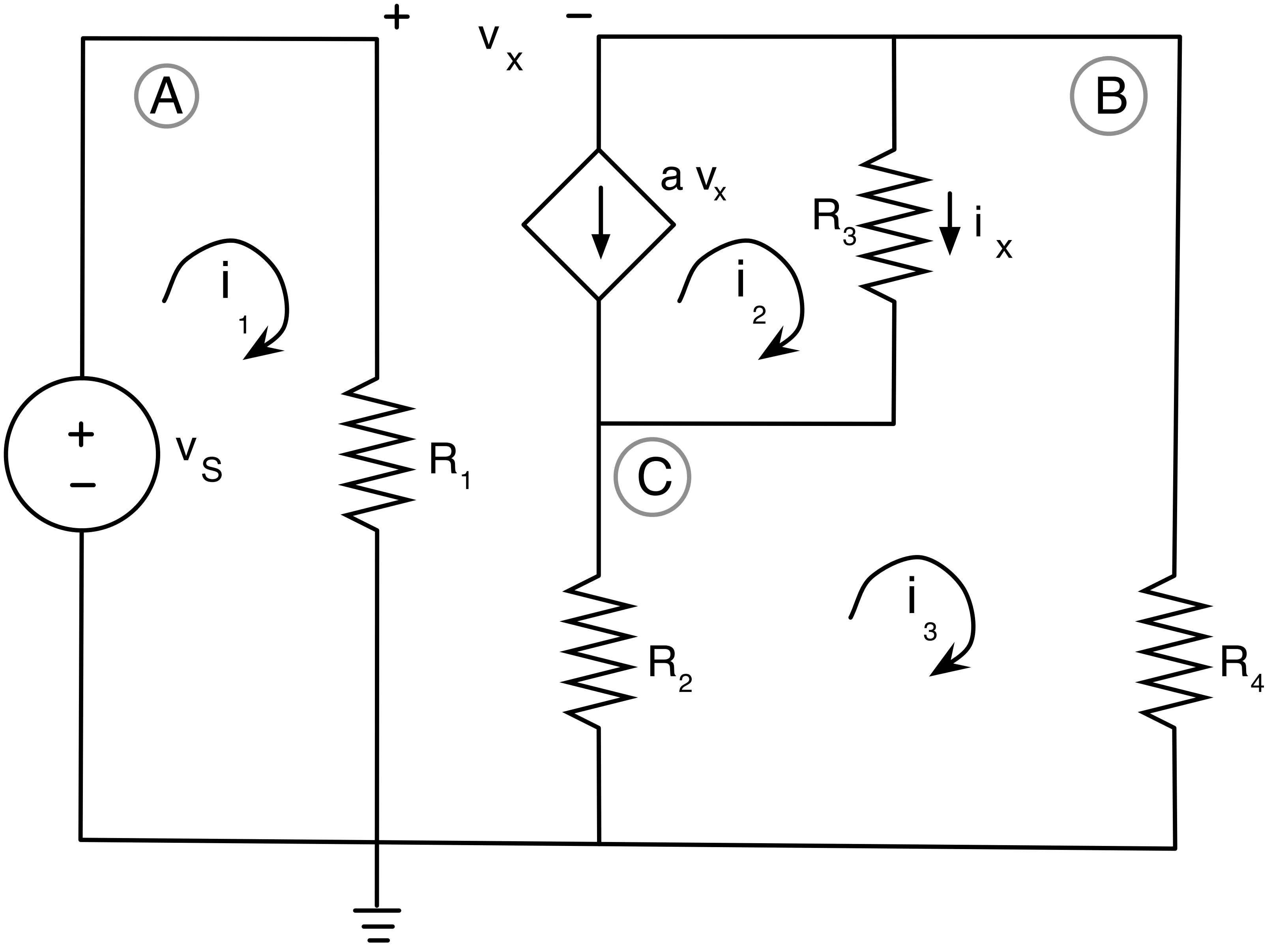


+
v_x
−
A
B
C
$a\,v_x$
R_3
i_x
i_1
i_2
i_3
v_S
R_1
R_2
R_4

1._ Part I.

For node-voltage equations, we know the presence of the voltage source is a problem. However, based on the location of ground, we can use Method 2 to state

$$V_A = V_S$$ [+1.5 point]

Therefore, we just need to write KCL equations for nodes B and C.

