

Active Circuits: Life gets interesting

Active cct elements – operational amplifiers (OP-AMPS) and transistors

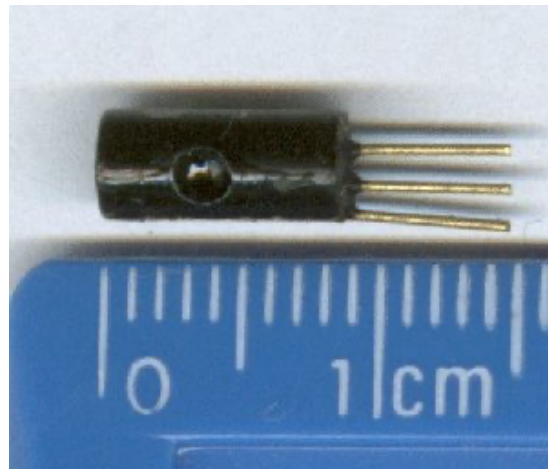
Devices which can inject power into the cct

External power supply – normally comes from connection to the voltage supply “rails”

Capable of linear operation – amplifiers

and nonlinear operation – typically switches

Triodes, pentodes, transistors



Active Cct Elements

Amplifiers – linear & active

Signal processors

Stymied until 1927 and Harold Black

Negative Feedback Amplifier

Control rescues communications

Telephone relay stations manageable
against manufacturing variability

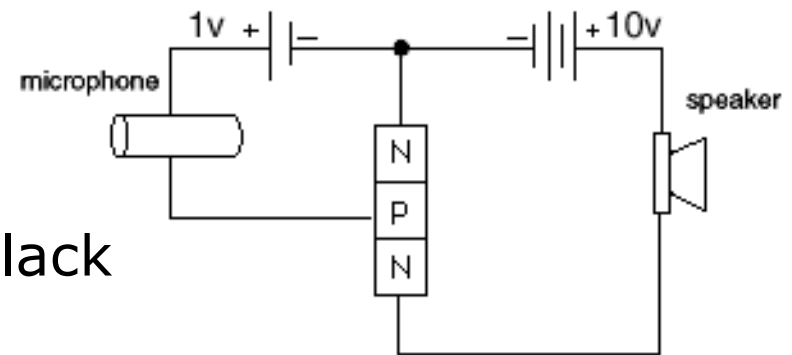
Linearity

Output signal is proportional to the input signal

Note distinction between signals and systems which
transform them

Yes! Just like your stereo amplifier

Idea – controlled current and voltage sources



A Brief Aside - Transistors

Bipolar Junction Transistors

Semiconductors – doped silicon

n-doping: mobile electrons

Si doped with Sb, P or As

p-doping: mobile holes

Si doped with B, Ga, In

Two types npn and pnp

Heavily doped Collector and Emitter

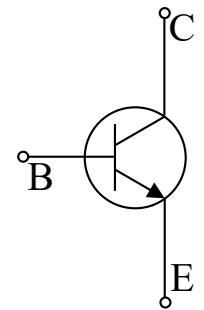
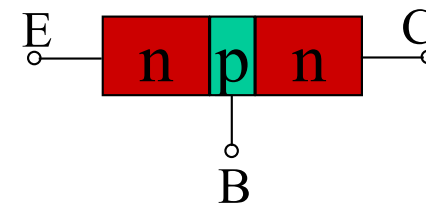
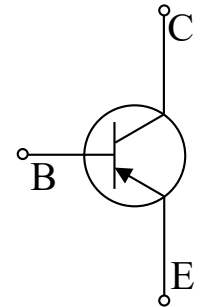
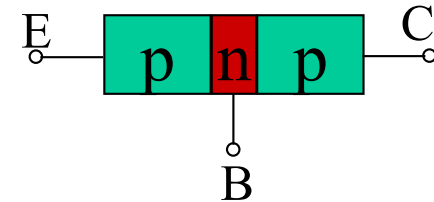
Lightly doped Base and very thin

Collector and Emitter thick and dopey

Need to bias the two junctions properly

Then the base current modulates a strong C→E current

$$\text{Amplification } i_C = \beta i_B$$



Transistors

Common Emitter Amplifier Stage

Biasing resistors R_1 and R_2

Keep transistor junctions biased in amplifying range

Blocking capacitors C_{B1} and C_{B2}

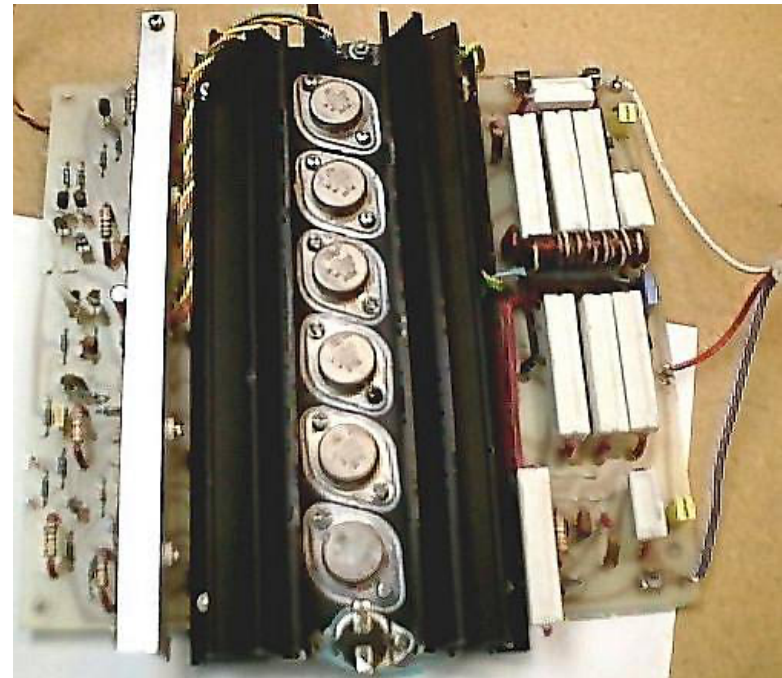
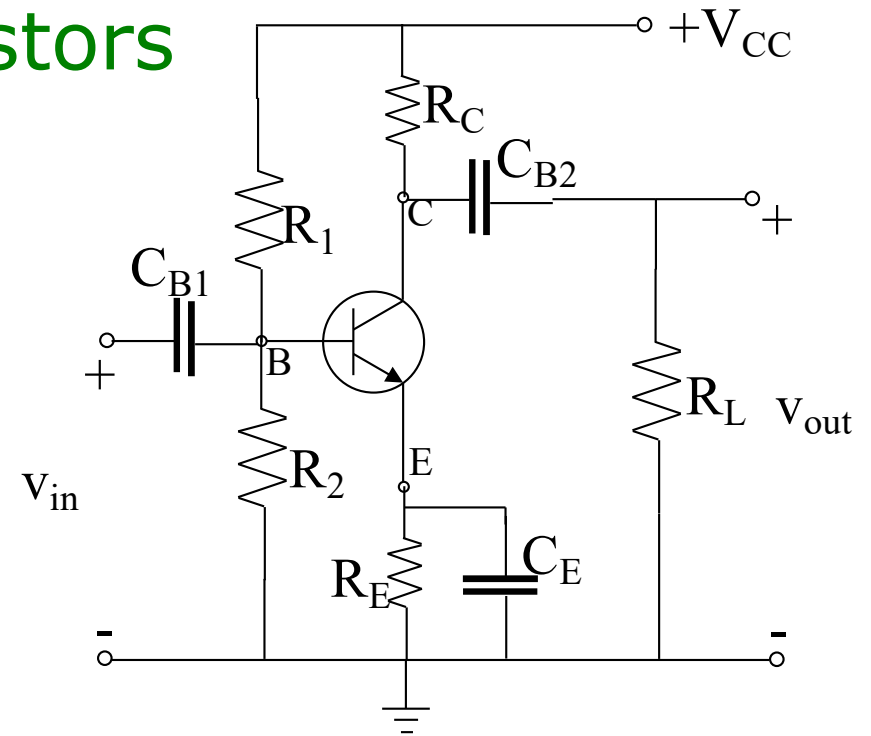
Keep dc currents out

Feedback capacitor C_E

Grounds emitter at high frequencies

Small changes in v_{in}

Produce large changes in v_{out}

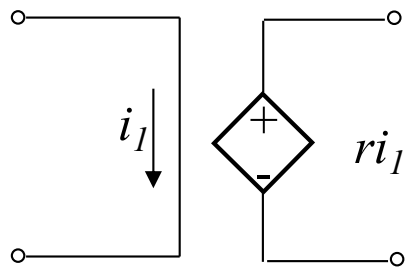


Linear Dependent Sources

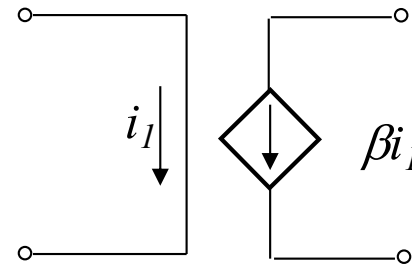
Active device models in linear mode

Transistor takes an input voltage v_i and produces an output current $i_o = gv_i$ where g is the gain

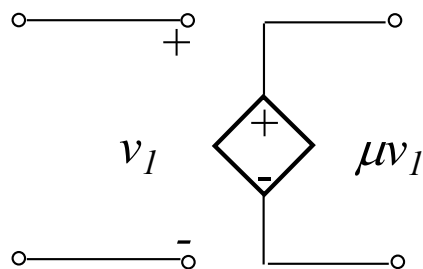
This is a linear voltage-controlled current source VCCS



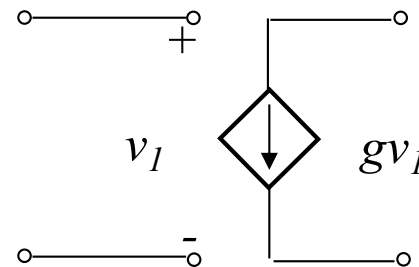
CCVS r transresistance



CCCS β current gain



VCVS μ voltage gain



VCCS g transconductance

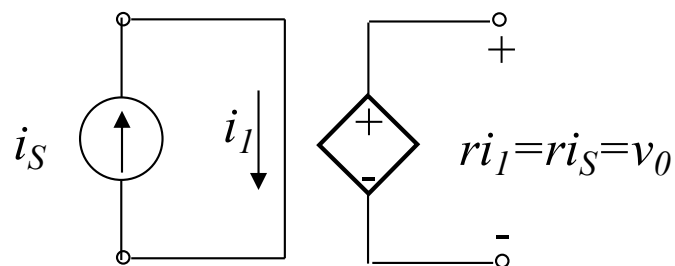
Linear dependent source (contd)

Linear dependent sources are parts of active cct models – they are not separate components

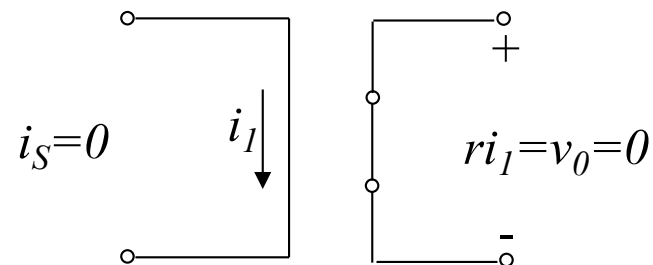
But they allow us to extend our cct analysis techniques to really useful applications

This will become more critical as we get into dynamic ccts

Dependent elements change properties according to the values of other cct variables



Source on

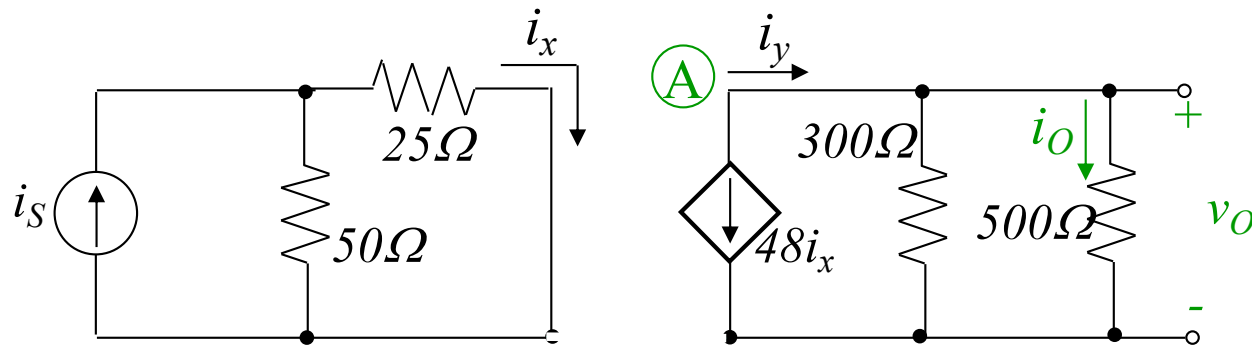


Source off

Cct Analysis with Dependent Sources

Golden rule – do not lose track of control variables

Find i_O , v_O and P_O for the 500Ω load



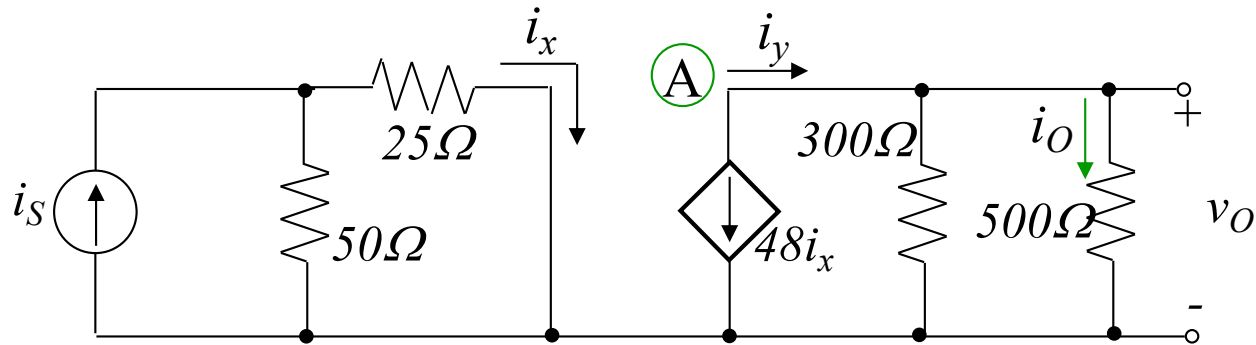
Current divider on LHS $i_x = \frac{2}{3}i_S$

