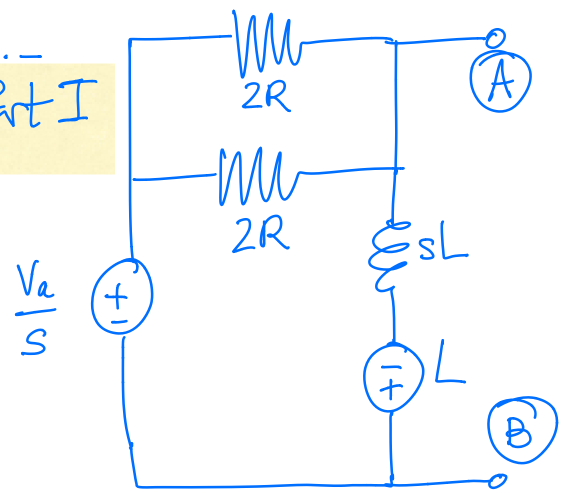
1.-

Part I

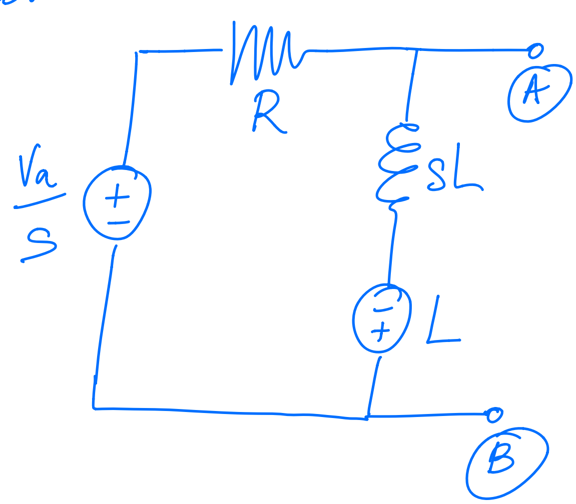


(Here, we have used $\mathcal{L}(V_a u(t)) = \frac{V_a}{s}$ and the information that $i_L(0) = 1A$)

[+1 point for correct initial condition; +1 point for correct overall circuit]

Part II

To find the open circuit voltage, we combine the two 2R impedances in parallel to obtain



[+0.5 point]

The voltage drop across the sL-impedance is, by voltage division,

$$V_{sL}(s) = \frac{sL}{R+sL}\left(\frac{V_a}{s} + L\right)$$

[+0.5 point]

Therefore,

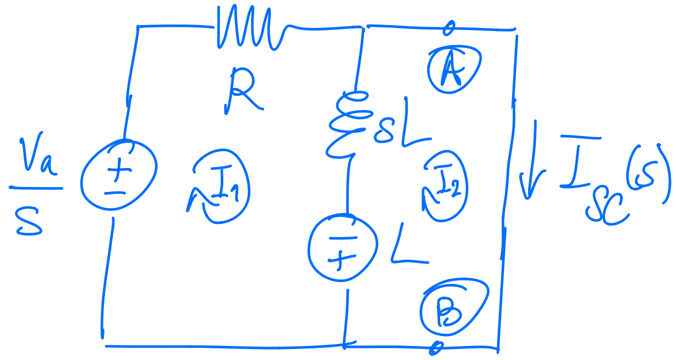
$$V_{AB}(s) = V_{sL}(s) - L = \frac{sL}{R+sL}\,\frac{V_a}{s} + L\left(\frac{sL}{R+sL} - 1\right)$$

$$= \frac{V_a L}{R+sL} + L\,\frac{\cancel{sL} - R - \cancel{sL}}{R+sL} = \frac{L(V_a - R)}{R+sL}$$

[+1 point]

Part III

We have



We set up mesh current analysis equations to obtain the short-circuit current

Mesh 1: $R\,I_1(s) + sL\left(I_1(s) - I_2(s)\right) = \frac{V_a}{s} + L$

Mesh 2: $sL\left(I_2(s) - I_1(s)\right) = -L$ [+1 point]

