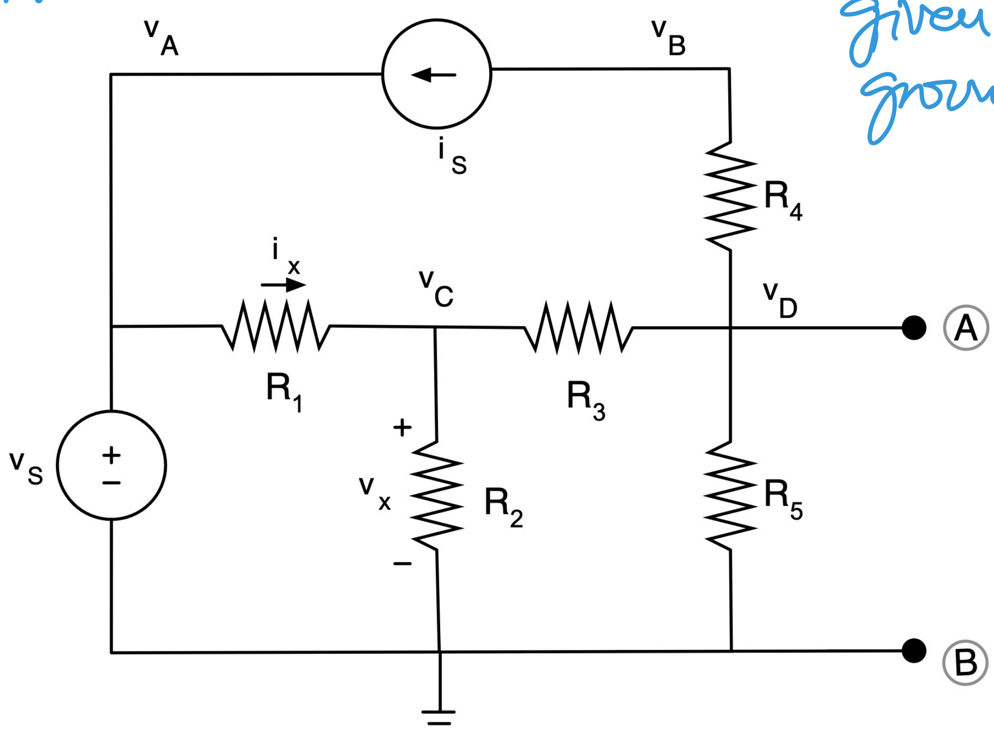
1.– Part I

To use node-voltage analysis, we must take care of the presence of the voltage source using one of the three methods discussed in class:

1) source transformation
2) grounding a node conveniently
3) supernode

We cannot use 1) because the voltage source is not in series with a resistor (even if it was, the statement of the question explicitly rules out modifying the circuit, which also discards source transformation). 2) is instead applicable, because the ground has been chosen in a convenient way. One could also use 3) (because it is always applicable) but that would overcomplicate things given where ground is located.



[+1 point]

So we settle on using method 2, which gives

$$V_A = V_S$$

[+1 point]

And we write KCL eqs for nodes B, C, & D. We do it by inspection, which is faster.

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & G_4 & 0 & -G_4 \\ -G_1 & 0 & G_1+G_2+G_3 & -G_3 \\ 0 & -G_4 & -G_3 & G_3+G_4+G_5 \end{pmatrix} \begin{pmatrix} V_A \\ V_B \\ V_C \\ V_D \end{pmatrix} = \begin{pmatrix} V_S \\ -i_S \\ 0 \\ 0 \end{pmatrix}$$

Either or are worth [+4 points]

(Here, we have used the short-hand notation $G_i = \frac{1}{R_i}$)

This gives 4 eqs in 4 unknowns, written in matrix form. If you substitute $V_A = V_S$ in the other 3 eqs, then you can also express this as 3 eqs in 3 unknowns.

$$\begin{pmatrix} G_4 & 0 & -G_4 \\ 0 & G_1+G_2+G_3 & -G_3 \\ -G_4 & -G_3 & G_3+G_4+G_5 \end{pmatrix} \begin{pmatrix} V_B \\ V_C \\ V_D \end{pmatrix} = \begin{pmatrix} -i_S \\ G_1 V_S \\ 0 \end{pmatrix}$$