KCL@B : $G_3(v_B - v_C) + G_4 v_B + a v_x = 0$ [+1 point]

KCL@ C : $G_2 v_C + G_3 (v_C - v_B) = a v_x$ [+1 point]

(Here, we have used the notation $G_i = \frac{1}{R_i}$ for convenience).

The presence of the dependent source means that we need one more equation. Looking at the circuit, we see that

$$v_x = v_A - v_B$$ [+1.5 point]

Therefore, we have 4 equations in 4 unknowns, v_A, v_B, v_C, v_x. Alternatively, we can also express this as

$$G_3(V_B - V_C) + G_4 V_B + aV_X = 0$$
$$G_2 V_C + G_3(V_C - V_B) - aV_X = 0$$
$$V_X = V_S - V_B$$

that is, 3 equations in 3 unknowns,
V_B, V_C, V_X. [+1 point]

Part II.

By looking at the circuit, we have

$$i_X = G_3(V_B - V_C)$$ [+0.5 point]
$$i_1 = G_1 V_A$$ [+0.5 point]
$$i_2 = -aV_X = -a(V_A - V_B)$$ [+0.5 point]
$$i_3 = G_4 V_B$$ [+0.5 point]

3.- Part I.

We are asked to use two inverting op-amps and one inverting summer to realize

$$V_o = 2V_{in} - 27$$

with the devices in our toolbox. One possibility is to decompose this operation as