Therefore, $R\,I_1(s) = \frac{V_a}{s} \Rightarrow I_1(s) = \frac{V_a}{sR}$

And hence $sL\,I_2(s) = -L + \cancel{s}L \cdot \frac{V_a}{\cancel{s}R} \Rightarrow$

$$\Rightarrow I_2(s) = \frac{-R + V_a}{Rs} = I_{sc}(s)$$ [+1 point]

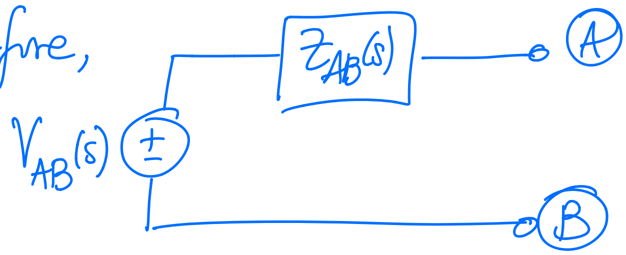
Part IV

Given that we know $V_{OC}(s)$ (from Part II) and $I_{SC}(s)$ (from Part III), we conclude

$$Z_{AB}(s) = \frac{V_{OC}(s)}{I_{SC}(s)} = \frac{L\cancel{(V_a-R)}}{R+sL} \cdot \frac{Rs}{\cancel{V_a-R}} = \frac{RLs}{R+sL}$$

[+1 point]

Therefore,



[+1 point]

Part V

We have

$$V_{AB}(s) = \frac{L(V_a-R)}{R+sL}$$

Zero-input corresponds to zeroing the input (i.e., $V_a = 0$), so

$$V_{zi}(s) = \frac{-RL}{R+sL}$$ [+0.5 point]

Therefore,

$$V_{zs}(s) = \frac{LV_a}{R+sL}$$ [+0.5 point]

So the break-down is $V_{AB}(s) = \overbrace{\frac{LV_a}{R+sL}}^{zs} + \overbrace{\frac{-RL}{R+sL}}^{zi}$

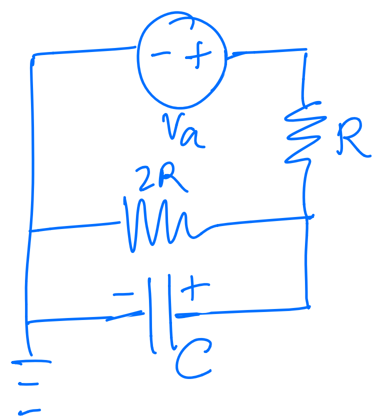
The forced response corresponds to the terms with the same poles as the input $\frac{V_a}{s}$. Looking at our expression for $V_{AB}(s)$, we deduce

$$V_{fr}(s) = 0 \quad \text{and} \quad V_{nr}(s) = V_{AB}(s).$$

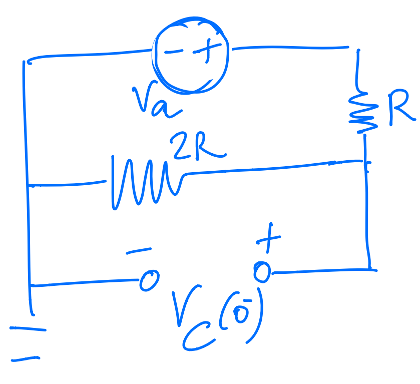
[+0.5 point] [+0.5 point]

2.- Part I

Since the circuit is kept in position A for a very long time, we need to consider



V_a is constant. Under DC excitation, we know the capacitor behaves like an open circuit.
Therefore, we are looking at [+1 point]