Part II

In terms of the node voltages, we have

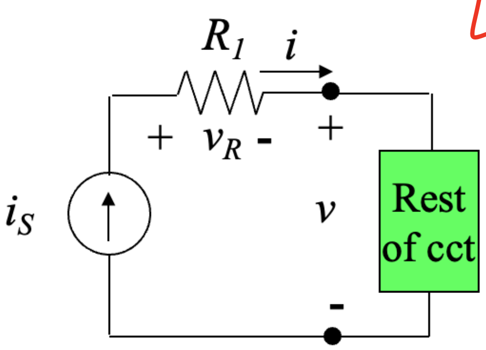
$$v_x = V_C \qquad \text{[+1 point]}$$

$$i_x = G_4(V_A - V_C) \qquad \text{[+1 point]}$$

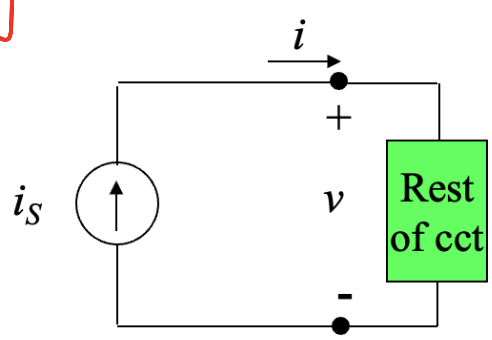
Part III

The resistor R_4 is in series with the current source. From what we know from class, a current source in series with a resistor is equivalent, from the point of view of the rest of the circuit, to just having the current source. This is depicted in the diagram

[+1 point]



$i = i_S$

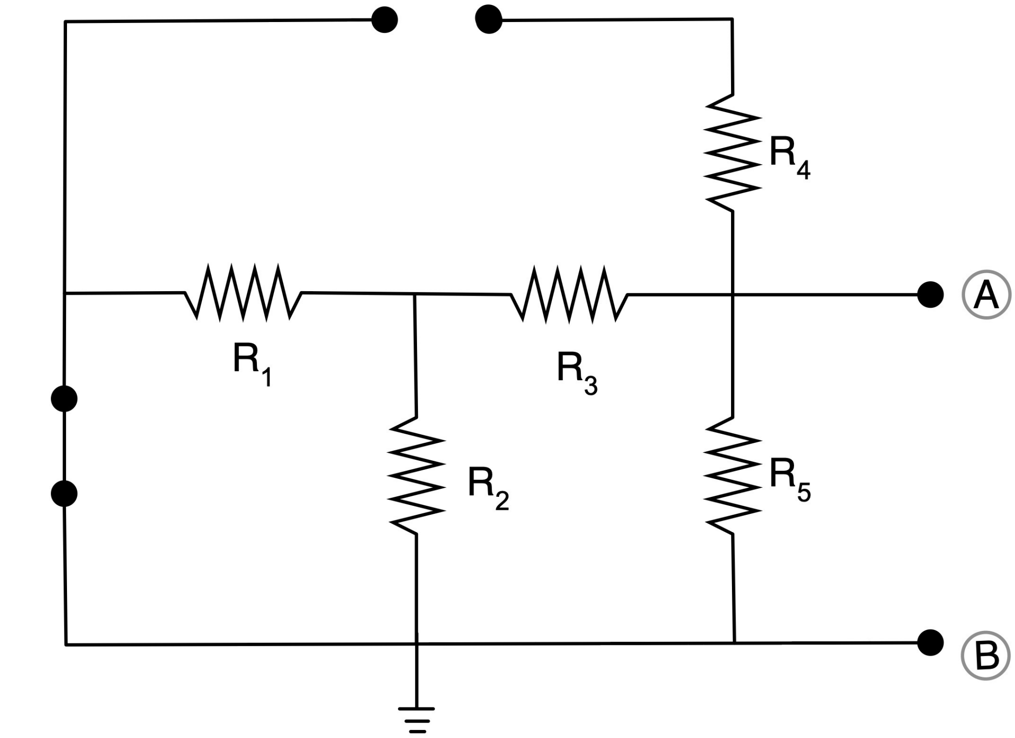


This means that, if the technician replaced the resistor R_4 in our circuit by a short circuit, then nothing changed in the rest of the circuit, so the values of v_x and i_x remained the same.