Current divider on RHS $i_O = \frac{3}{8}(-48)i_x = -18i_x = -12i_S$

Ohm's law $v_O = i_O 500 = -6000i_S$

Power $p_O = i_O v_O = 72,000i_S^2$

Analysis with dependent sources



Power provided by ICS

$$p_S = (50 \parallel 25) i_S^2 = \frac{50}{3} i_S^2$$

Power delivered to load

$$72000 i_S^2$$

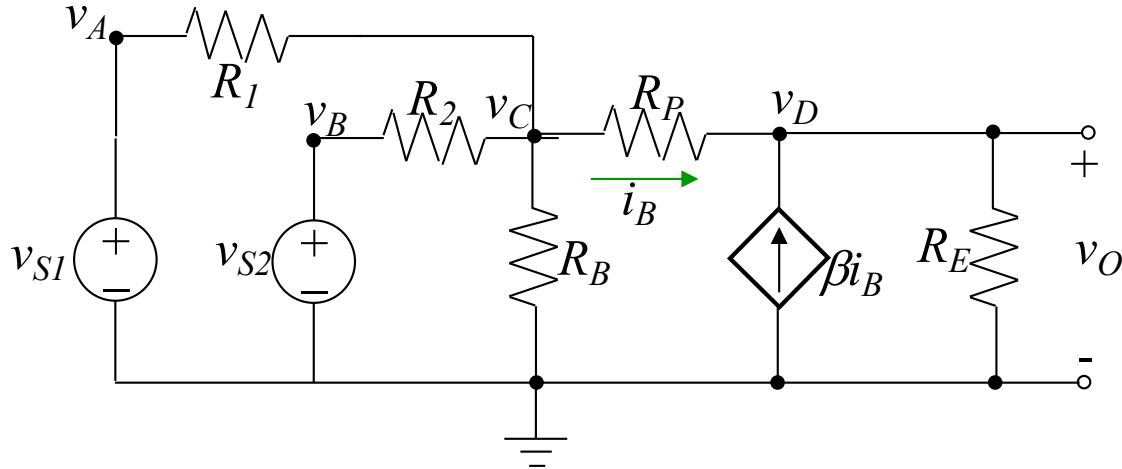
Power gain

$$G = \frac{p_O}{p_S} = \frac{72000 i_S^2}{\frac{50}{3} i_S^2} = 4320$$

Where did the energy come from?

External power supply

Nodal Analysis with Dependent Source



KCL at node C $G_1(v_C - v_{S1}) + G_2(v_C - v_{S2}) + G_B v_C + G_P(v_C - v_D) = 0$

KCL at node D $G_P(v_D - v_C) + G_E v_D - \beta i_B = 0$

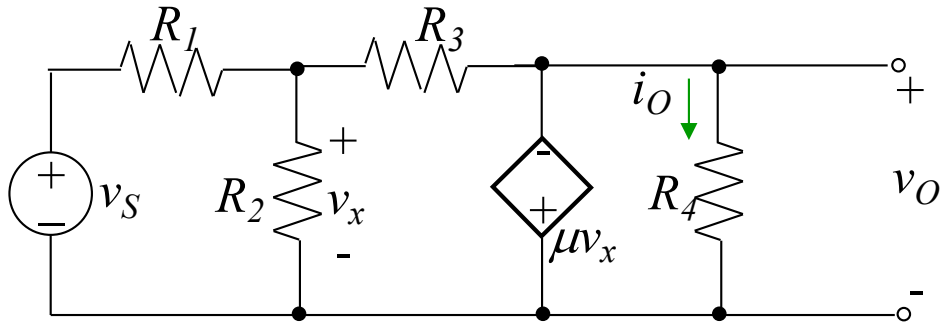
CCCS element description $i_B = G_P(v_C - v_D)$

Substitute and solve

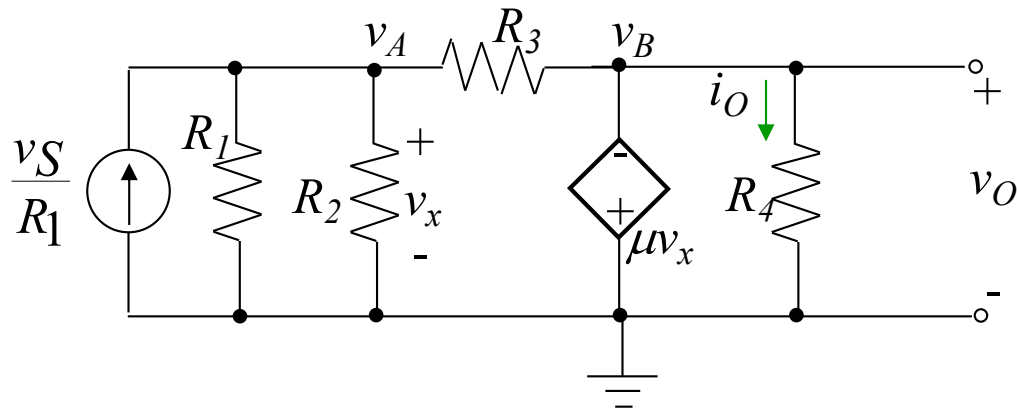
$$(G_1 + G_2 + G_B + G_P)v_C - G_P v_D = G_1 v_{S1} + G_2 v_{S2}$$

$$-(\beta + 1)G_P v_C + [(\beta + 1)G_P + G_E]v_D = 0$$

T&R, 5th ed, Example 4-3 p 148



Find v_O in terms of v_S
What happens as $\mu \rightarrow \infty$?



Node A:

$$(G_1 + G_2 + G_3)v_A - G_3v_B = G_1v_S$$

Node B:

$$v_B = -\mu v_x = -\mu v_A$$

Solution:

$$v_O = v_B = -\mu v_A = \left(\frac{-\mu G_1}{G_1 + G_2 + (1 + \mu)G_3} \right) v_S$$

For large gains μ : $(1 + \mu)G_3 \gg G_1 + G_2$

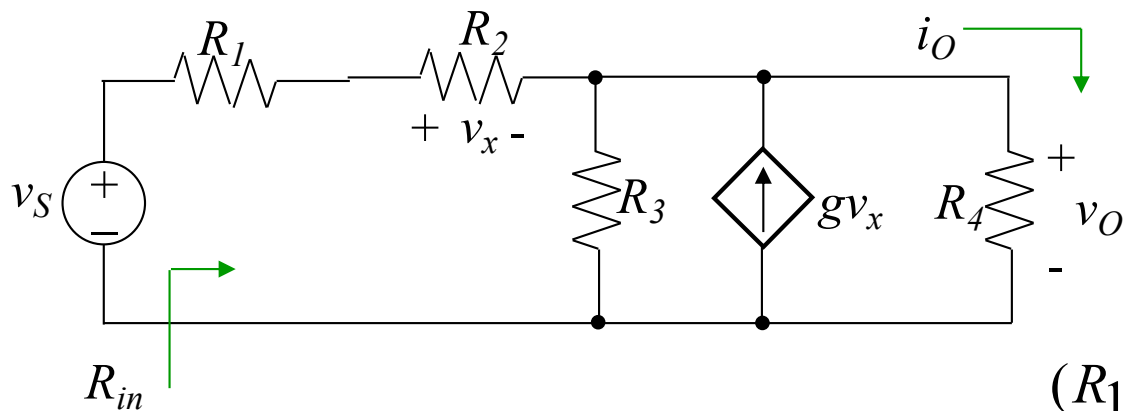
$$v_O \approx \left[\frac{-\mu G_1}{(1 + \mu)G_3} \right] v_S \approx -\frac{R_3}{R_1} v_S$$

This is a model of an inverting op-amp

Mesh Current Analysis with Dependent Sources

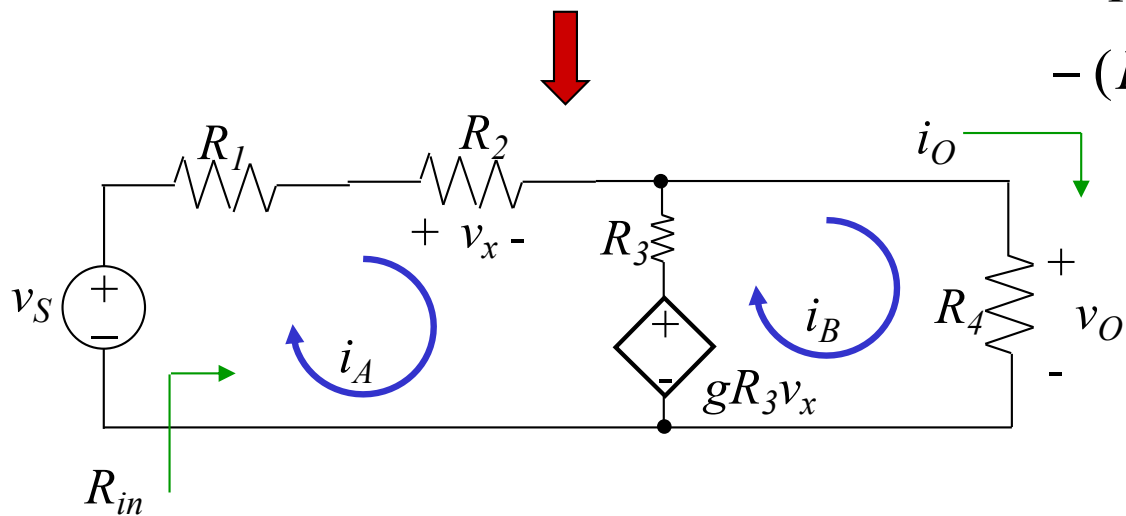
Dual of Nodal Analysis with dependent sources

Treat the dependent sources as independent and sort out during the solution



$$(R_1 + R_2 + R_3 + gR_2R_3)i_A - R_3i_B = v_S$$

$$-(R_3 + gR_2R_3)i_A + (R_3 + R_4)i_B = 0$$

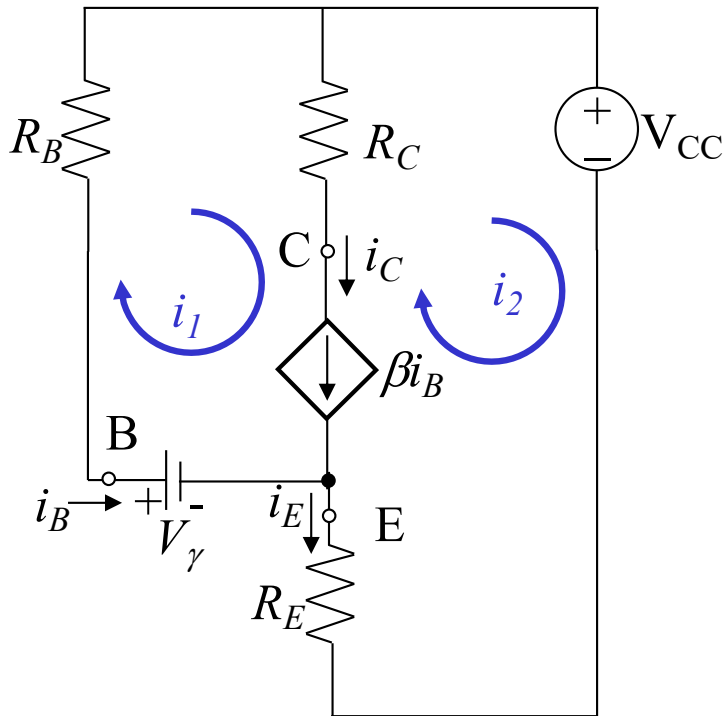


T&R, 5th ed, Example 4-5 BJT transistor

Needs a supermesh

Current source in two loops
without R in parallel

Supermesh = entire outer loop



Supermesh equation

$$i_2 R_E - V_\gamma + i_1 R_B + V_{CC} = 0$$

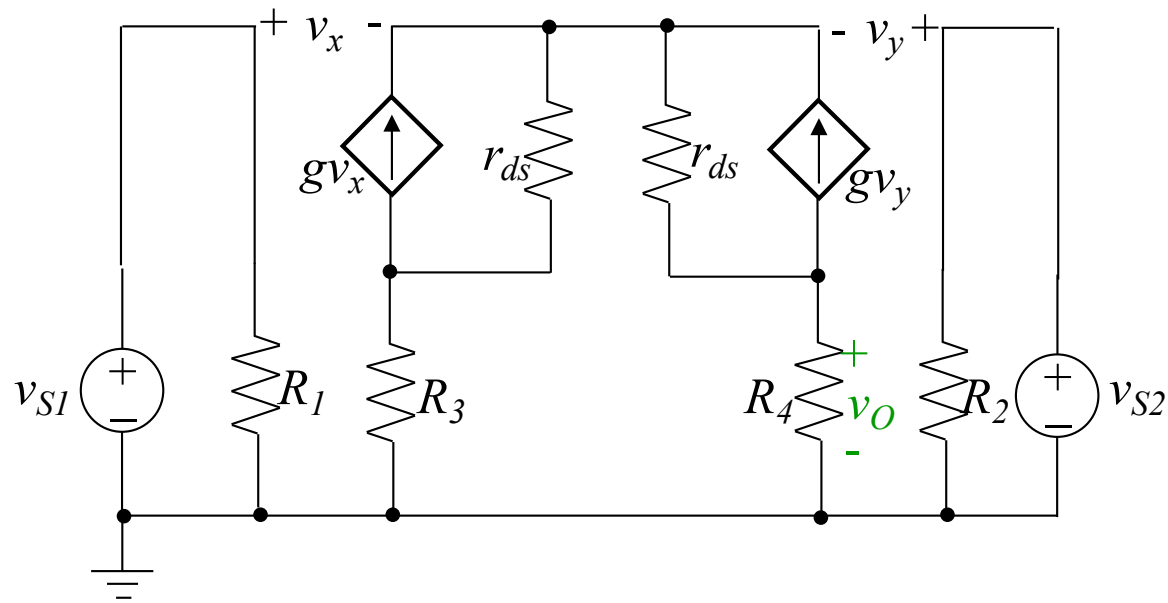
Current source constraint

$$i_1 - i_2 = \beta i_B$$

Solution

$$i_B = -i_1 = \frac{V_{CC} - V_\gamma}{R_B + (\beta + 1)R_E}$$

T&R, 5th ed, Example 4-6 Field Effect Transistor



Since cct is linear $v_O = K_1 v_{S1} + K_2 v_{S2}$

Solve via superposition

First v_{S1} on and v_{S2} off, then v_{S1} off and v_{S2} on

This gives K_1 and K_2

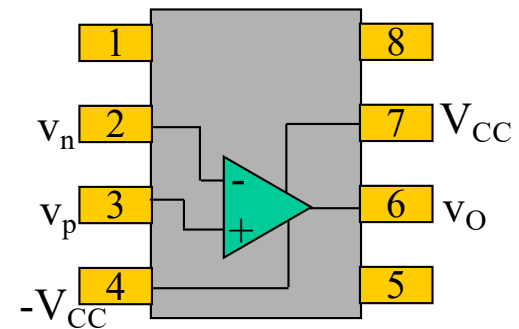
Operational Amplifiers - OpAmps

Basic building block of linear analog circuits

Package of transistors, capacitors, resistors, diodes in a chip

Five terminals

- Positive power supply V_{CC}
- Negative power supply $-V_{CC}$
- Non-inverting input v_p
- Inverting input v_n
- Output v_o