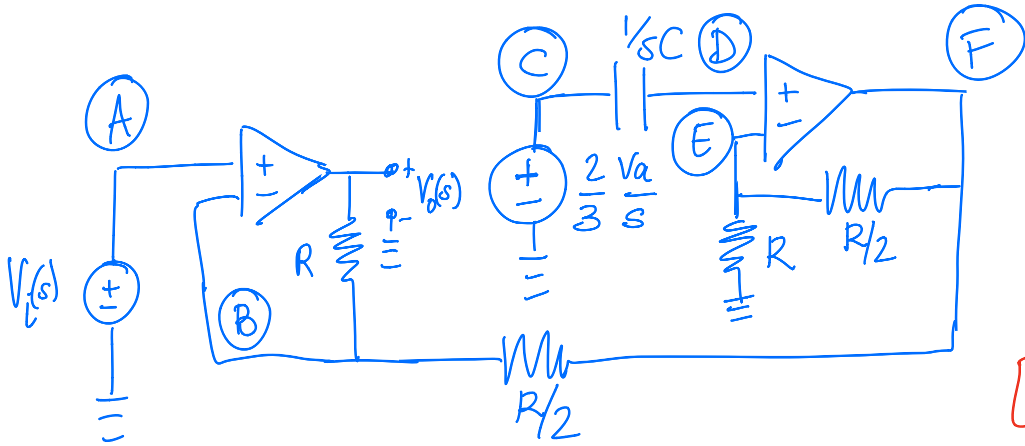


By looking at the plot, we see that $V_C(0^-)$ is equal to the voltage drop across the 2R resistor.
Using voltage division, [+1 point]

$$V_C(0^-) = \frac{2R}{2R+R} V_a = \frac{2}{3} V_a$$

Part II

We use the initial condition to transform the circuit into the s-domain.



[+1 point]

We use nodal analysis to express the output response transform $V_o(s)$ as a function of $V_i(s)$ and Va.

Looking at the plot, we know

[+0.5 point]

$$V_A(s) = V_i(s)$$

$$V_C(s) = \frac{2}{3}\frac{Va}{s} = V_D(s) \quad (\text{b/c of } I_P = 0) \text{ [+0.5 point]}$$

– ideal op-amp –

Because of ideal op-amp conditions,

$$V_A(s) = V_B(s) \qquad \text{[+0.5 point]}$$

$$V_D(s) = V_E(s) \qquad \text{[+0.5 point]}$$

We only need to write KCL for nodes B and E.

KCL@B

$$\frac{1}{R}\left(V_B(s)-V_o(s)\right)+\frac{2}{R}\left(V_B(s)-V_F(s)\right)=0$$ [+0.5 point]

KCL@E

$$\frac{1}{R}\left(V_E(s)\right)+\frac{2}{R}\left(V_E(s)-V_F(s)\right)=0$$ [+0.5 point]

Solving the last equation,

$$V_F(s)=\frac{3}{2}V_E(s)=\frac{3}{2}V_D(s)=\frac{\cancel{3}}{\cancel{2}}\cdot\frac{\cancel{2}}{\cancel{3}}\frac{V_a}{s}=\frac{V_a}{s}$$

Solving KCL@B,

$$V_o(s)=3V_B(s)-2V_F(s)=$$

$$=3V_i(s)-2\frac{V_a}{s}$$ [+1 point]

Part III

With $V_i(s)=\dfrac{1}{(s+2)^2}$ and $V_a=2$, [+1 point]

we have

$$V_o(s)=\frac{3}{(s+2)^2}-\frac{4}{s}$$ [+1 point]

Therefore,

$$v_0(t) = (3te^{-2t} - 4)u(t)$$ [+1 point]

Part IV

$$(v_0)_{fr}(t) = (3te^{-2t})u(t)$$ [+0.5 extra point]

$$(v_0)_{nr}(t) = -4u(t)$$ [+0.5 extra point]

$$(v_0)_{zs}(t) = (3te^{-2t})u(t)$$ [+0.5 extra point]

$$(v_0)_{zi}(t) = -4u(t)$$ [+0.5 extra point]

3.- Part I

$$T(j\omega) = \frac{500j\omega}{-\omega^2 + 520j\omega + 10000}$$

$$|500j\omega| = 500\omega$$

$$\angle 500j\omega = \frac{\pi}{2}$$

[+0.5 point]

$$|10^4 - \omega^2 + 520j\omega| = \sqrt{(10^4-\omega^2)^2 + 520^2\omega^2} =$$

$$= \sqrt{10^8 + \omega^4 - 2\cdot 10^4\omega^2 + 270400\,\omega^2}$$

$$= \sqrt{10^8 + \omega^4 + 250400\omega^2}$$

$$= \sqrt{(25\cdot 10^4 + \omega^2)(4\cdot 10^2 + \omega^2)}$$

$$\angle(10^4 - \omega^2 + 520j\omega) = \arctan\frac{520\omega}{10^4-\omega^2}$$

[+0.5 point]

