MAE140 - Linear Circuits - Fall 20
Midterm #1, October 29

Instructions

(i) Prior to the exam, you must have completed the Academic Integrity Pledge at `https://academicintegrity.ucsd.edu/forms/form-pledge.html`

(ii) The exam is open book. You may use your class notes and textbook.

(iii) Collaboration is not permitted. Your answers must be the result only of your own work.

(iv) The exam has 2 questions for a total of 20 points.

(v) You have from 2:00pm to 3:20pm. Please allow sufficient time to post your answers in Canvas.

(vi) If there is any clarification needed for a statement, post your question in the "Discussions" tab of the class Canvas webpage ("Clarifications on question statements of midterm1")

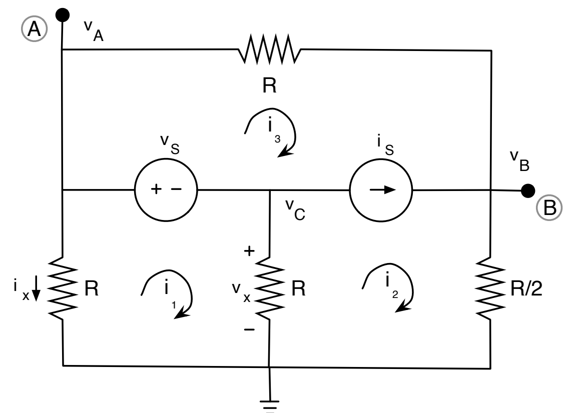


Figure 1: Circuit for all questions.

Good luck!

1. **Circuit analysis**

Part I: [5 points] Formulate node-voltage or mesh-current equations for the circuit in Figure 1. Use the node labels provided in the figure. Clearly indicate the final equations and circuit variable unknowns. The final equations in matrix form must depend only on unknown node-voltages or mesh-currents. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: Part I:
[Solution via node-voltage analysis:] There are four nodes in this circuit. The ground node has already been chosen for us. Unfortunately, with this choice, the ground node is not directly connected to the voltage source, so we cannot use method 2 to take care of it. Since they do not let us redraw the circuit, we have to use a supernode. **(+ 1 point)**
The equation defining the supernode is

$$v_A - v_C = v_s \qquad \textbf{(+ 1 point)}$$

KCL for the supernode takes the form

$$Gv_A + G(v_A - v_B) + Gv_C = -i_s \qquad \textbf{(+ 1 point)}$$

(where we are using the short-hand notation $G = 1/R$).
Finally, we write KCL for node B,

$$G(v_B - v_A) + 2Gv_B = i_s \qquad \textbf{(+ 1 point)}$$

We can write everything together as a system of 3 equations in 3 unknowns v_A, v_B, v_C,

$$\begin{pmatrix} 1 & 0 & -1 \\ 2G & -G & G \\ -G & 3G & 0 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} v_s \\ -i_s \\ i_s \end{pmatrix} \qquad \textbf{(+ 1 point)}$$

[Solution via mesh-current analysis:] There are three meshes in this circuit. The current source belongs to two meshes, instead of one. Without redrawing the circuit, we are forced to use a supermesh.

(+ 1 point)

Consequently, we set

$$i_2 - i_3 = i_s \qquad \textbf{(+ 1 point)}$$

And we write KVL for the supermesh as

$$Ri_3 + \frac{R}{2}i_2 + R(i_2 - i_1) = v_s \qquad \textbf{(+ 1 point)}$$

Finally, we write KVL for mesh 1 to get

$$v_s + R(i_1 - i_2) + Ri_1 = 0 \qquad \textbf{(+ 1 point)}$$

This gives us a total of 3 equations in the 3 mesh current unknowns i_1, i_2, i_3. In matrix form, we can write this as

$$\begin{pmatrix} 0 & 1 & -1 \\ -R & \frac{3R}{2} & R \\ 2R & -R & 0 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} i_s \\ v_s \\ -v_s \end{pmatrix} \qquad \textbf{(+ 1 point)}$$

Part II: [3 points] Provide expressions for the mesh currents i_1, i_2, and i_3 in terms of the node voltages.

Solution: Part II: The mesh currents can be expressed in terms of the node voltages by using Ohm's law. For instance, i_1 is the current passing through the R resistor at the bottom left. Likewise, i_2 is the current passing through the $R/2$ resistor. Finally, i_3 is the current passing through the R resistor at the top. Therefore,

$$i_1 = G(-v_A) \qquad \textbf{(+ 1 point)}$$
$$i_2 = 2Gv_B \qquad \textbf{(+ 1 point)}$$
$$i_3 = G(v_A - v_B) \qquad \textbf{(+ 1 point)}$$

Part III: [2 points] Provide expressions for the voltage v_x and the current i_x in terms of node voltages.