[+1 point]

2._ Part I

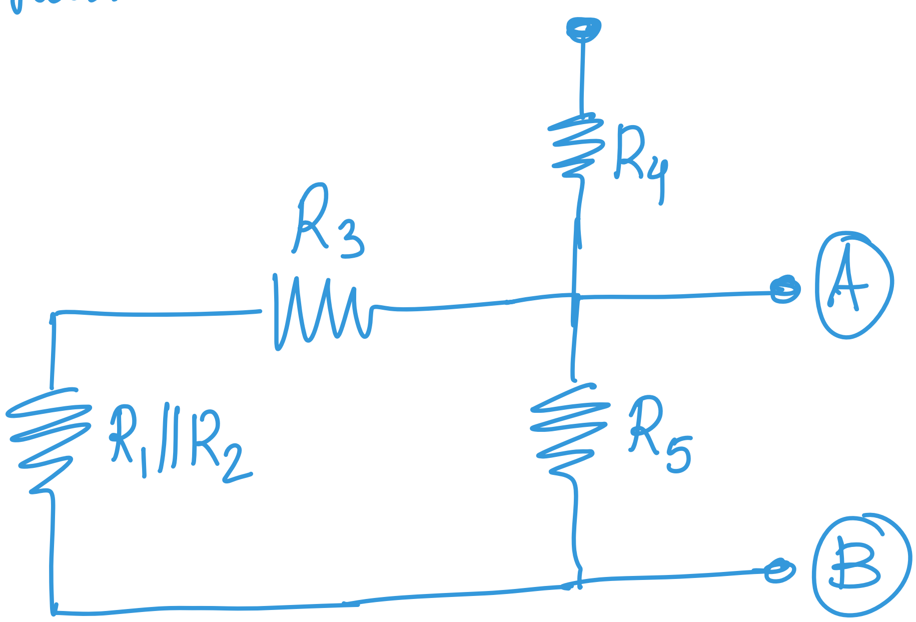
We turn off all the sources in the circuit and obtain the circuit below



[+0.5 point]

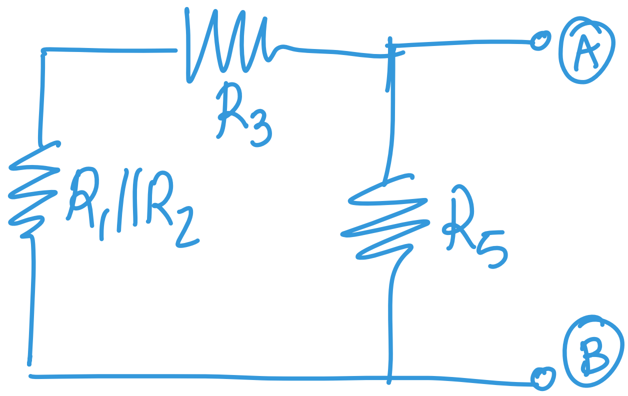
where the voltage source gets replaced by a short circuit, and the current source by an open circuit. [+0.5 point]

Next, we use association of resistors to simplify it further. Note that R_1 and R_2 are in parallel, so



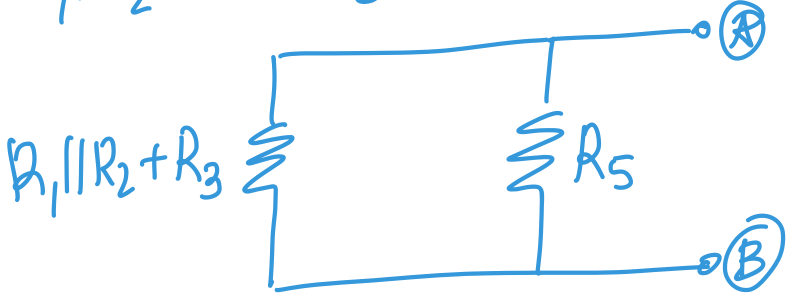
[+0.5 point]

Moreover, there is no current going through R_4 (because of the open circuit), so it is as if that resistor was not there.