$$v_o = -(27 - 2v_{in}) =$$

$$= -(-(-27) - 2v_{in})$$

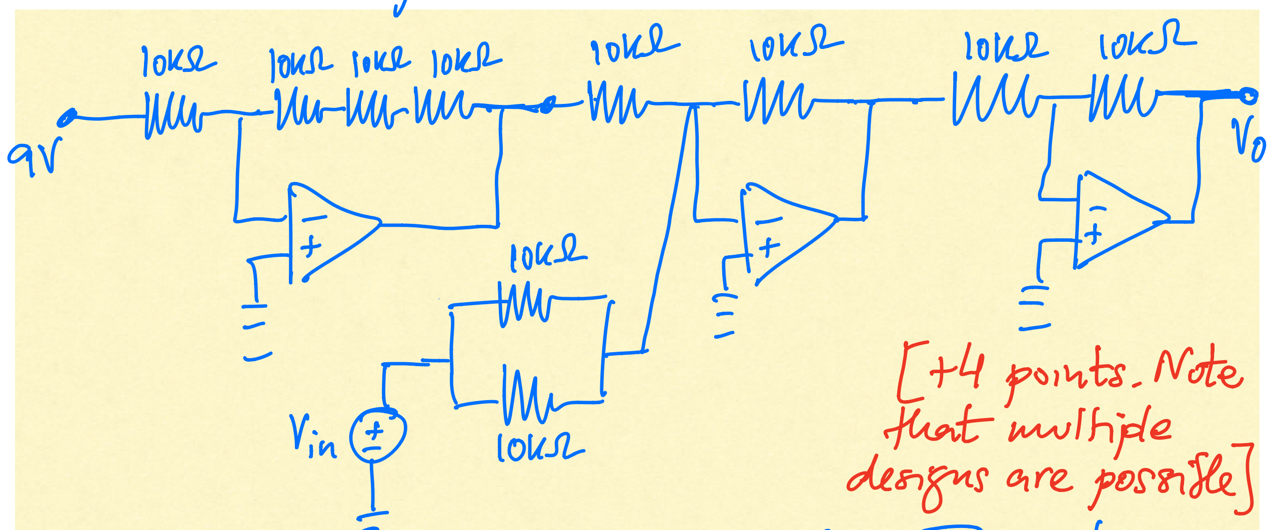
$$= -(-(-3\cdot 9) - 2v_{in})$$

inverting op-amp

inverting summer

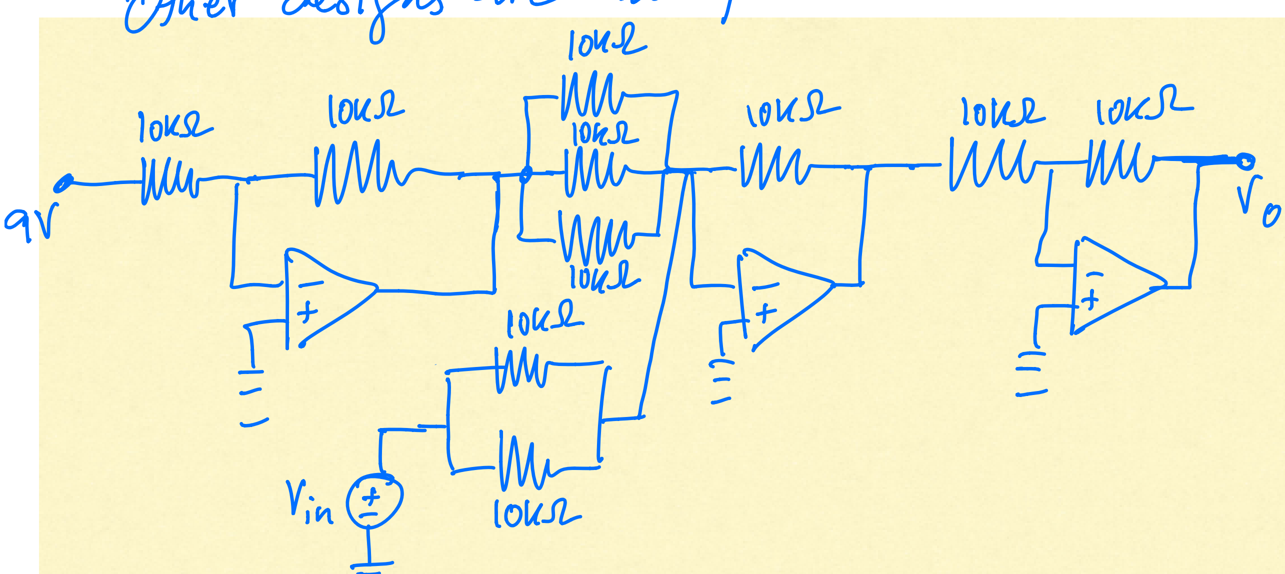
inverting opamp

This corresponds to the following design



[+4 points. Note that multiple designs are possible]

Other designs are also possible. For instance,



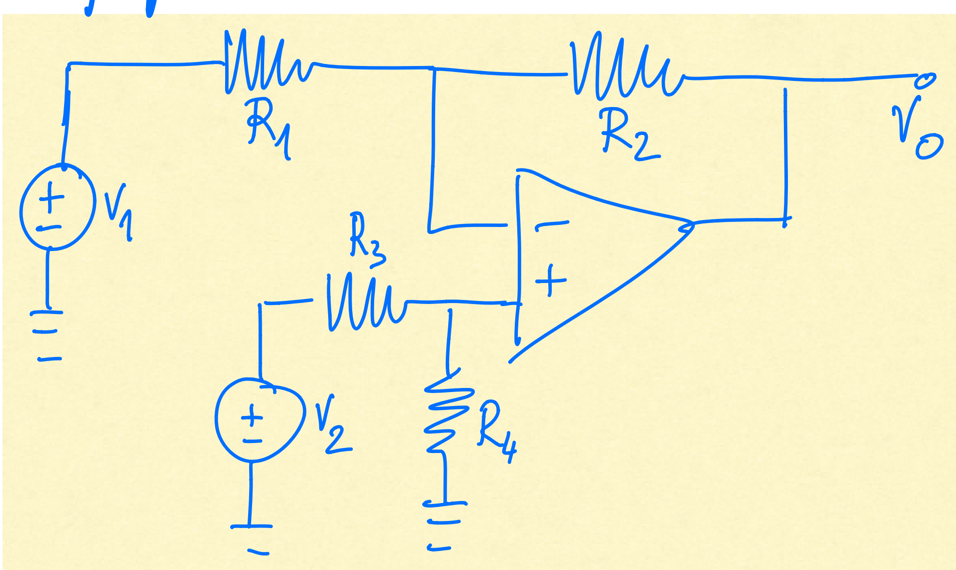
which realizes the operation

$$V_0 = -\left(-3\cdot(-9) - 2V_{in}\right).$$

Part II.