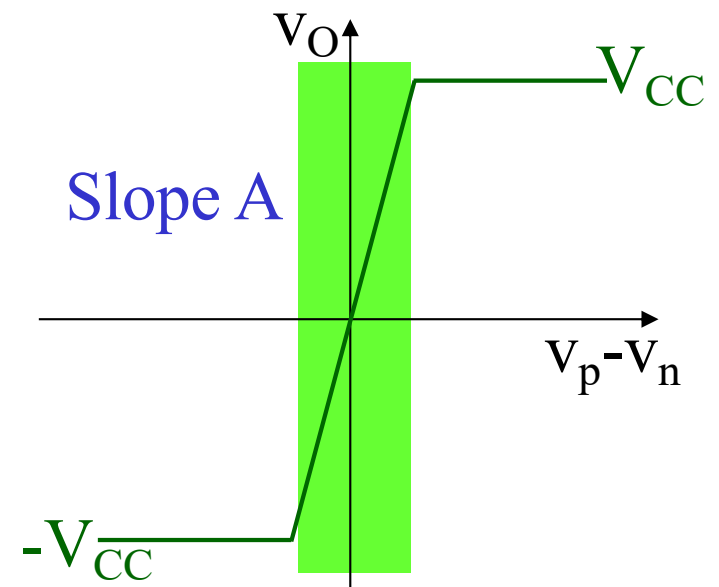
Linear region of operation

$$v_o = A(v_p - v_n)$$

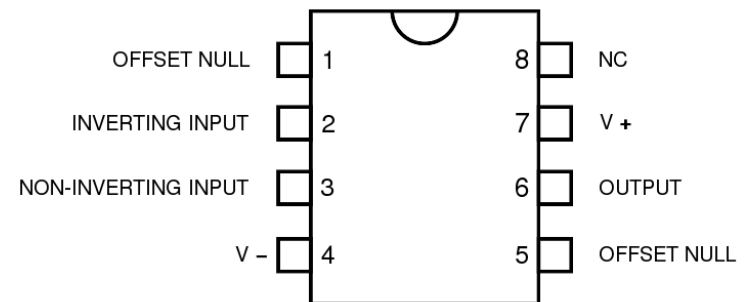
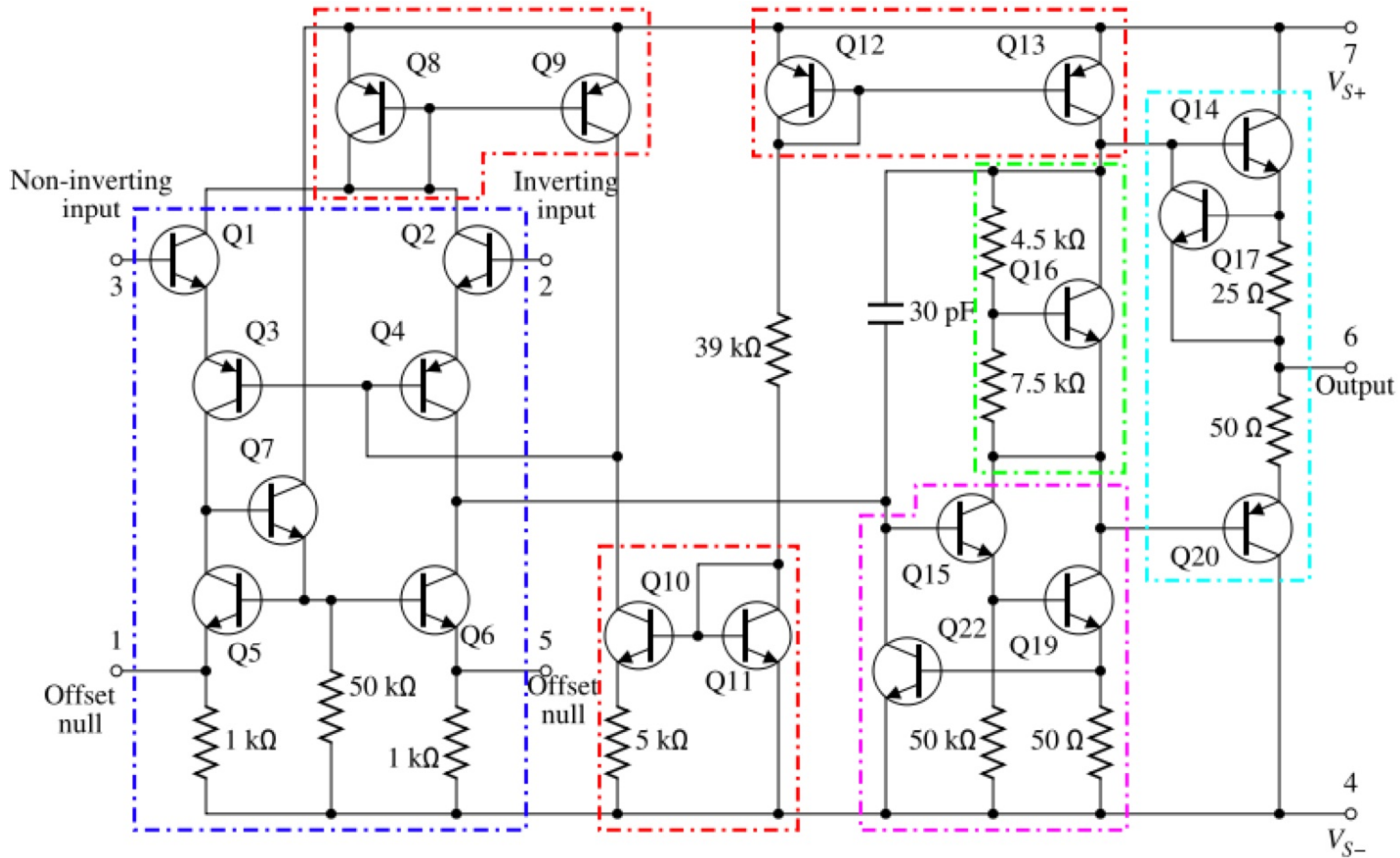
Ideal behavior

$$10^5 < A < 10^8$$

Saturation at $V_{CC}/-V_{CC}$ limits range



Real OpAmp (u741)



Ideal OpAmp

Equivalent linear circuit

Dependent source model

$$10^6 < R_I < 10^{12} \Omega \quad \infty \Omega$$

$$10 < R_O < 100 \Omega \quad 0 \Omega$$

$$10^5 < A < 10^8 \quad \infty$$

Need to stay in linear range

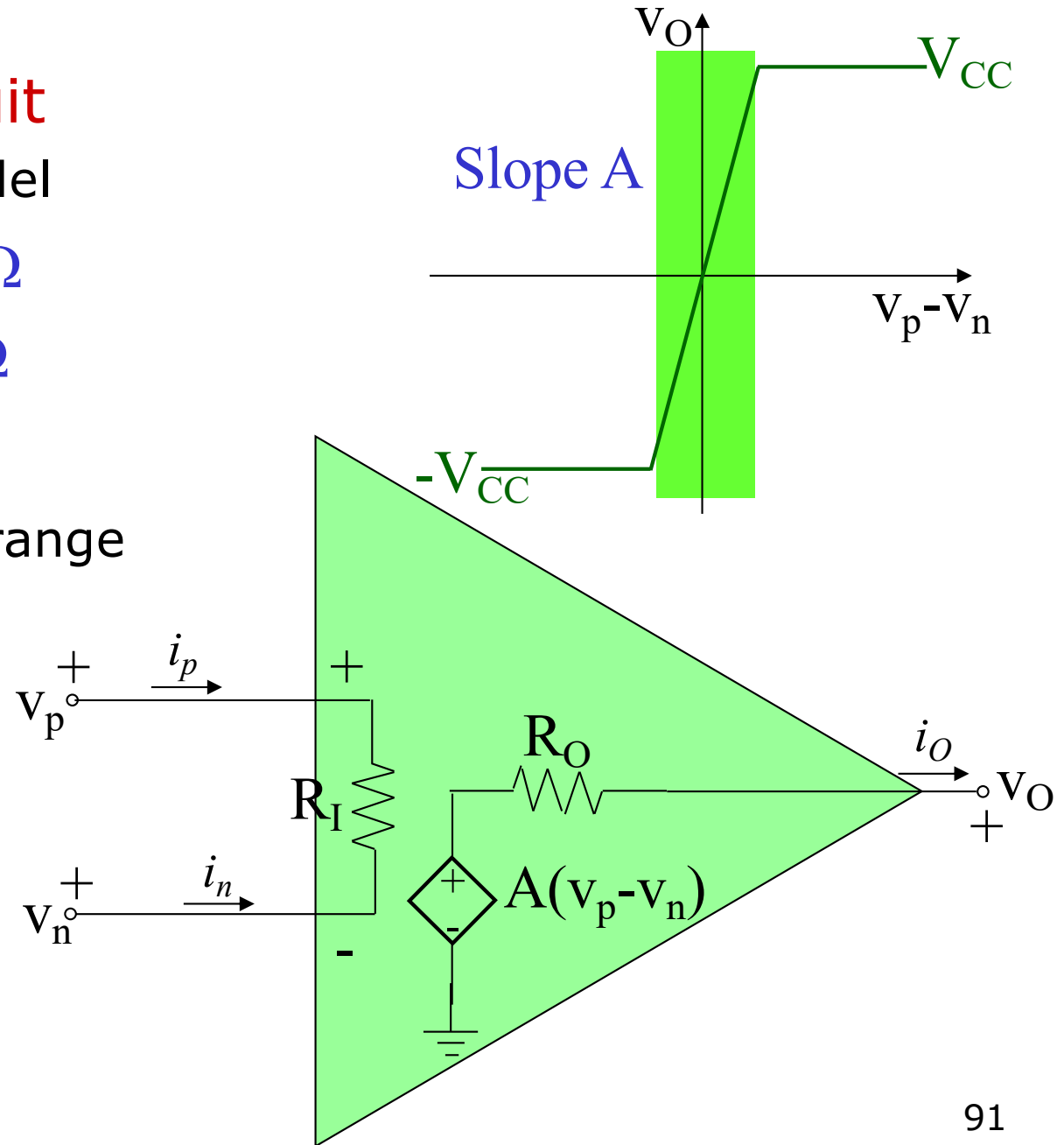
$$-V_{CC} \leq v_O \leq V_{CC}$$

$$-\frac{V_{CC}}{A} \leq v_p - v_n \leq \frac{V_{CC}}{A}$$

Ideal conditions

$$v_p = v_n$$

$$i_p = i_n = 0$$



Non-inverting OpAmp - Feedback

What happens now?

Voltage divider feedback

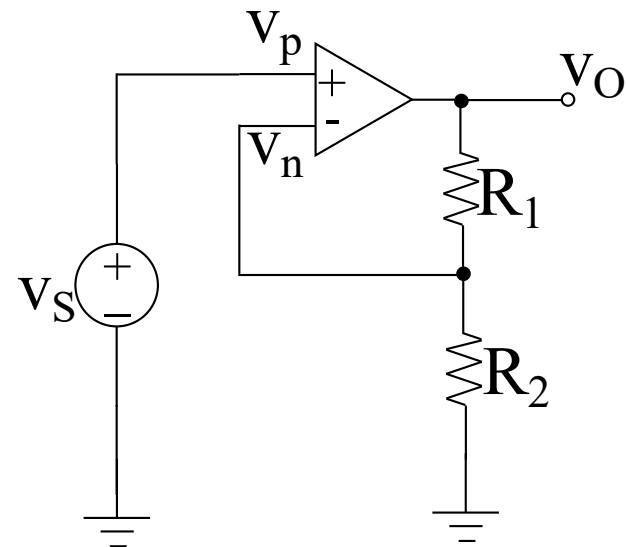
$$v_n = \frac{R_2}{R_1 + R_2} v_O$$

Operating condition $v_p = v_S$

$$v_O = \frac{R_1 + R_2}{R_2} v_S$$

Linear non-inverting amplifier

$$\text{Gain } K = \frac{R_1 + R_2}{R_2}$$



With dependent source model

$$v_O = \frac{R_I A (R_1 + R_2) + R_2 R_O}{R_I (A R_2 + R_O + R_1 + R_2) + R_2 (R_1 + R_O)} v_S$$

T&R, 5th ed, Example 4-13

Analyze this

$$i_p = 0$$

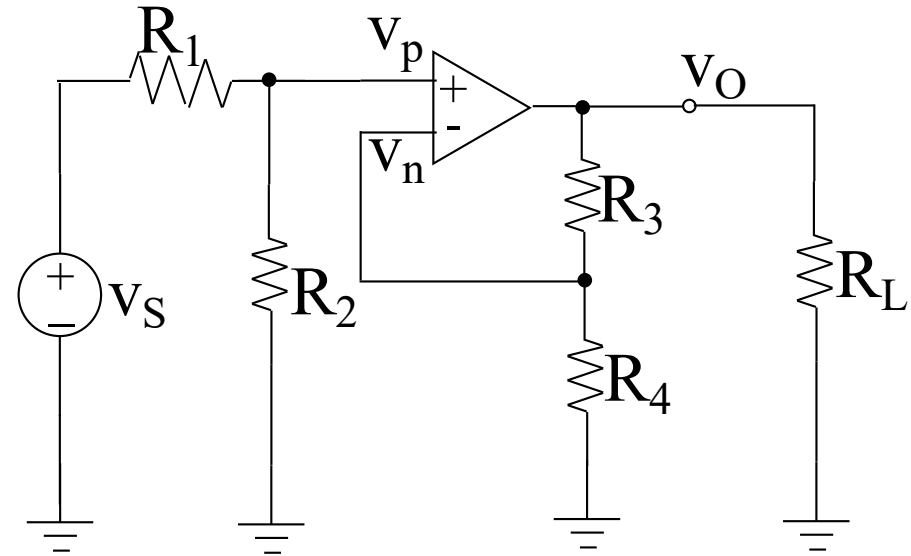
$$K_S = \frac{v_p}{v_S} = \frac{R_2}{R_1 + R_2}$$