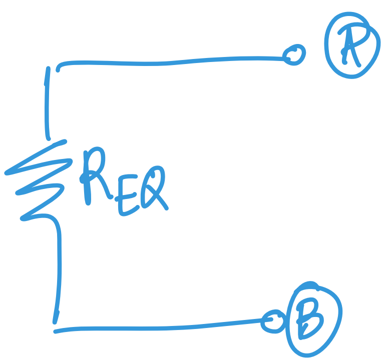
[+0.5 point]

$R_1 || R_2$ and R_3 are in series, so we simplify as



[+0.5 point]

Finally, the two remaining resistors are in parallel, so



$$R_{EQ} = (R_1 || R_2 + R_3) || R_5 =$$

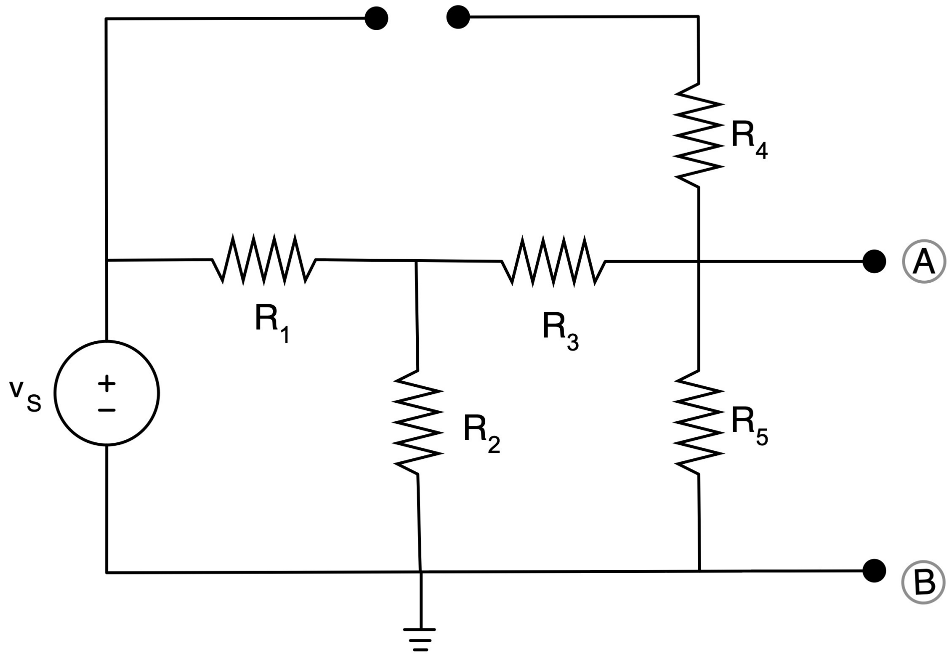
$$= \frac{R_5 \cdot \left(\frac{R_1 R_2}{R_1 + R_2} + R_3 \right)}{R_5 + R_3 + \frac{R_1 R_2}{R_1 + R_2}} =$$

$$= \frac{R_5 \cdot (R_1 R_2 + R_3 (R_1 + R_2))}{(R_1 + R_2)(R_3 + R_5) + R_1 R_2}$$ [+0.5 point]