Solution: Part III: In terms of the node voltages, we have

$$v_x = v_C \qquad \textbf{(+ 1 point)}$$
$$i_x = Gv_A \qquad \textbf{(+ 1 point)}$$

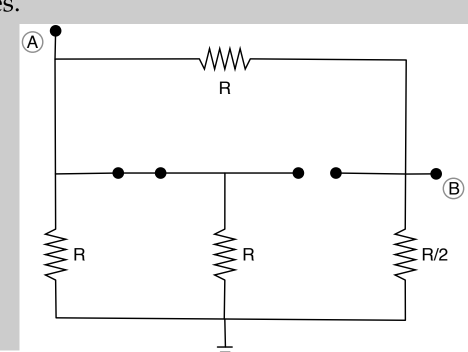
2. **Linearity and Equivalent circuits**

Part I: [2 points] Turn off all the sources in the circuit of Figure 1 and find the equivalent resistance as seen from terminals (A) and (B).

Solution: Part I: We start by switching off the sources.

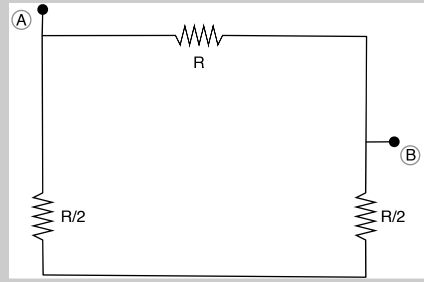
We substitute the voltage source by a short circuit, and the current source by an open circuit. Then, we get the circuit on the right

(+ 0.5 point)



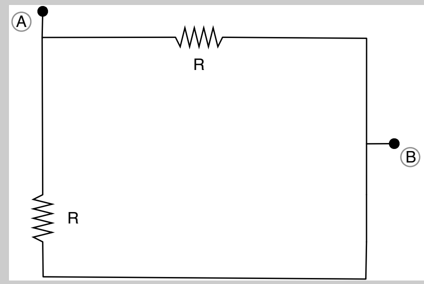
The two R resistors at the bottom left corner are in parallel, so we combine them

(+ 0.5 point)



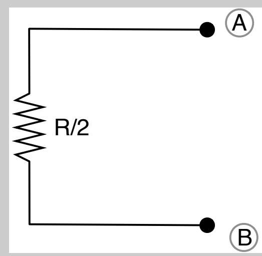
The two $R/2$ resistors are in series, so we combine them too

(+ 0.5 point)



Finally, the two remaining resistors are in parallel, to yield

(+ 0.5 point)

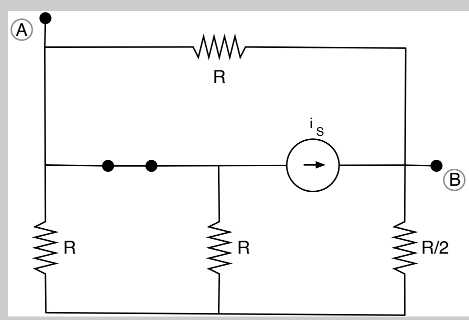


Part II: [3 points] Turn off the voltage source and compute the open-circuit voltage as seen from terminals (A) and (B) using association of resistors and current division.

Solution: Part II:

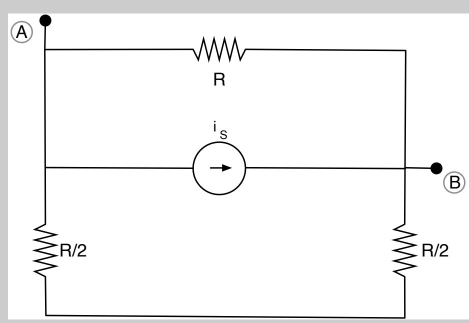
We switch off the voltage source, substituting it by a short circuit. Then, we get the circuit on the right

(+ 1 point)