Therefore,

$$|T(j\omega)| = \frac{500\omega}{\sqrt{(25\cdot 10^4+\omega^2)(4\cdot 10^2+\omega^2)}}$$ [+1 point]

$$\angle T(j\omega) = \frac{\pi}{2} - \arctan\frac{520\omega}{10^4-\omega^2}$$ [+1 point]

Part II

DC gain corresponds to $\omega = 0$

$$|T(j0)| = \frac{0}{10^4} = 0$$ [+0.5 point]

∞-freq gain corresponds to $\omega = \infty$

$$|T(j\infty)| = \lim_{\omega \to \infty} |T(j\omega)| = 0$$ [+0.5 point]

The corresponding values of the phase function

$$\angle T(j0) = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$ [+0.5 point]

$$\angle T(j\infty) = \frac{\pi}{2} - \pi = -\frac{\pi}{2}$$ [+0.5 point]

Based on the fact that $|T(j0)| = 0 = |T(j\infty)|$, one might suspect we are dealing with a bandpass filter. In fact,

$$T(s) = \underbrace{\frac{s}{s+20}}_{\text{high-pass}} \cdot \underbrace{\frac{500}{s+500}}_{\text{low-pass}}$$

Cut-off frequency of high-pass is $\omega_1 = 20$ rad/s

Cut-off frequency of low-pass is $\omega_2 = 500$ rad/s

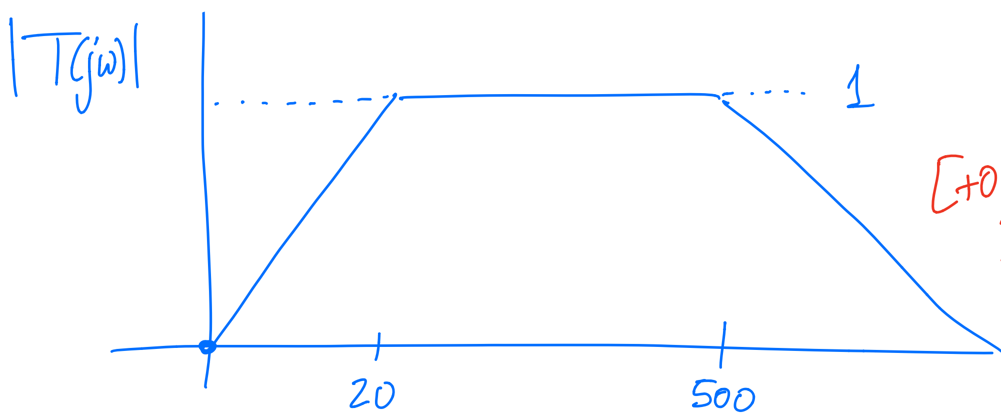
[This would also be ok: derive $|T(j\omega)|$ to find $T_{max} = \frac{25}{26}$ at $\omega_{max} = 100$, and then setting $|T(j\omega_c)| = \frac{25}{26} \cdot \frac{1}{\sqrt{2}}$, which gives solutions

$\omega_{c_1} \simeq 18.57$

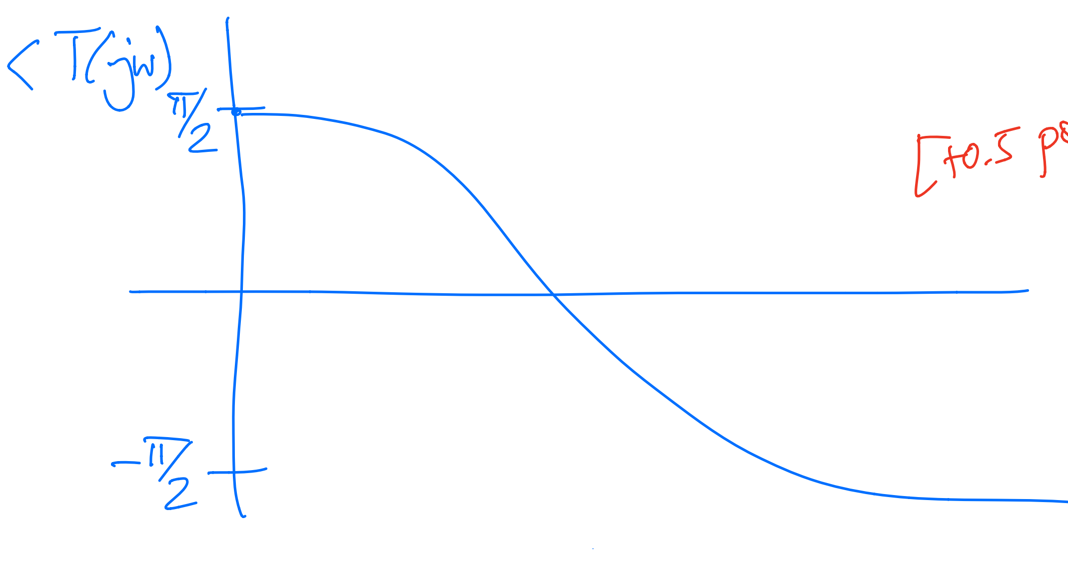
$\omega_{c_2} \simeq 538.57$]