Here we are asked to use only one op-amp. Given the operation we have to realize, the candidate to try is the **differential amplifier.**

[+1 point]



We know that $V_0 = -\frac{R_2}{R_1}V_1 + \frac{R_4}{R_3+R_4}\cdot\frac{R_1+R_2}{R_1}V_2$

If we set $V_1 = 9V$ and $V_2 = V_{in}$, we need

$$\frac{R_2}{R_1} = 3 \qquad \frac{R_4}{R_3+R_4} \cdot \frac{R_1+R_2}{R_1} = 2$$

Using the first equation in the second, this can also be expressed as

$$\frac{R_2}{R_1} = 3 \qquad \frac{R_4}{R_3+R_4} = \frac{1}{2}$$ [+1 point]

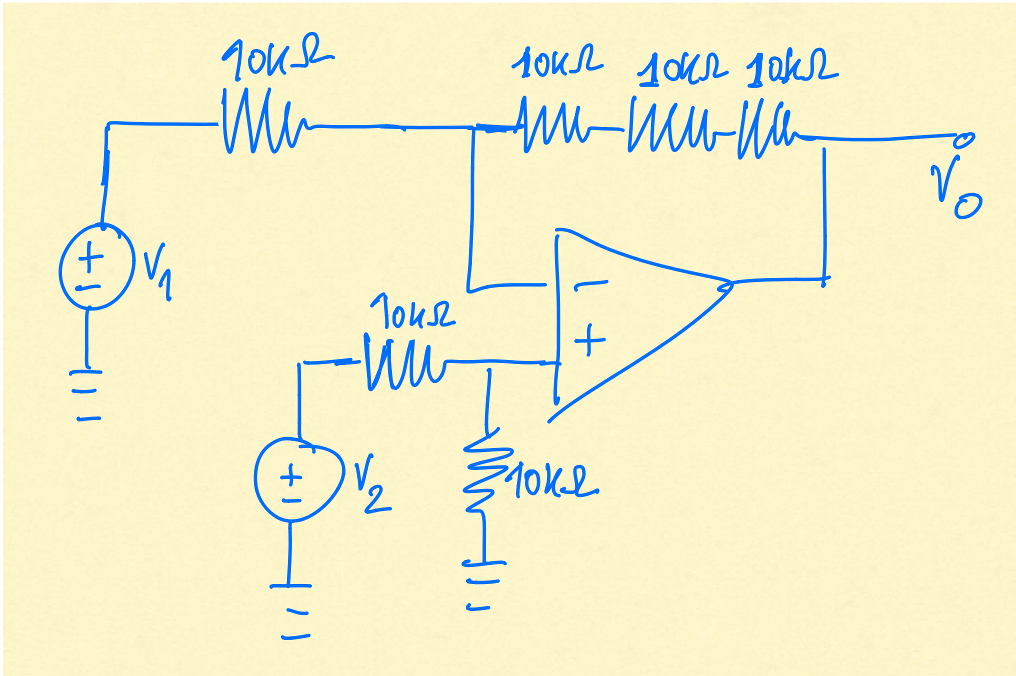
So for instance, we can make this happen with