Ideal OpAmp has zero output resistance

R_L does not affect v_O

$$K_{AMP} = \frac{v_O}{v_p} = \frac{R_3 + R_4}{R_4}$$

$$K_{Total} = K_S K_{AMP} = \frac{v_O}{v_S} = \left[\frac{R_2}{R_1 + R_2} \right] \left[\frac{R_3 + R_4}{R_4} \right]$$



Voltage Follower - Buffer

Feedback path

$$v_n = v_o$$

Infinite input resistance

$$i_p = 0, \quad v_p = v_s$$

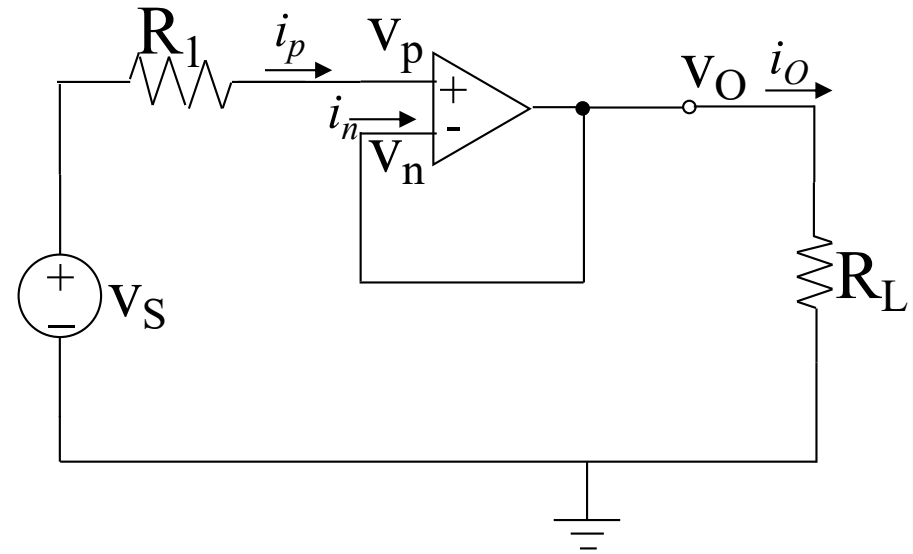
Ideal OpAmp

$$v_p = v_n$$

$$v_o = v_s \quad i_o = \frac{v_o}{R_L}$$

Loop gain is 1

Power is supplied from the Vcc/-Vcc rails



OpAmp Ccts – inverting amplifier

Input and feedback applied at same terminal of OpAmp

R_2 is the feedback resistor

So how does it work?

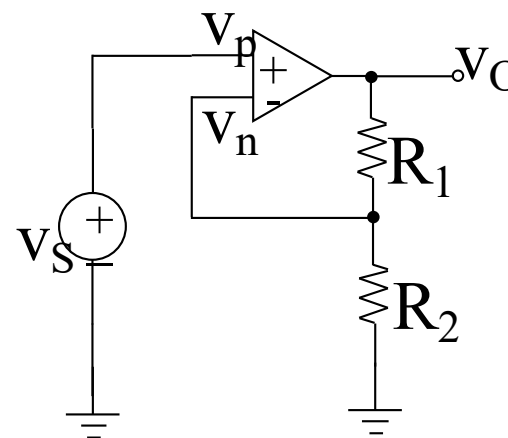
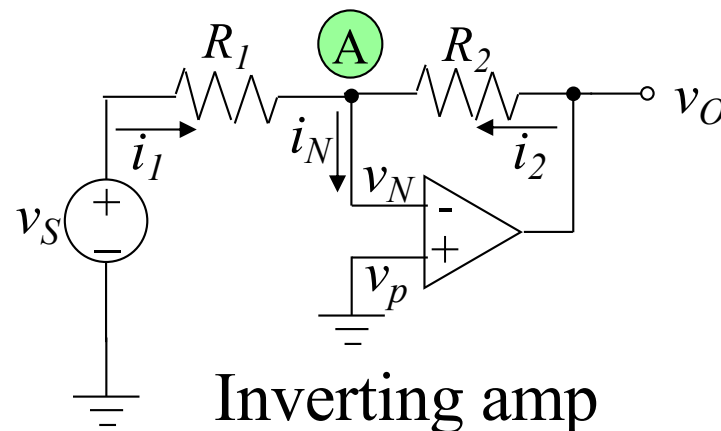
KCL at node A

$$\frac{v_N - v_S}{R_1} + \frac{v_N - v_O}{R_2} + i_N = 0$$

$$i_N = 0, v_N = v_p = 0$$

$$v_O = -\frac{R_2}{R_1} v_S$$

$v_O = -K v_S$ hence the name



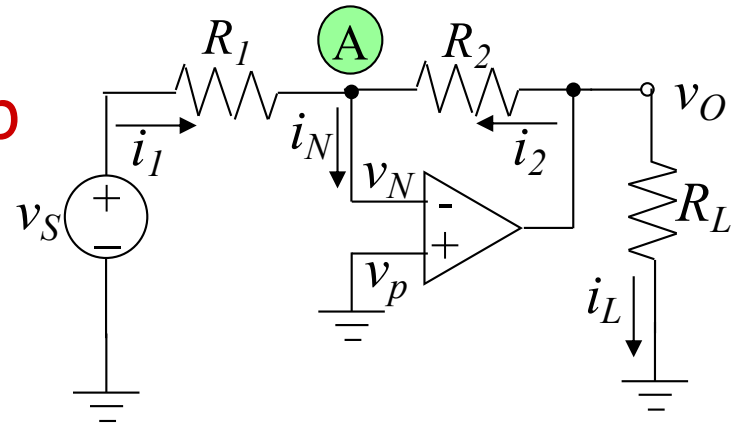
Inverting Amplifier (contd)

Current flows in the inverting amp

$$i_1 = \frac{v_S}{R_1}, \quad R_{in} = R_1$$

$$i_2 = \frac{v_O}{R_2} = \frac{-v_S}{R_1} = -i_1$$

$$i_L = \frac{v_O}{R_L} = -\frac{R_2}{R_1} \times \frac{1}{R_L} \times v_S$$



OpAmp Analysis – T&R, 5th ed, Example 4-14

Compute the input-output relationship of this cct

Convert the cct left of the node A to its Thévenin equivalent

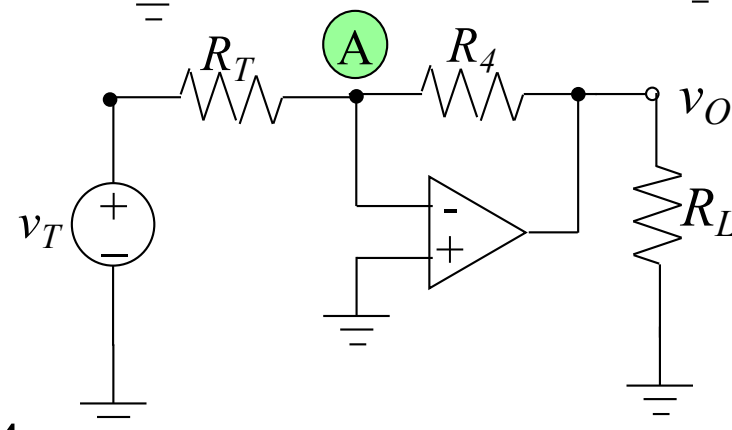
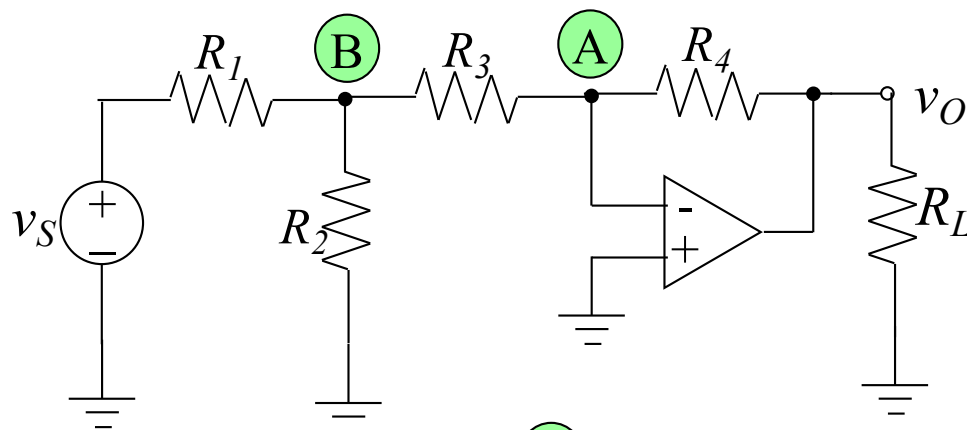
$$v_T = v_{OC} = \frac{R_2}{R_1 + R_2} v_S$$

$$R_T = R_{in} = R_3 + \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1 + R_2}$$

Note that this is not the inverting amp gain times the voltage divider gain

There is interaction between the two parts of the cct (R_3)

This is a feature of the inverting amplifier configuration



$$\begin{aligned} v_O &= -\frac{R_4}{R_T} v_T \\ &= -\left[\frac{R_4 (R_1 + R_2)}{R_1 R_2 + R_1 R_3 + R_2 R_3} \right] \left[\frac{R_2}{R_1 + R_2} \right] v_S \\ &= -\frac{R_2 R_4}{R_1 R_2 + R_1 R_3 + R_2 R_3} v_S \end{aligned}$$

Summing Amplifier - Adder



So what happens?

Node A is effectively grounded

$$v_n = v_p = 0$$

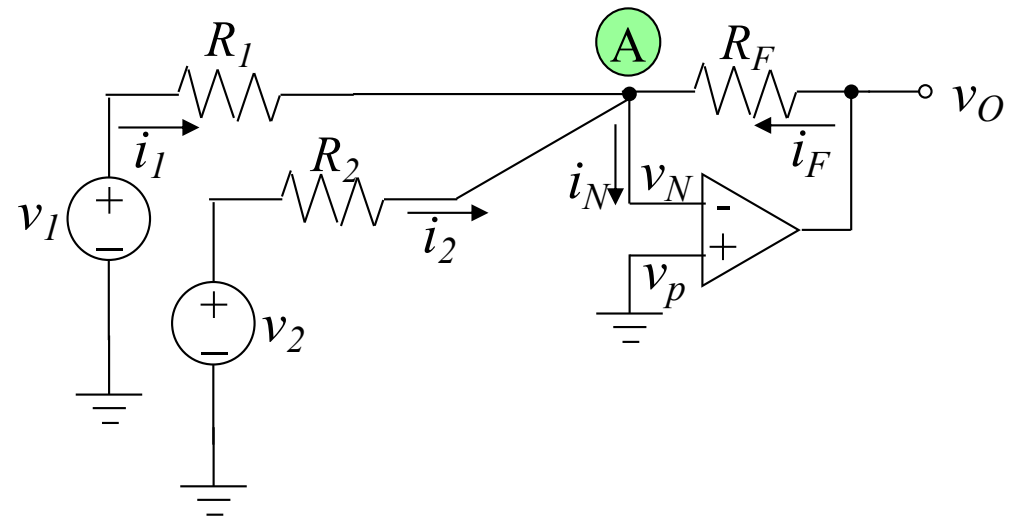
Also $i_N = 0$ because of R_{in}

So

$$i_1 + i_2 + i_F = 0$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_O}{R_F} = 0$$

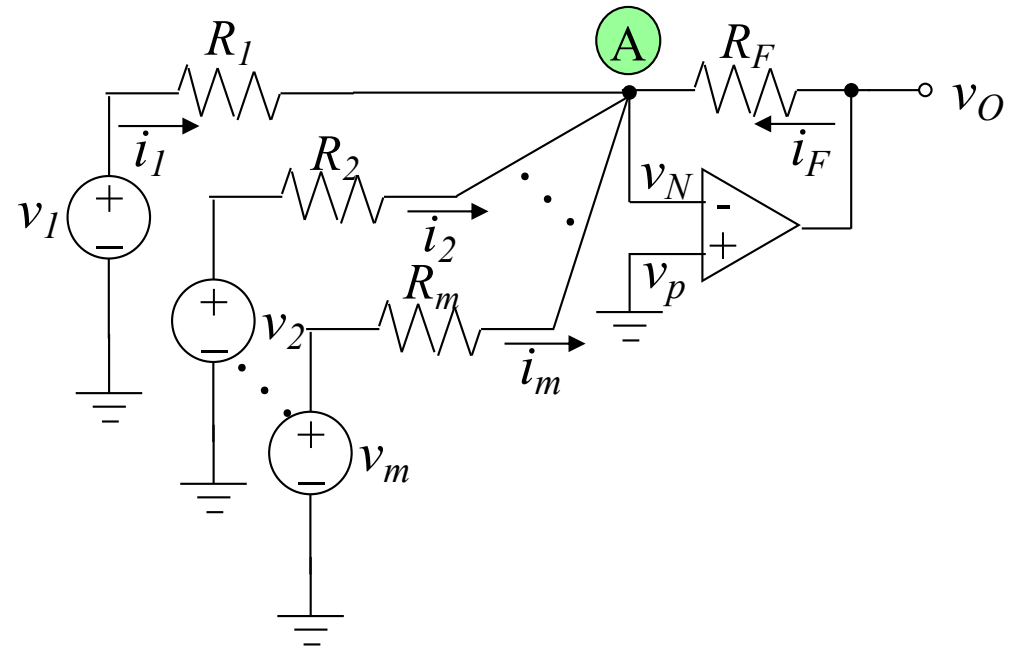
This is an inverting summing amplifier



$$v_O = \left(-\frac{R_F}{R_1} \right) v_1 + \left(-\frac{R_F}{R_2} \right) v_2$$
$$= -K_1 v_1 - K_2 v_2$$

Ever wondered about audio mixers? How do they work?