[+1 point]

Part III Sketch of plots

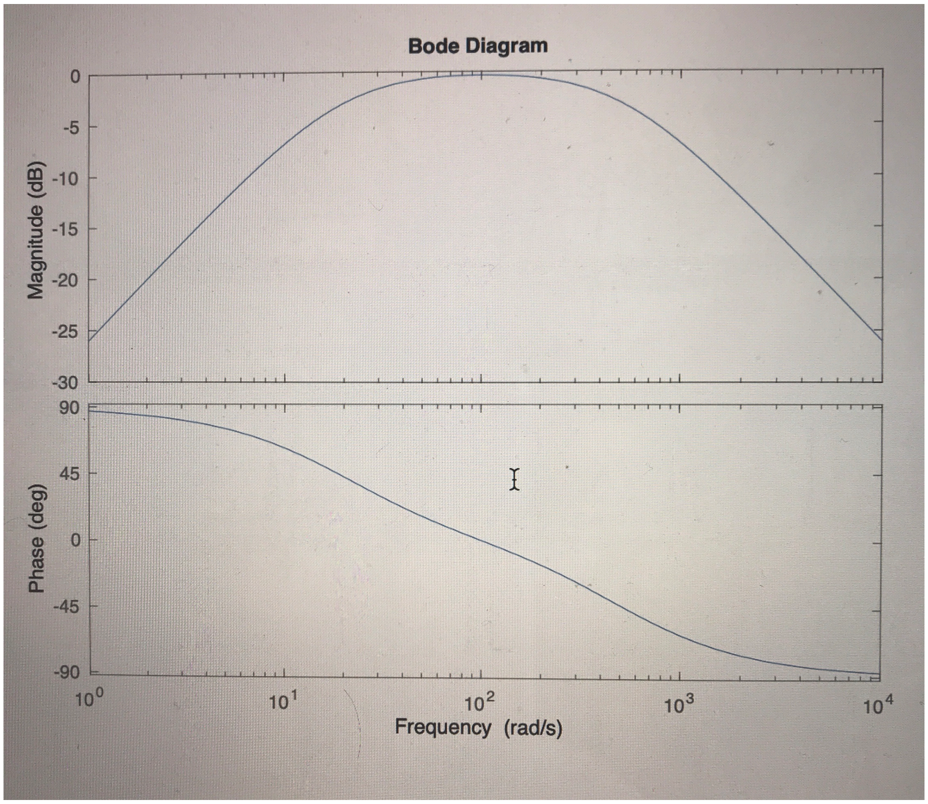


[+0.5 point]



[+0.5 point]

Bode plot from matlab



This is a bandpass filter. [+1 point]

Part IV

$$V_i(t) = \cos\left(300t + \frac{\pi}{4}\right)$$

We know that

$$V_o^{ss}(t) = |T(j300)| \cos\left(300t + \frac{\pi}{4} + \angle T(j300)\right)$$ [+1 point]