$R_1 = 10k\Omega$

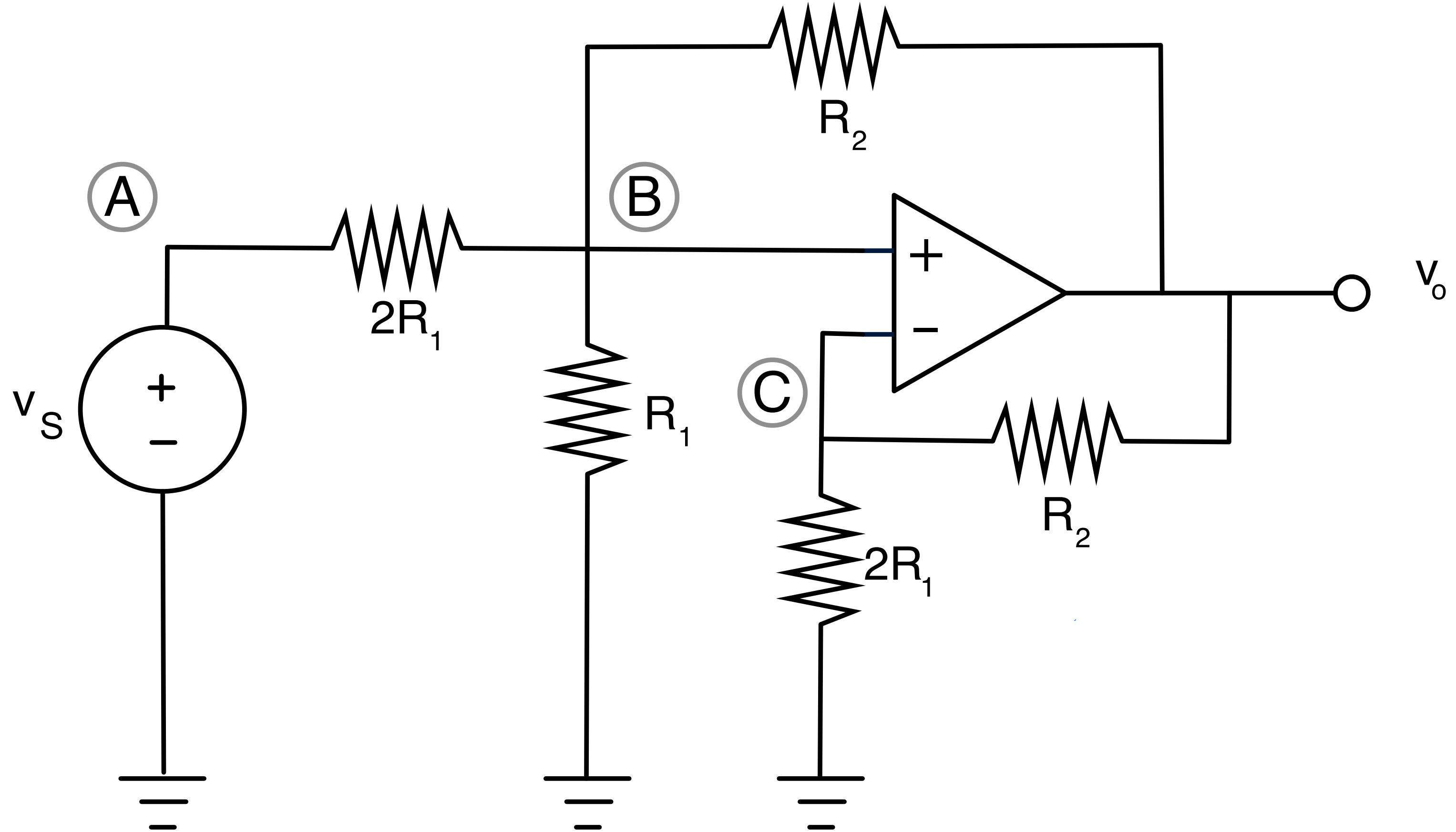
$R_2 = 10 + 10 + 10 = 30k\Omega$

$R_4 = R_3 = 10k\Omega$ [+1 point]

So our design is as follows.



[+1 point]



R2
A
B
2R1
+
−
vo
vS
+
−
R1
C
R2
2R1

2.- Part I.

Labels are already in place for the nodes, so we use them to set up node-voltage eqs.

$$V_A = V_S$$ [+1 point]

KCL@B $$\frac{1}{2R_1}(V_B - V_A) + \frac{1}{R_1}V_B + \frac{1}{R_2}(V_B - V_o) = 0$$ [+1.5 point]

KCL@C $$\frac{1}{2R_1}V_C + \frac{1}{R_2}(V_C - V_0) = 0$$ [+1.5 point]

where we have used $i_p = i_N = 0$ for the input currents to the op-amp. From ideal conditions, we also know

$$V_B = V_C$$ [+1 point]

Next, we solve for V_o to determine which engineer was correct. Substituting,

$$\left(\frac{1}{2R_1} + \frac{1}{R_1} + \frac{1}{R_2}\right)V_B = \frac{1}{2R_1}V_S + \frac{1}{R_2}V_o$$

$$\left(\frac{1}{2R_1} + \frac{1}{R_2}\right)V_B = \frac{1}{R_2}V_o$$

Therefore

$$\frac{1}{R_1} V_B = \frac{1}{2R_1} V_S$$

Hence $V_B = \dfrac{V_S}{2}$

Finally

$$V_0 = R_2 \cdot \frac{R_2 + 2R_1}{2R_1 R_2} \cdot \frac{V_S}{2} =$$

$$= \frac{R_2 + 2R_1}{4R_1} V_S$$

[+1 point]

So the second engineer was correct.

Part II.

With the known resistor values, we have

$$V_0 = \frac{20+10}{40} V_S = \frac{3}{4} V_S$$

The op-amp does not saturate if

$$-V_{CC} \leq V_0 \leq V_{CC}$$

[+1 point]

Therefore

$$-6 \le \frac{3}{4} v_s \le 6$$

Hence

$$-8 \le v_s \le 8.$$

[+1 point]