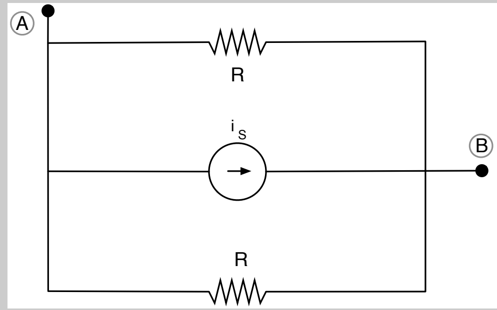
We combine the two R-resistors in parallel to obtain

(+ 0.5 point)



We combine the two $R/2$-resistors in series to obtain

(+ 0.5 point)



Using current division, we determine that

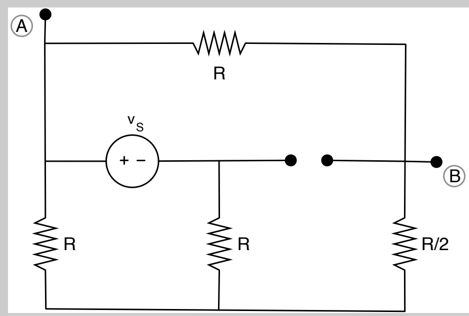
$$v_{AB} = R\left(-\frac{1/R}{1/R+1/R}i_s\right) = -\frac{R}{2}i_s \qquad \textbf{(+ 1 point)}$$

Part III: [3 points] Turn off the current source and compute the open-circuit voltage as seen from terminals (A) and (B) using association of resistors, source transformations, and voltage division.

Solution: Part III:

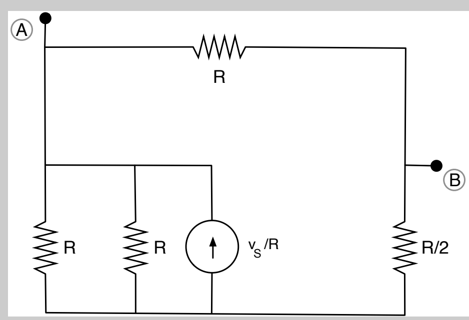
We switch off the current source, substituting it by an open circuit. Then, we get the circuit on the right

(+ 0.5 point)



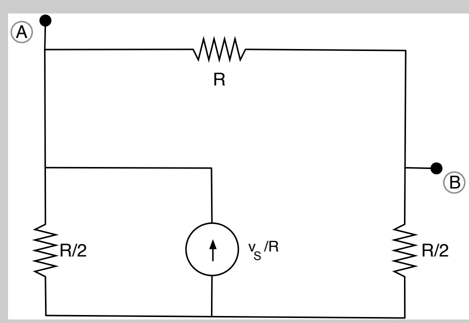
To be able to combine resistors, we first use source transformation to obtain

(+ 0.5 point)



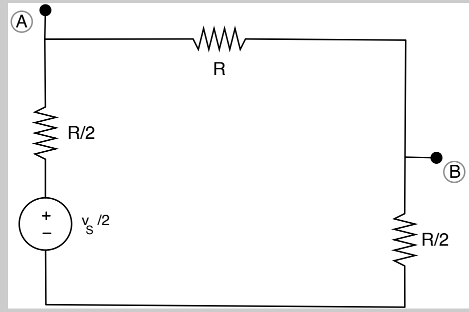
Now we combine the two R-resistors in parallel to obtain

(+ 0.5 point)



Now we use again source transformation to obtain

(+ 0.5 point)



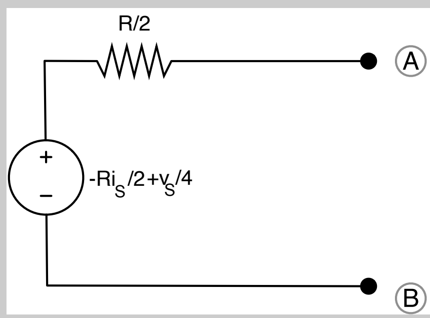
Using voltage division, we determine

$$v_{AB} = \frac{R}{R + R/2 + R/2} \frac{v_s}{2} = \frac{v_s}{4}$$

(+ 1 point)

Part IV: [2 points] Use your answers to Parts I-III to determine the Thévenin equivalent of the circuit as seen from terminals (A) and (B).

Solution: Part IV: We have computed the equivalent resistance from terminals (A) and (B) with all sources turned off in Part I. By superposition, using the answers in Parts II and III, we know that the open-circuit voltage as seen from terminals (A) and (B) is $v_{OC} = -\frac{R}{2} i_s + \frac{v_s}{4}$. Therefore, the Thévenin equivalent of the circuit is simply



(+ 2 points)