Now,

$$|T(j300)| \simeq 0.8556$$

$$\angle T(j300) \simeq -0.474$$

Therefore,

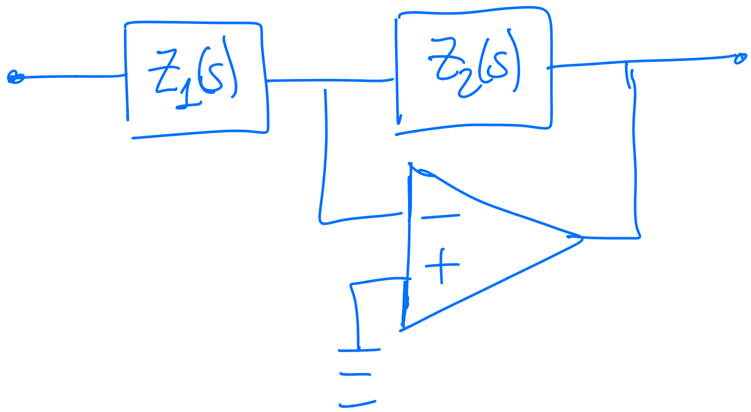
$$V_o^{ss}(t) = 0.8556 \cdot \cos(300t + 0.3115)$$

[+1 point]

4.- Part I

Based on the given decomposition, we try a design with inverting op-amps, since

[+1 point]



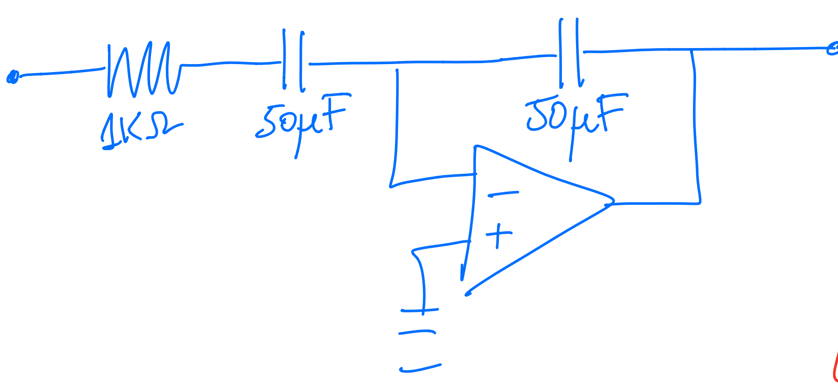
$$T(s) = -\frac{Z_2(s)}{Z_1(s)}$$

Since we cannot use inductors, we rewrite

$$T_1(s) = \frac{-20}{s+20} = \frac{-20/s}{1+20/s} = \frac{-20000}{1000+\frac{20000}{s}} =$$

$$= \frac{-\frac{1}{5\cdot10^{-5}s}}{1000+\frac{1}{5\cdot10^{-5}s}}$$

Therefore, we design

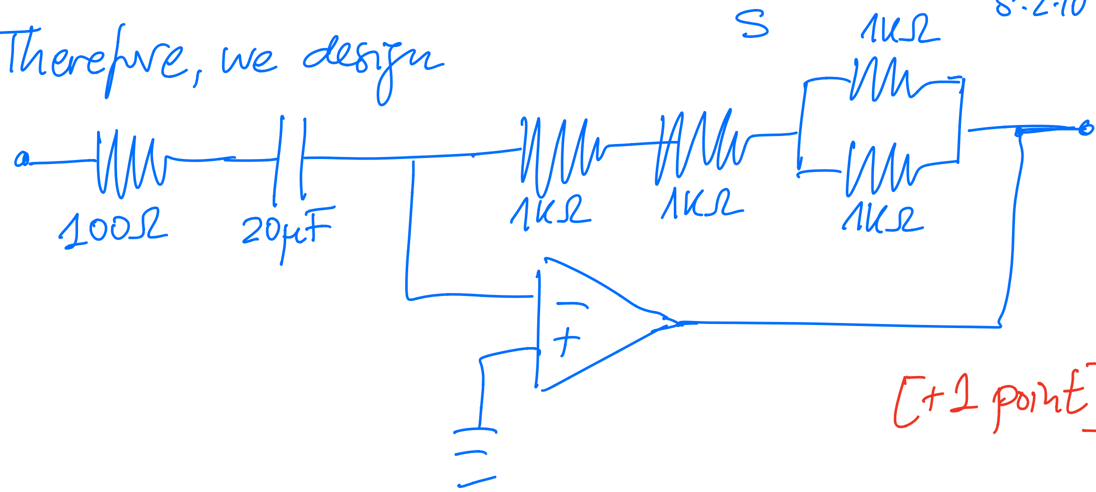


[+1 point]