Mixing desk – Linear ccts



Virtual ground at v_n

Currents add
Summing junction

$$v_O = \left(-\frac{R_F}{R_1}\right)v_1 + \left(-\frac{R_F}{R_2}\right)v_2 + \dots + \left(-\frac{R_F}{R_m}\right)v_m$$

$$= K_1v_1 + K_2v_2 + \dots + K_mv_m$$

Permits adding signals to create a composite

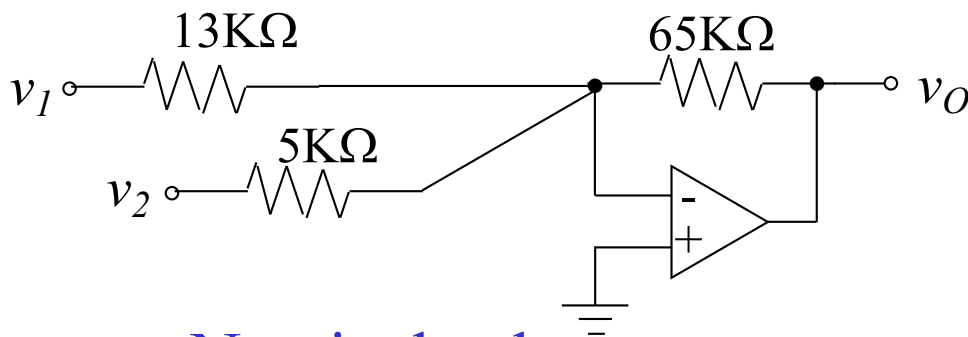
Strings+brass+woodwind+percussion

Guitars+bass+drums+vocal+keyboards

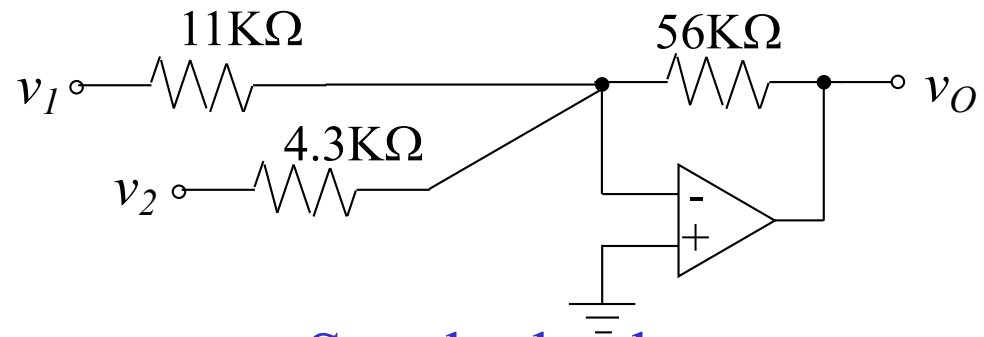
T&R, 5th ed, Design Example 4-15

Design an inverting summer to realize $v_O = -(5v_1 + 13v_2)$

Inverting summer with $\frac{R_F}{R_1} = 5, \frac{R_F}{R_2} = 13$



Nominal values



Standard values

If $v_1 = 400\text{mV}$ and $V_{CC} = \pm 15\text{V}$ what is max of v_2 for linear opⁿ?

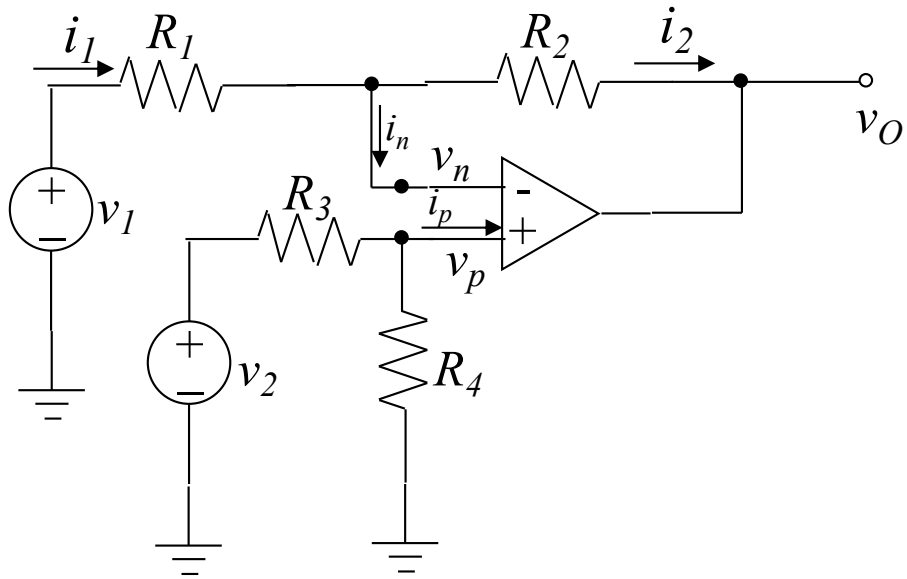
Need to keep $v_O > -15\text{V}$

$$-15 < -(5v_1 + 13v_2)$$

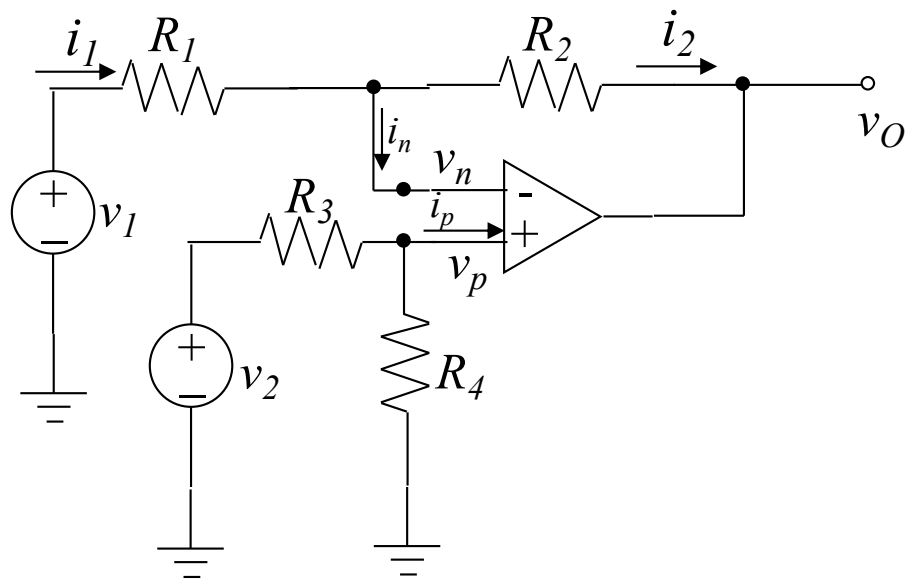
$$15 > 5v_1 + 13v_2$$

$$v_2 < \frac{15 - 5 \times 0.4}{13} = 1\text{V}$$

OpAmp Circuits – Differential Amplifier



OpAmp Circuits – Differential Amplifier



Use superposition to analyze

$v_2=0$: inverting amplifier

$$v_{O1} = -\frac{R_2}{R_1} v_1$$

$v_1=0$: non-inverting amplifier plus voltage divider

$$v_{O2} = \left[\frac{R_4}{R_3 + R_4} \right] \left[\frac{R_1 + R_2}{R_1} \right] v_2$$

$$v_O = v_{O1} + v_{O2}$$

$$= -\left[\frac{R_2}{R_1} \right] v_1 + \left[\frac{R_4}{R_3 + R_4} \right] \left[\frac{R_1 + R_2}{R_1} \right] v_2$$

$$= -K_1 v_1 + K_2 v_2$$

K_1 inverting gain

K_2 non-inverting gain

T&R, 5th ed, Exercise 4-13

What is v_o ?

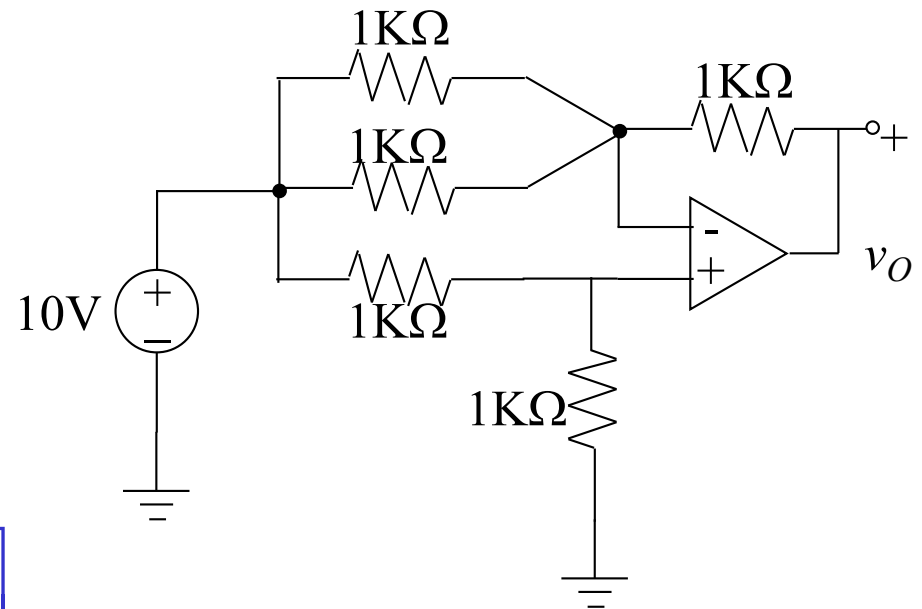
This is a differential amp

v_1 is 10V, v_2 is 10V

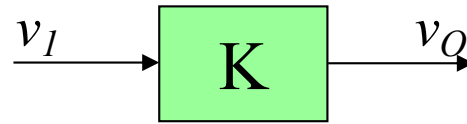
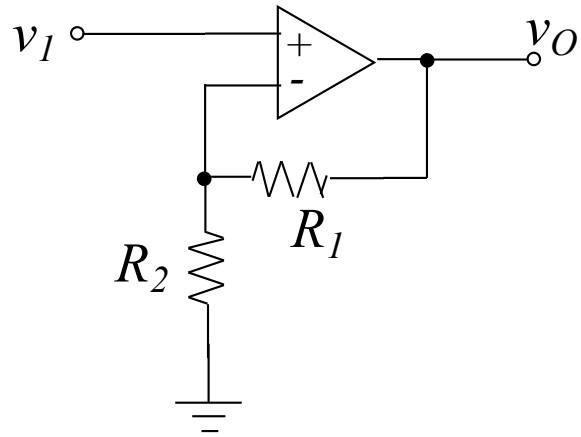
$R_1 = 1\text{K}\Omega \parallel 1\text{K}\Omega = 500\Omega$

$R_2 = R_3 = R_4 = 1\text{K}\Omega$

$$\begin{aligned} v_o &= K_1 v_1 + K_2 v_2 \\ &= -\frac{R_2}{R_1} v_1 + \left[\frac{R_1 + R_2}{R_1} \right] \left[\frac{R_4}{R_3 + R_4} \right] v_2 \\ &= -20 + 3 \cdot \frac{1}{2} \cdot 10 = -5V \end{aligned}$$

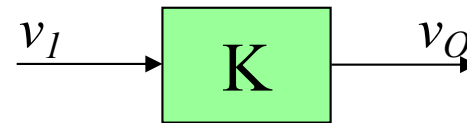
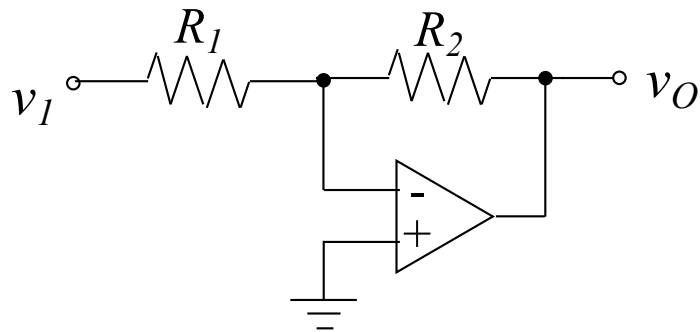


Lego Circuits



$$K = \frac{R_1 + R_2}{R_2}$$

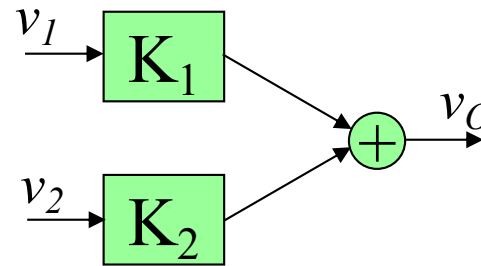
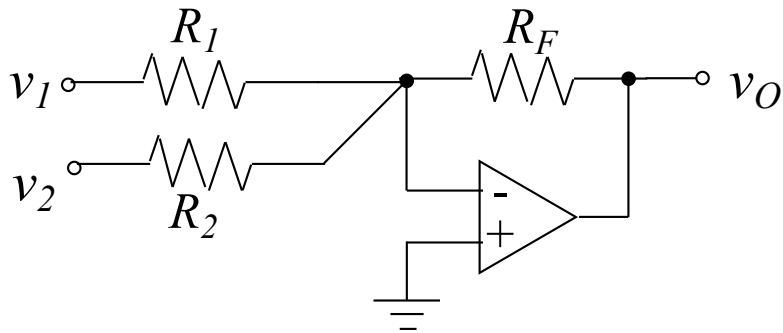
Non-inverting amplifier



$$K = -\frac{R_2}{R_1}$$

Inverting amplifier

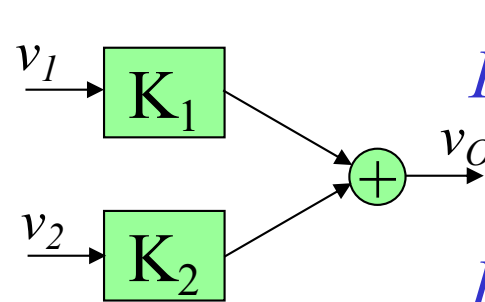
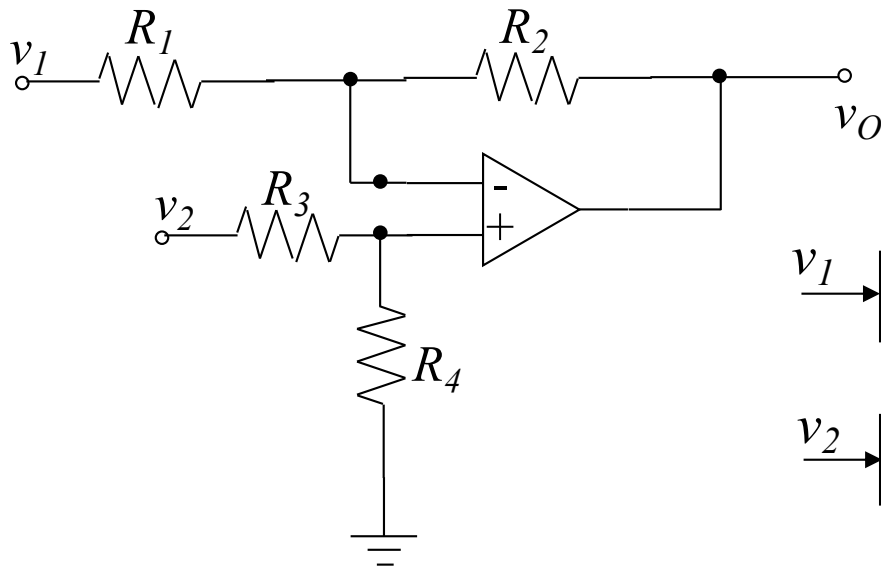
Lego Circuits (contd)