$= 3\Omega$

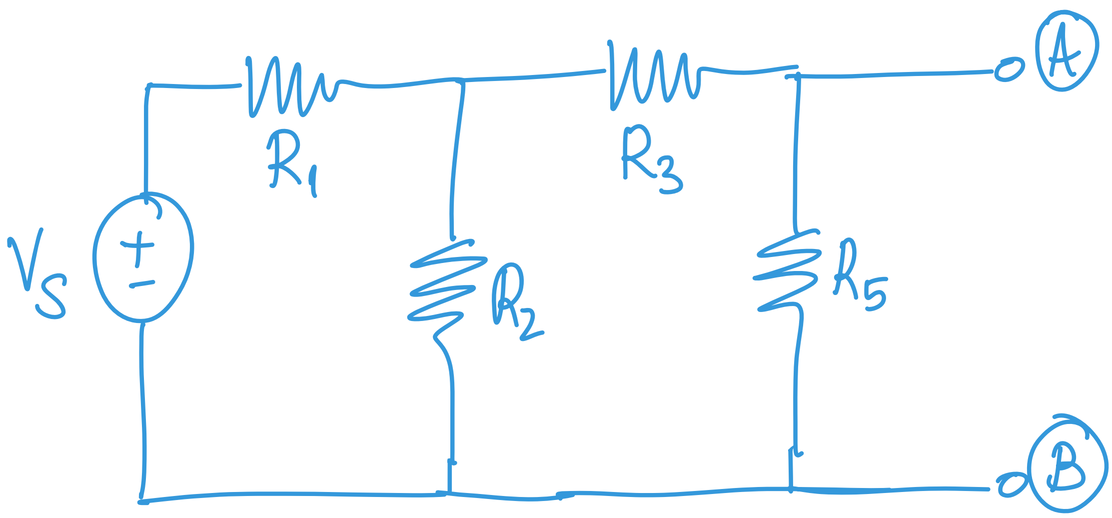
Part II

We turn off the current source, substituting it by an open circuit as



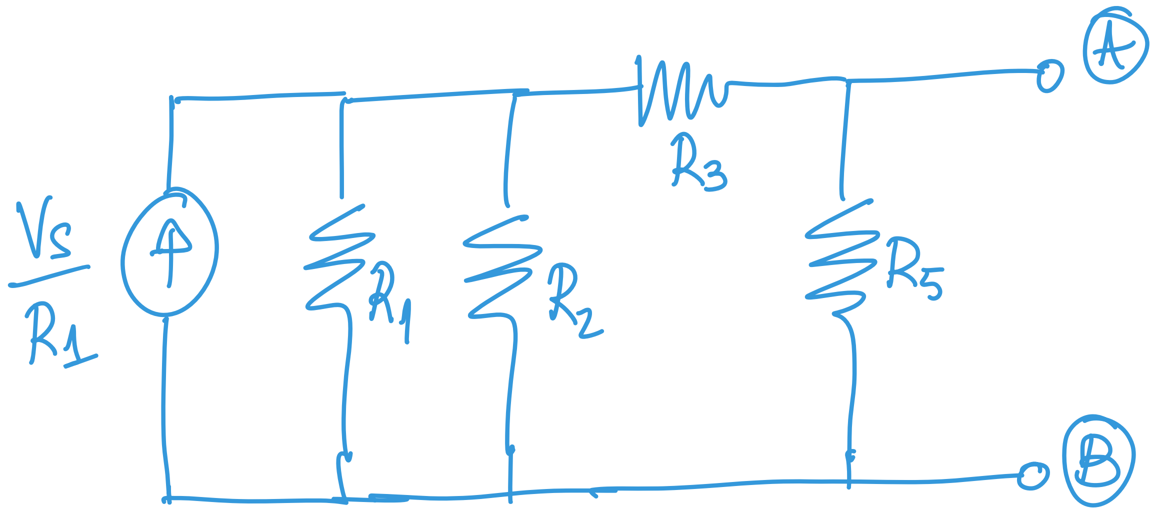
[+0.5 point]