Regarding $T_2(s)$, we write

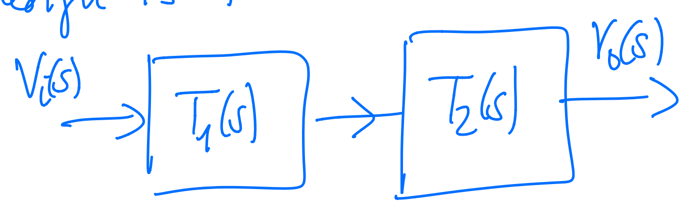
$$T_2(s) = -\frac{25s}{s+500} = -\frac{2500}{100+\frac{50000}{s}} = -\frac{2500}{100+\frac{1}{s\cdot 2\cdot 10^{-5}}}$$

Therefore, we design



[+1 point]

Our design is then



The chain rule applies: zero-output impedance of stage 1 means stage 2 does not load it.

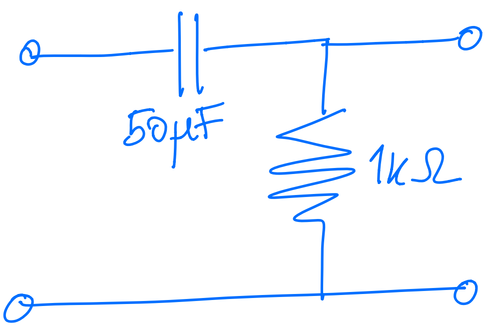
[+1 point]

Part II

Given the decomposition, we attempt a design with voltage dividers and a voltage follower. Since we cannot use inductors, we rewrite

$$T_1(s) = \frac{s}{s+20} = \frac{1}{1+\frac{20}{s}} = \frac{1000}{1000+\frac{1}{s\cdot 5\cdot 10^{-5}}}$$

Therefore we design



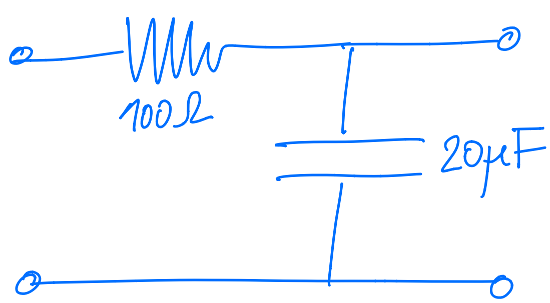
[+1 point]

On the other hand,

$$T_2(s) = \frac{500}{s+500} = \frac{500/s}{1+500/s} = \frac{50000/s}{100+50000/s} =$$

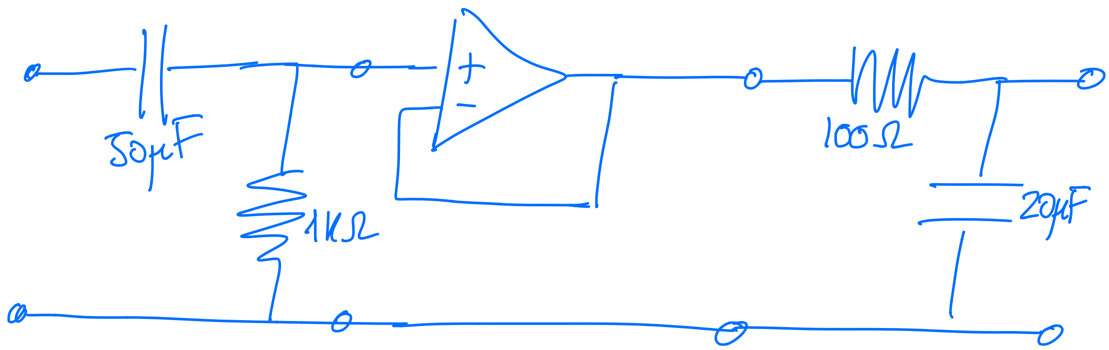
$$= \frac{\frac{1}{s\cdot 2\cdot 10^{-5}}}{100 + \frac{1}{s\cdot 2\cdot 10^{-5}}}$$

Therefore, we design



[+1 point]

Simply connecting the two voltage dividers in series would <u>not</u> work [since the second stage would load the first, disrupting the chain rule]. This is why we use a voltage follower in the middle (leveraging the fact that it has ∞-input impedance and 0-output impedance). Our final design is [+1 point]



Part III

Given the decomposition, we also employ voltage division in this design, but this time paired with a non-inverting op-amp.