$$K_1 = -\frac{R_F}{R_1}$$

$$K_2 = -\frac{R_F}{R_2}$$

Inverting summer

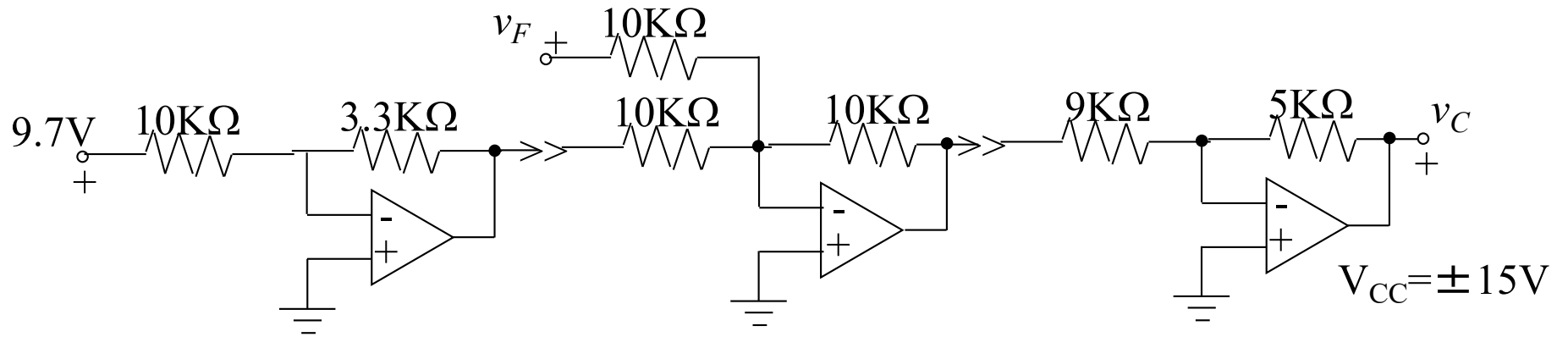


$$K_1 = -\frac{R_2}{R_1}$$

$$K_2 = \left(\frac{R_1 + R_2}{R_1} \right) \left(\frac{R_4}{R_3 + R_4} \right)$$

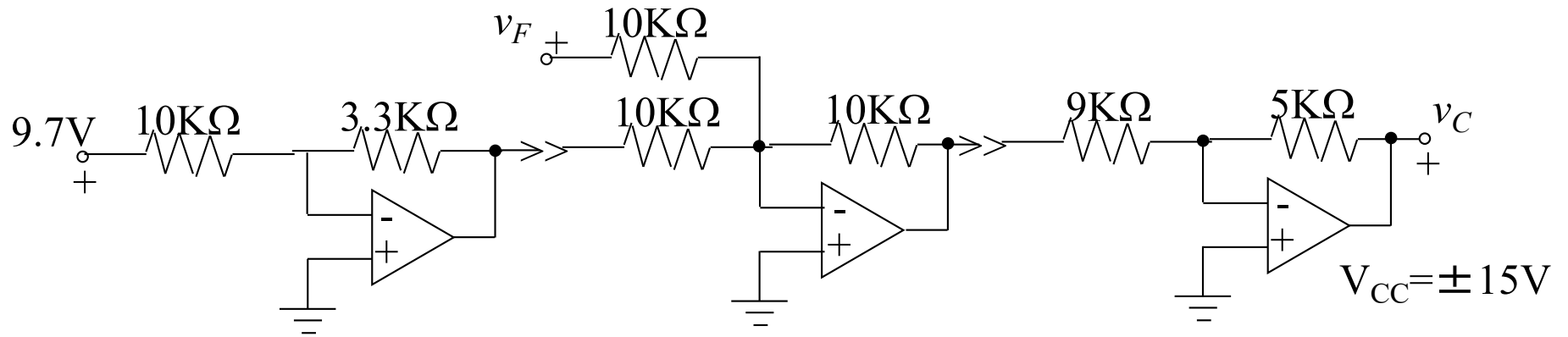
Differential amplifier

T&R, 5th ed, Example 4-16: OpAmp Lego

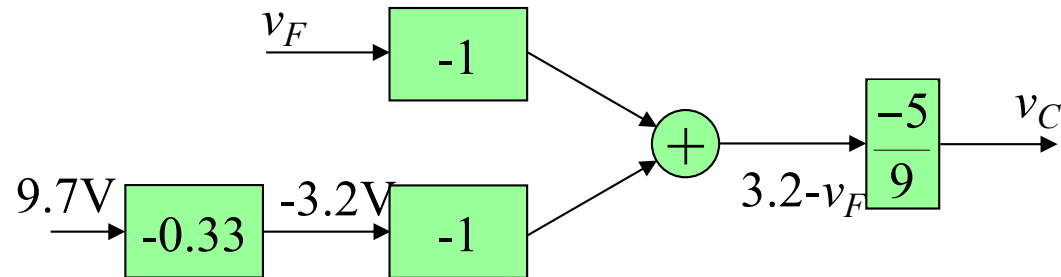


So what does this circuit do?

Example 4-16: OpAmp Lego



So what does this circuit do?



It converts tens of °F to tens of °C

Max current drawn by each stage is 1.5mA

OpAmp Cct Analysis

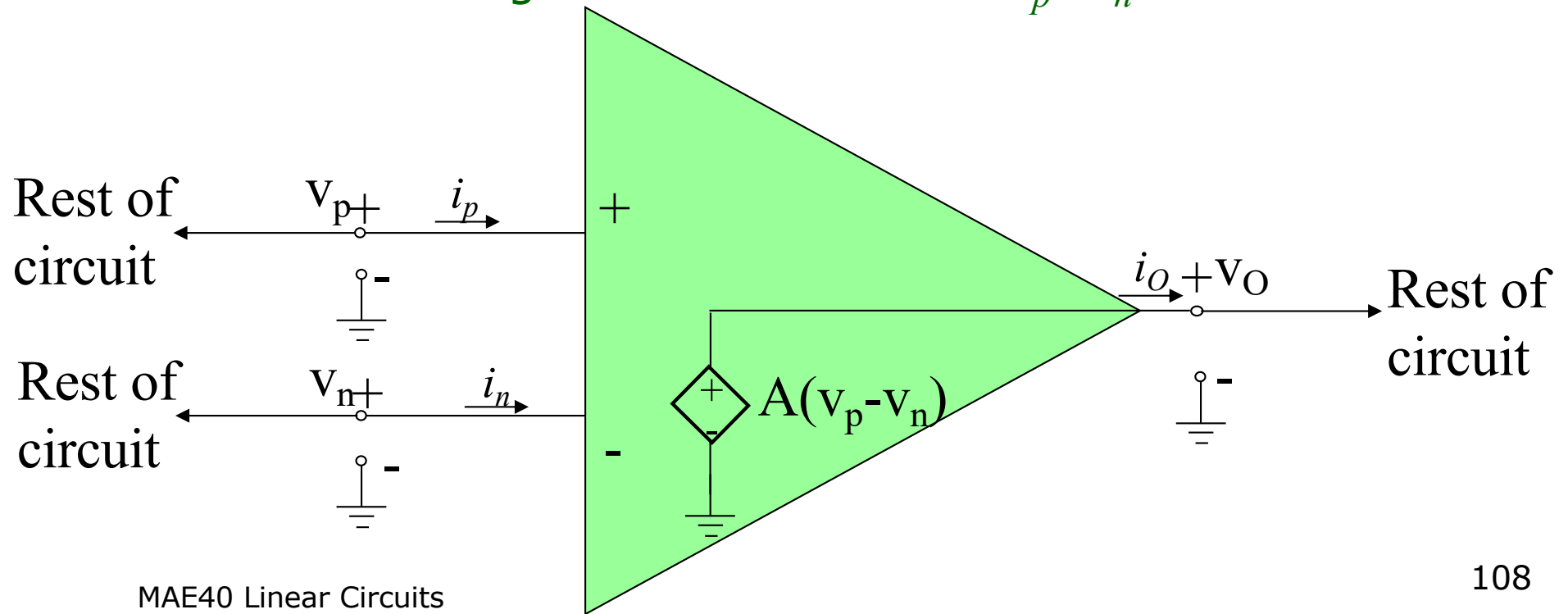
What if circuit is not simple interconnection of basic building blocks? OpAmp Nodal Analysis

Use dependent voltage source model

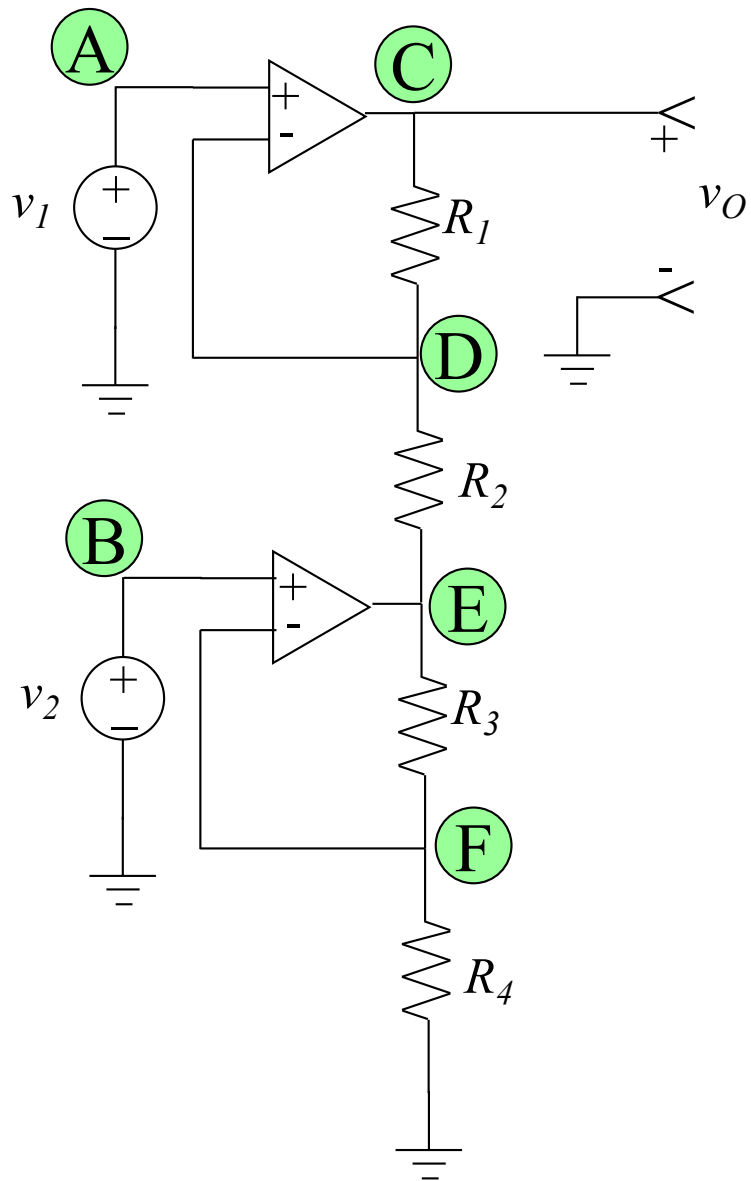
Identify node voltages

Formulate input node equations

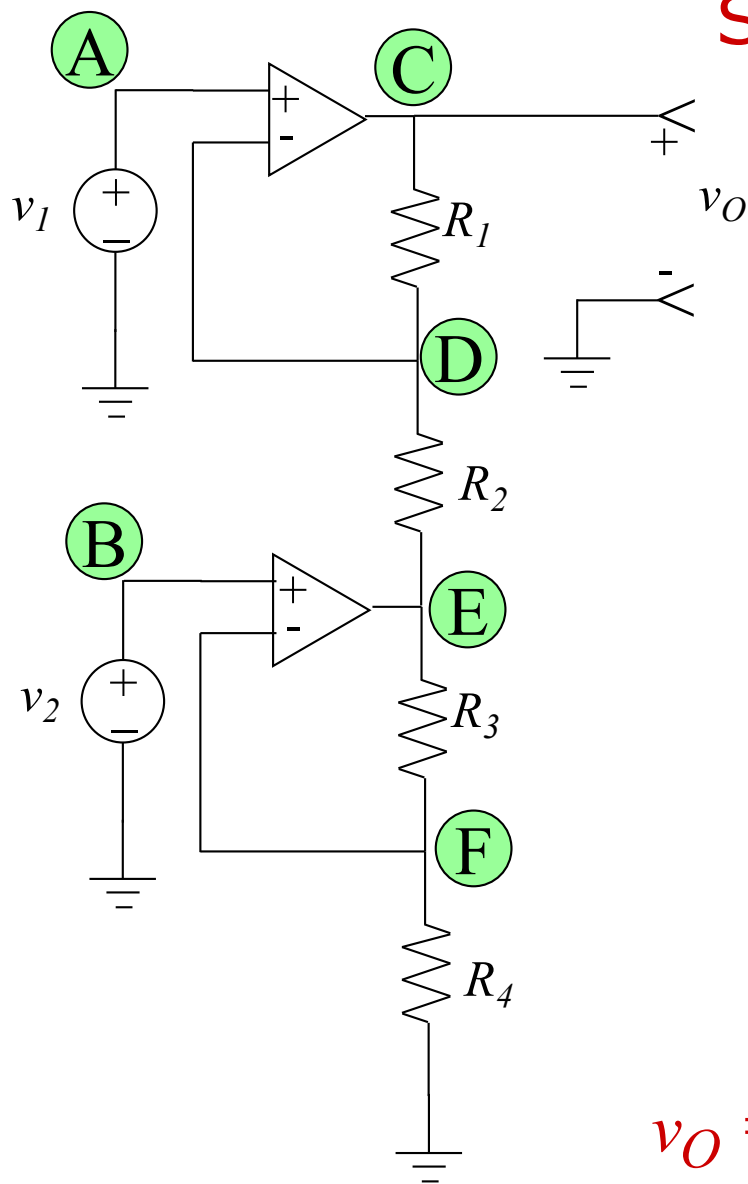
Solve using ideal characteristic $v_p = v_n$