Since there is no current flowing through the resistor R_4, we redraw this as



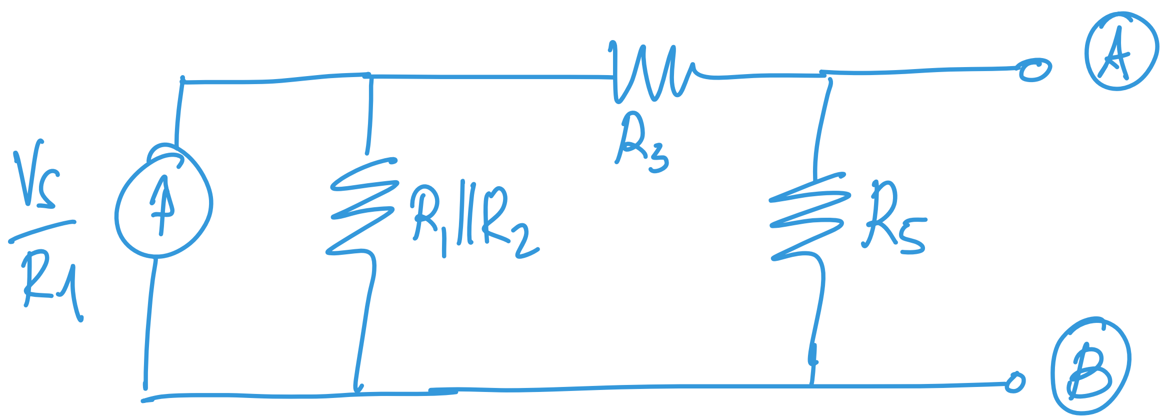
[+0.5 point]

Next, we use source transformation to plot



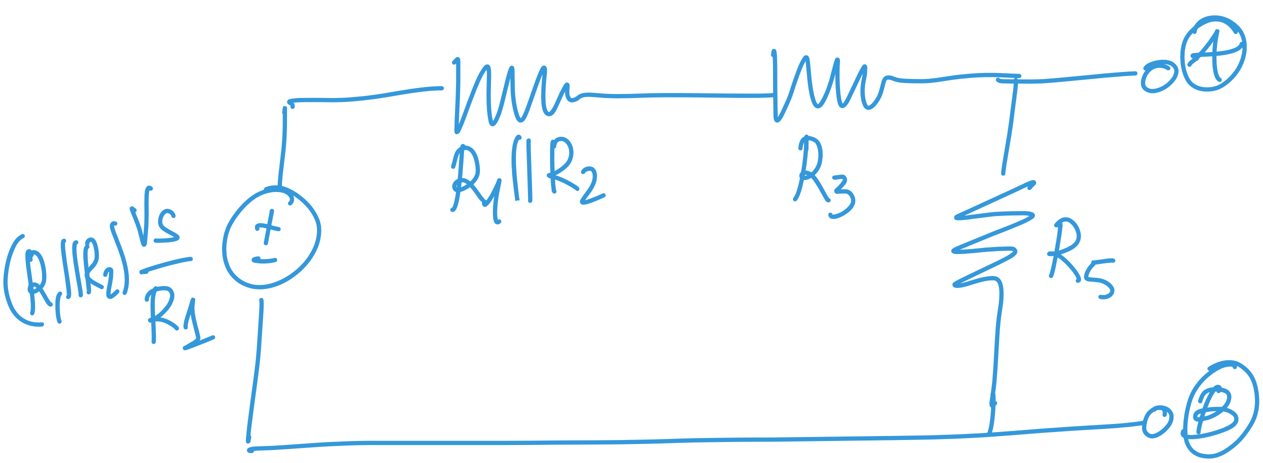
[+0.5 point]

Now, we combine the two resistors in parallel,



[+0.5 point]

And use one more source transformation,



Finally, we see that the open circuit voltage is the voltage drop seen by the resistor R_5, which can be computed by voltage division