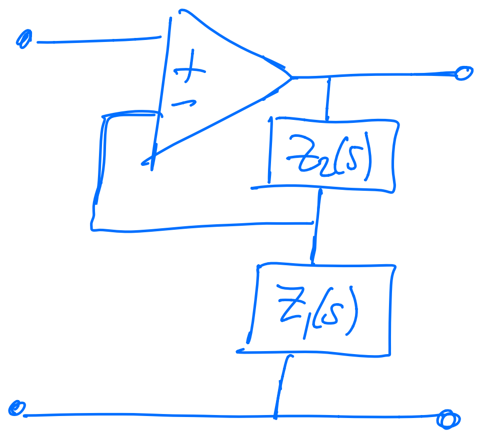
Note that

$$T_1(s) = \frac{20}{20+s} = \frac{\frac{1}{s \cdot 5 \cdot 10^{-5}}}{10^3 + \frac{1}{s \cdot 5 \cdot 10^{-5}}}$$ [+0.5 point]

$$T_2(s) = \frac{s}{s+500} = \frac{100}{100 + \frac{1}{s \cdot 2 \cdot 10^{-5}}}$$

[+0.5 point]

Finally, we can realize $T_3(s) = 25$ with a non-inverting opamp



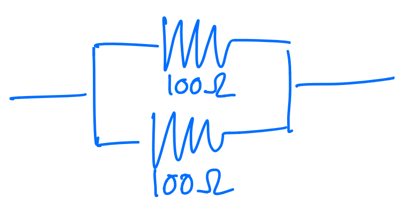
by setting

$$\frac{Z_1(s) + Z_2(s)}{Z_1(s)} = 25 \iff Z_2(s) = 24\, Z_1(s)$$

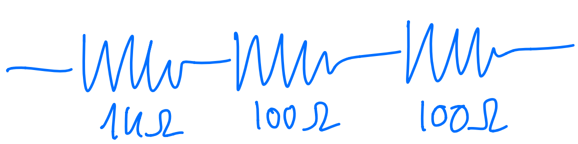
We can make this happen with values

$Z_1(s) = 50$ & $Z_2(s) = 1200$

So we choose



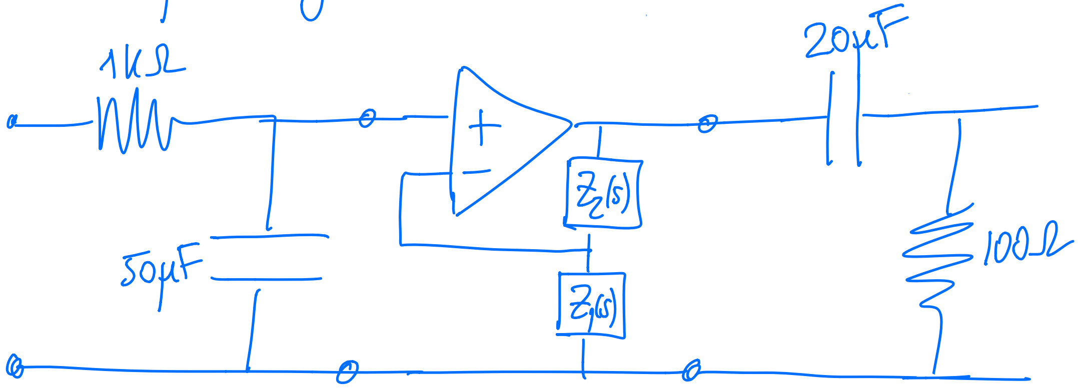
as $Z_1(s)$ and



as $Z_2(s)$

[+1 point]

Consequently, our design is



Again, the ∞-input impedance and 0-output impedance of the op-amp ensures that the chain rule applies. [+1 point]