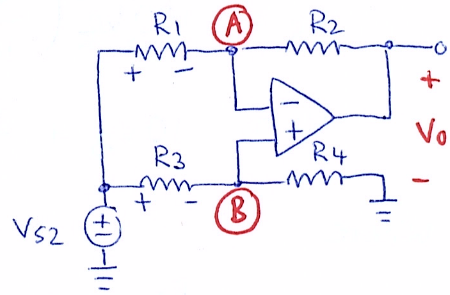


not relevant

$V_B = 0$ so the circuit behaves like an inverting amplifier:

$$V_{o,1} = -\frac{R_2}{R_1} V_{s1} = -2 V_{s1}$$

2) Turn off v_1

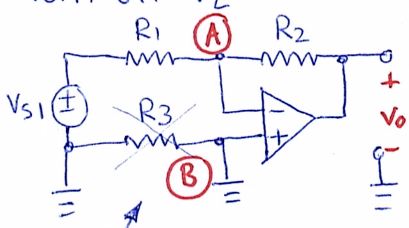


Since $V_A = V_B$ we have $V_{o,2} = 0$

$$\Rightarrow \boxed{V_o = V_{o,1} + V_{o,2} = -2 V_{s1}} \quad \text{(switch open)}$$

Case 2 (switch closed) circuit behaves like an inverting amplifier!

1) Turn off v_2

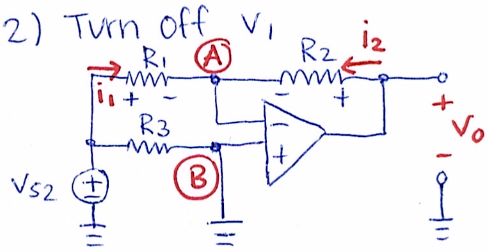


not relevant

This circuit behaves like an inverting amplifier.

$$V_{o,1} = -\frac{R_2}{R_1} V_{s1} = -2 V_{s1}$$

2) Turn off v_1



$V_A = V_B = 0\,V$ and using KCL we can write for node A

$$i_1 = -i_2$$

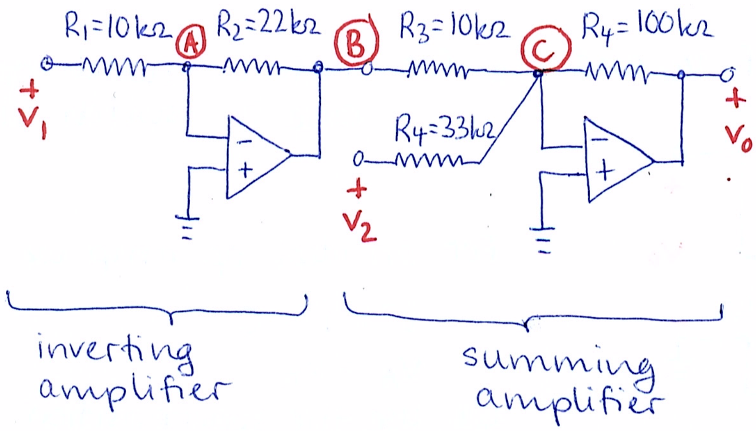
$$\frac{V_{s2}}{R_1} = -\frac{V_o}{R_2} \rightarrow V_{o,2} = -\frac{R_2}{R_1} V_{s2}$$

2

$$\Rightarrow \boxed{V_o = V_{o,1} + V_{o,2} = -2(V_{s1} + V_{s2})}$$

(switch closed)

4.43



This is essentially an inverting amplifier connected to a summing amplifier.
Starting on the far right we can write

$$V_o = -R_4\left(\frac{V_B}{R_3} + \frac{V_2}{R_4}\right) = -100\,k\Omega\left(\frac{V_B}{10k\Omega} + \frac{V_2}{33k\Omega}\right)$$

$$V_o = -10V_B - \frac{100}{33}V_2 \qquad (1)$$

where

$$V_B = -\frac{R_2}{R_1}V_1 = -\frac{22}{10}V_1 \qquad (2)$$

Substituting (2) in (1) yields

$$\boxed{V_o = 22V_1 - \frac{100}{33}V_2}$$