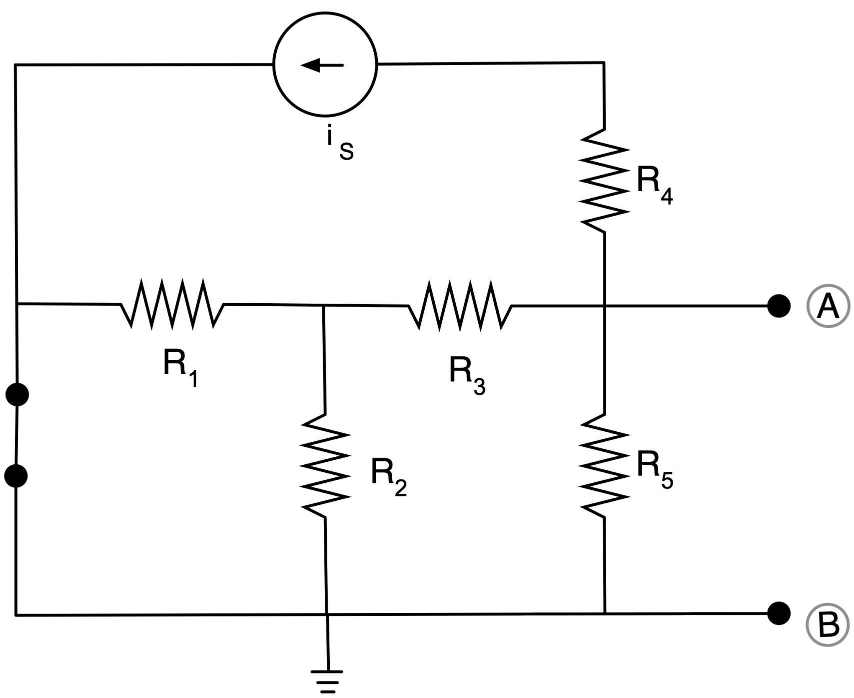
$$(V_{AB})_1 = \frac{R_5}{R_3+R_5+R_1 \| R_2} \cdot \frac{R_1 \| R_2}{R_1} V_S = \frac{R_2 R_5}{(R_3+R_5)(R_1+R_2)+R_1 R_2} \cdot V_S$$

$$= 2V$$

[+1 point]