OpAmp Analysis – T&R, 5th ed, Example 4-18



OpAmp Analysis – T&R, 5th ed, Example 4-18



Seemingly six non-reference nodes: A-E

Nodes A, B: connect to reference voltages v_1 and v_2

Node C, E: connected to OpAmp outputs (forget for the moment)

Node D: $(G_1 + G_2)v_D - G_1v_C - G_2v_E = 0$

Node F: $(G_3 + G_4)v_F - G_3v_E = 0$

OpAmp constraints

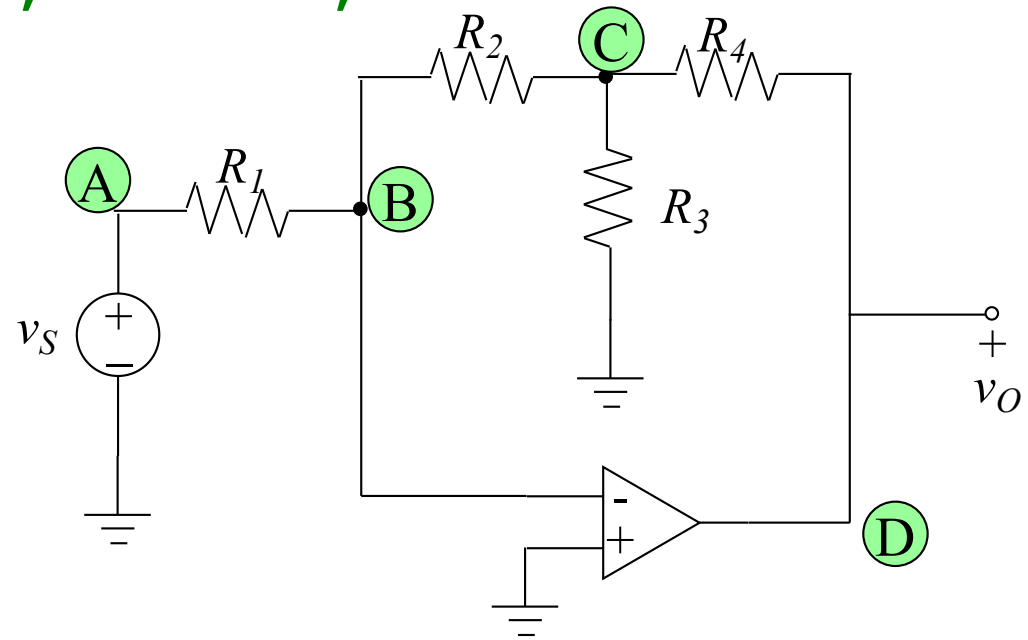
$$v_A = v_1 = v_D, \quad v_B = v_2 = v_F$$

$$G_1v_C + G_2v_E = (G_1 + G_2)v_1$$

$$G_3v_E = (G_3 + G_4)v_2$$

$$v_O = v_C = \left[\frac{G_1 + G_2}{G_1} \right] v_1 - \frac{G_2}{G_1} \left[\frac{G_3 + G_4}{G_3} \right] v_2$$

OpAmp Analysis – T&R, 5th ed, Exercise 4-14



OpAmp Analysis – T&R, 5th ed, Exercise 4-14

Node A: $v_A = v_S$

Node B:

$$(G_1 + G_2)v_B - G_1v_A - G_2v_C = 0$$

Node C:

$$(G_2 + G_3 + G_4)v_C - G_2v_B - G_4v_D = 0$$

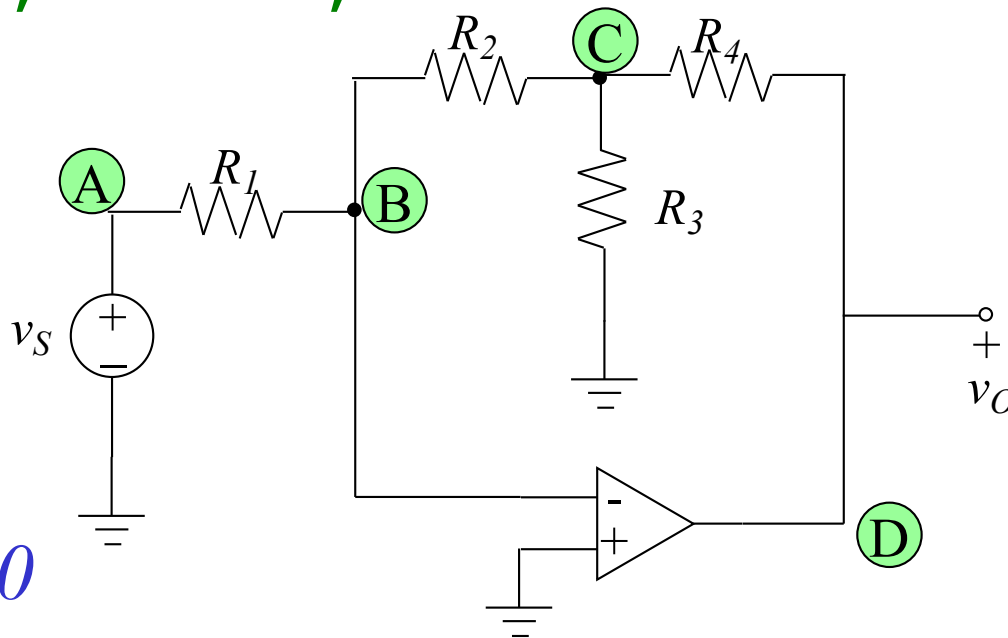
Constraints

$$v_B = v_p = v_n = 0$$

Solve

$$v_C = -\frac{G_1}{G_2}v_S$$

$$v_O = v_D$$



$$\begin{aligned} v_O &= \frac{(G_2 + G_3 + G_4)}{G_4} \cdot \frac{-G_1}{G_2} v_S \\ &= -\frac{(R_2R_3 + R_2R_4 + R_3R_4)}{R_1R_3} v_S \end{aligned}$$

OpAmp Circuit Design – the whole point

Given an input-output relationship design a cct to implement it

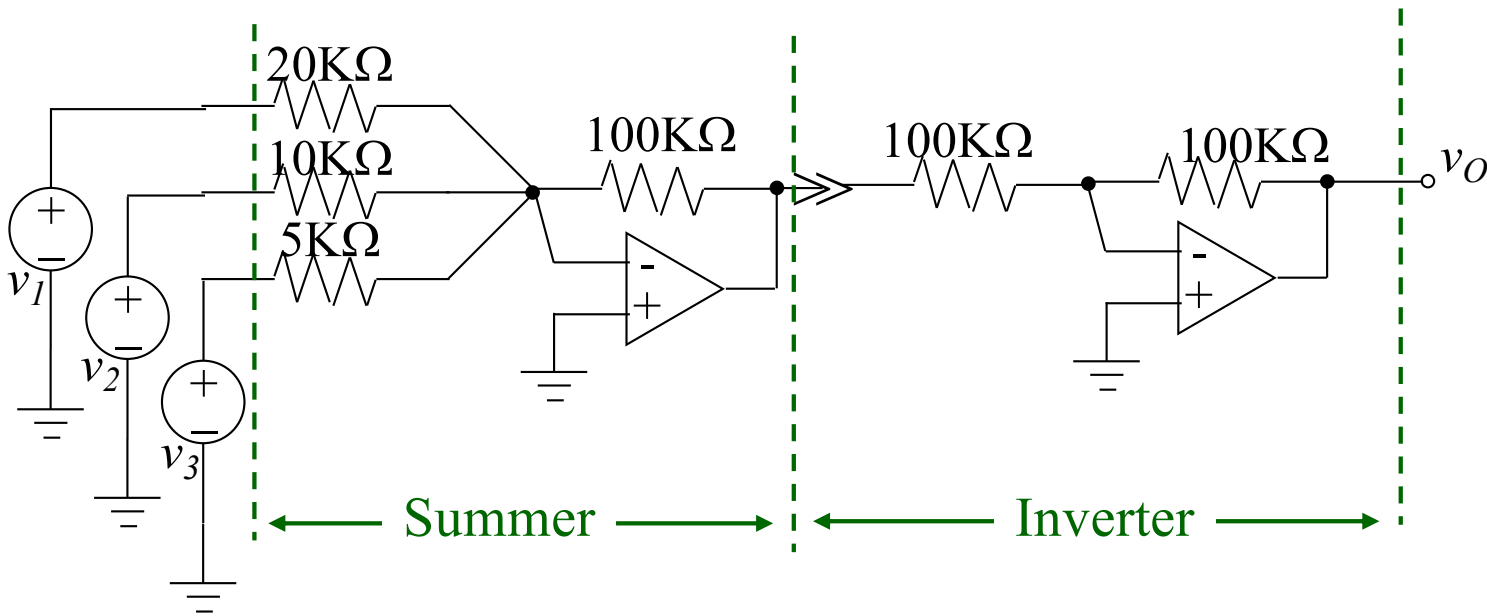
Build a cct to implement $v_O = 5v_1 + 10v_2 + 20v_3$

OpAmp Circuit Design – the whole point

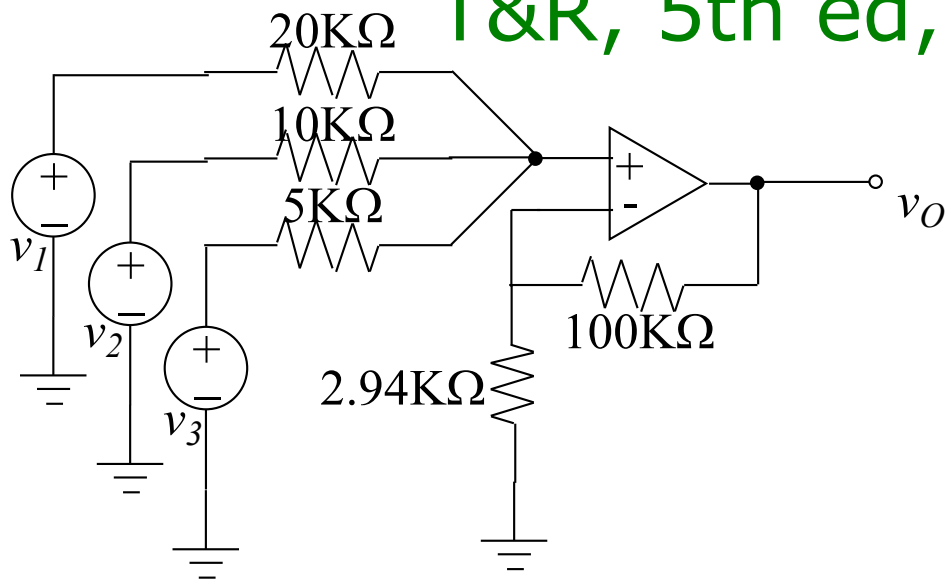
Given an input-output relationship design a cct to implement it

Build a cct to implement $v_O = 5v_1 + 10v_2 + 20v_3$

Inverting summer followed by an inverter

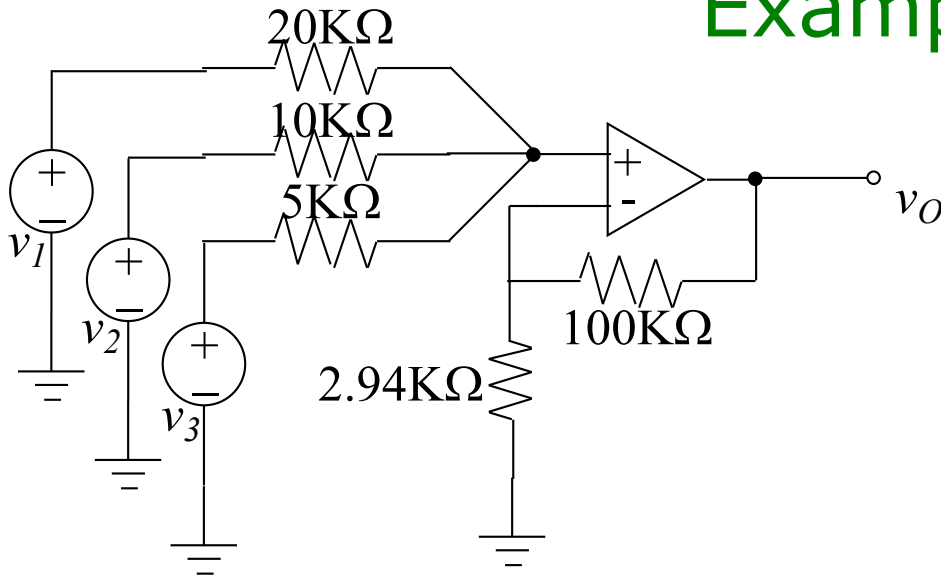


T&R, 5th ed, Example 4-21



How about this one?

Example 4-21



How about this one?

Non-inverting amp $v_p \rightarrow v_o$

$$v_o = K v_p = \frac{100 \cdot 10^3 + 2.94 \cdot 10^3}{2.94 \cdot 10^3} v_p = 35 v_p$$

KCL at p -node with $i_p = 0$

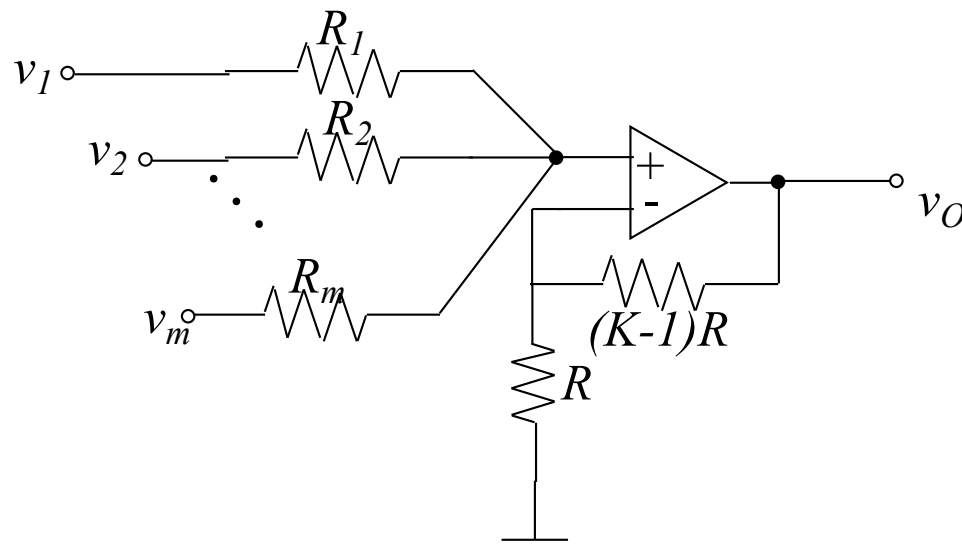
$$\frac{v_1 - v_p}{2 \cdot 10^4} + \frac{v_2 - v_p}{10^4} + \frac{v_3 - v_p}{0.5 \cdot 10^4} = 0$$

$$3.5 v_p = 0.5 v_1 + v_2 + 2 v_3$$

Non-inverting summer

Fewer elements than inv-summer + inverter

$$R_{eq} = R_1 \parallel R_2 \parallel R_3 \parallel \dots \parallel R_m$$



$$v_o = K \left[\frac{R_{eq}}{R_1} v_1 + \frac{R_{eq}}{R_2} v_2 + \dots + \frac{R_{eq}}{R_m} v_m \right]$$

Comparators – A Nonlinear OpAmp Circuit

We have used the ideal OpAmp conditions for the analysis of OpAmps in the linear regime

$$v_n = v_p, \quad i_n = i_p = 0 \quad \text{if} \quad A|v_p - v_n| \leq V_{CC}$$

What about if we operate with $v_p \neq v_n$?

That is, we operate outside the linear regime.

We saturate!!

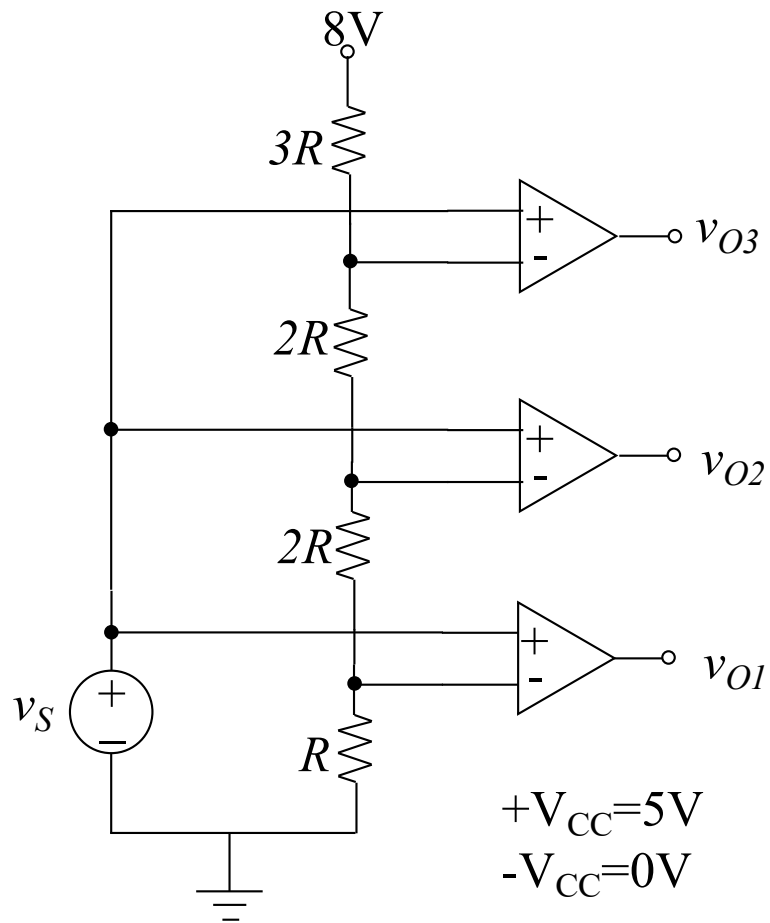
$$v_O = +V_{CC} \quad \text{if} \quad v_p > v_n$$

$$v_O = -V_{CC} \quad \text{if} \quad v_p < v_n$$

Without feedback, OpAmp acts as a comparator

There is one of these in every FM radio!

“Analog-to-digital converter” - comparators



“Analog-to-digital converter” - comparators

Current laws still work

$$i_p = i_n = 0$$

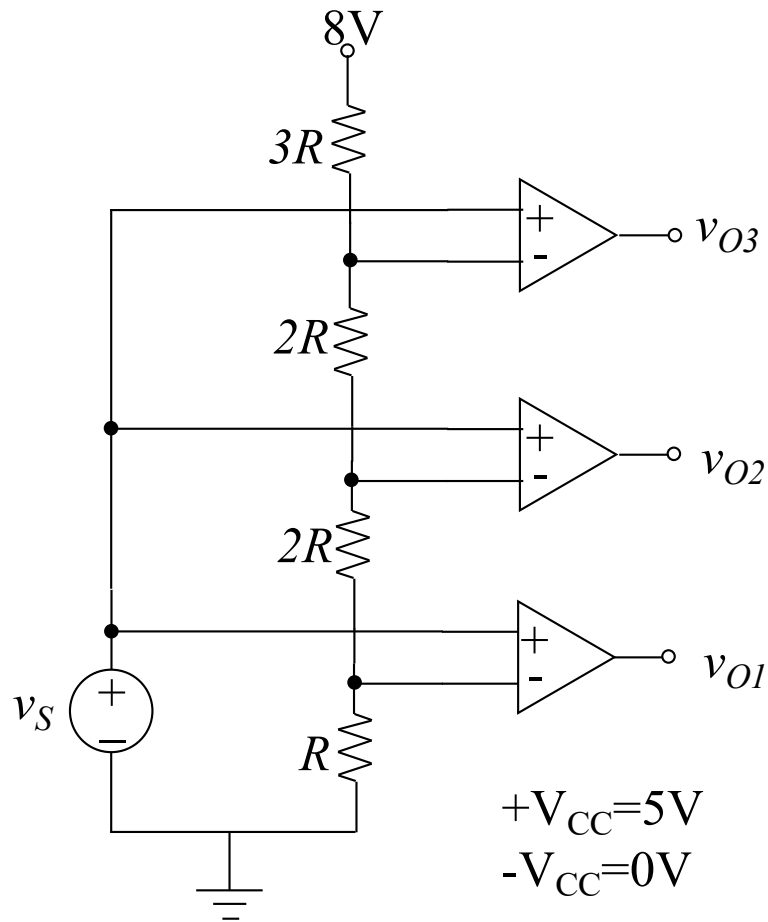
Parallel comparison

Flash converter

“3-bit” output

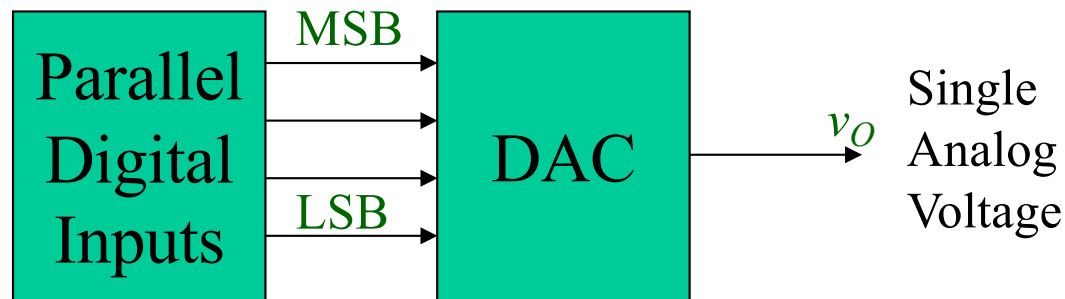
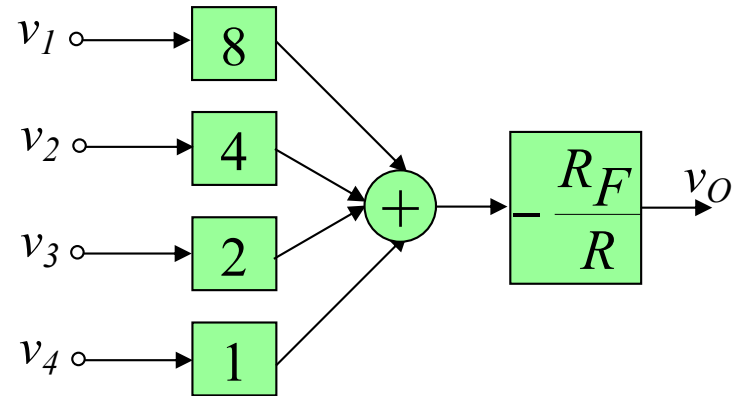
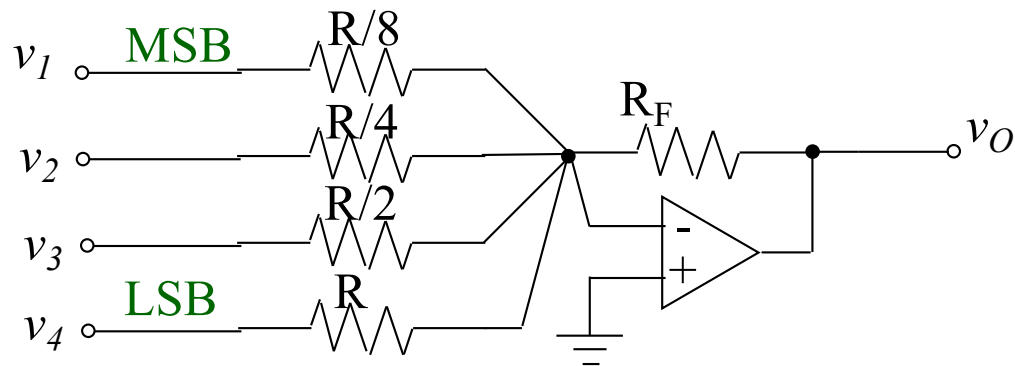
Not really how it is done

Voltage divider switched



Input	v_{O1}	v_{O2}	v_{O3}
$1 > v_S$	0	0	0
$3 > v_S > 1$	5	0	0
$5 > v_S > 3$	5	5	0
$v_S > 5$	5	5	5

Digital-to-analog converter



Conversion of digital data to analog voltage value

Bit inputs = 0 or 5V

Analog output varies between v_{min} and v_{max} in 16 steps

Signal Conditioning

Your most likely brush with OpAmps in practice

Signal – typically a voltage representing a physical variable

Temperature, strain, speed, pressure

Digital analysis – done on a computer after

Anti-aliasing filtering – data interpretation

Adding/subtracting an offset – zeroing

Normally zero of ADC is 0V

Scaling for full scale variation – quantization

Normally full scale of ADC is 5V

Analog-to-digital conversion – ADC

Maybe after a few more tricks like track and hold

Offset correction: use a summing OpAmp

Scaling: use an OpAmp amplifier

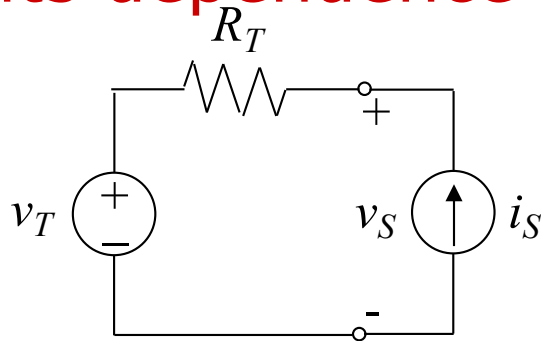
Anti-aliasing filter: use a dynamic OpAmp cct

Thévenin and Norton for dependent sources

Cannot turn off the ICSs and IVSs to do the analysis

This would turn off DCSs and DVSs

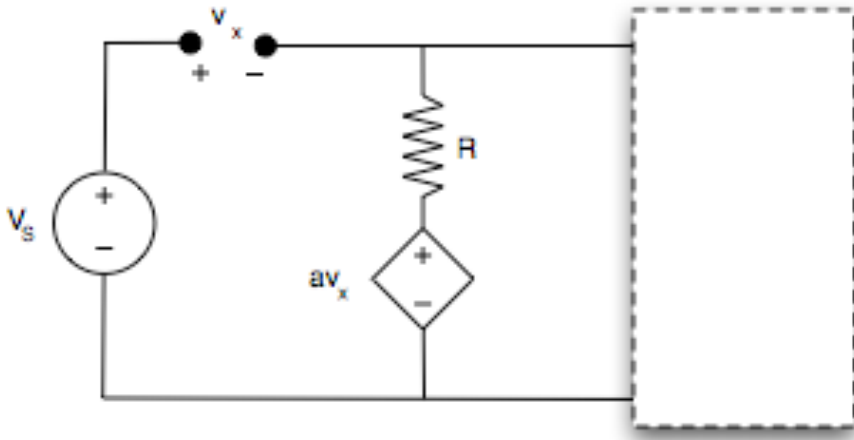
Connect an independent CS or VS to the terminal and compute the resulting voltage or current and its dependence on the source



Compute v_S in response to i_S : $v_S = v_T + i_S R_T$

Or just compute the open-circuit voltage and the short-circuit current

Thévenin and Norton for dependent sources



Thevenin equivalent circuit?

Open-circuit voltage

$$\left. \begin{aligned} v_{oc} &= v_s - v_x \\ v_{oc} &= v_R + av_x = av_x \end{aligned} \right\} \Rightarrow v_T = v_{oc} = \frac{a}{1+a} v_s$$

Thevenin resistance

$$R_T = \frac{v_{oc}}{i_{sc}} = \frac{1}{1+a} R$$

Short-circuit current

$$\left. \begin{aligned} 0 &= v_s - v_x \\ 0 &= -Ri_{sc} + av_x \end{aligned} \right\} \Rightarrow i_{sc} = \frac{a}{R} v_s$$

What would instead be the resistance obtained by turning off IVS?

Where to now?

Where have we been?

Nodal and mesh analysis

Thévenin and Norton equivalence

Dependent sources and active cct models

OpAmps and resistive linear active cct design

Where to now?

Capacitors and inductors (Ch.6)

Laplace Transforms and their use for ODEs and ccts (Ch.9)

s -domain cct design and analysis (Ch.10)

Frequency response (Ch.12) and filter design (Ch.14)

We will depart from the book more during this phase