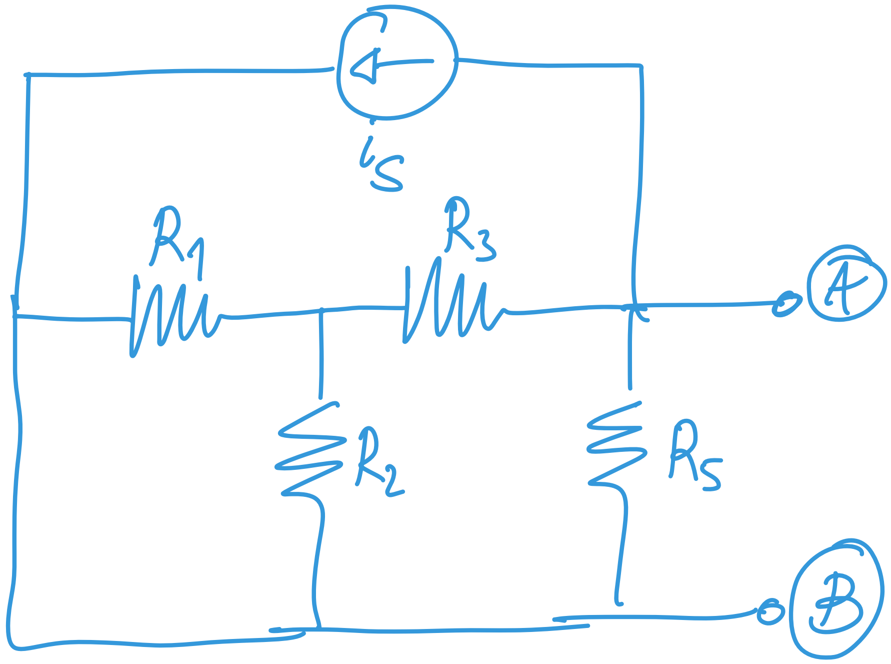
Part III

We turn off the voltage source, substituting it by a closed circuit as



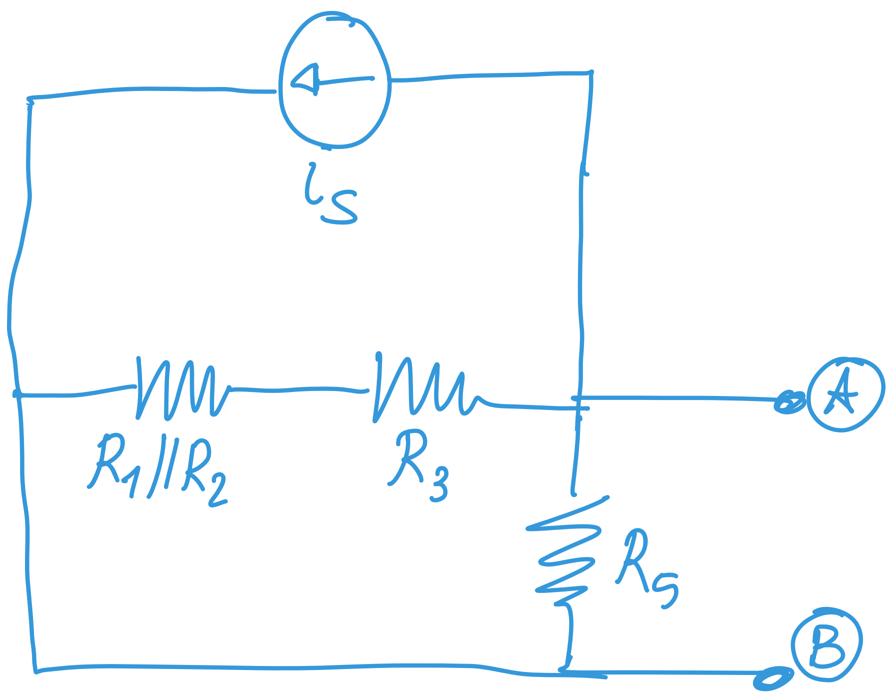
[+0.5 point]

Since the resistor R_4 is in series with the current source, we can simply redraw the equivalent circuit



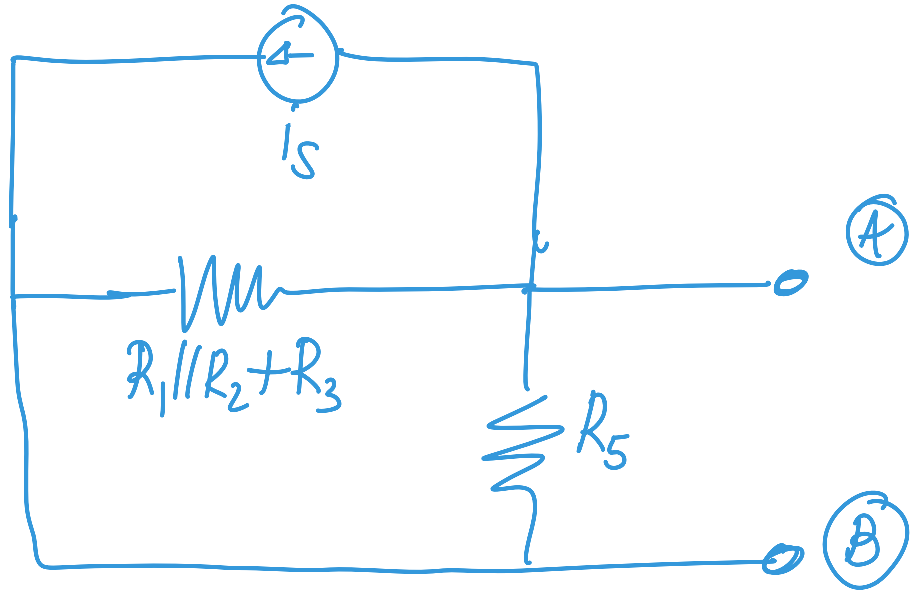
[+0.5 point]

We use the fact that R_1 and R_2 are in parallel to redraw the circuit as



[+0.5 point]

Next, we combine the resistors in series,



[+0.5 point]

Finally, we can use current division to obtain

$$(V_{AB})_2 = R_5 \cdot \frac{1/R_5}{1/R_5 + \frac{1}{R_1 \| R_2 + R_3}} (-i_S) =$$

$$= -\frac{(R_1R_2 + R_3(R_1+R_2))R_5}{R_1R_2 + (R_1+R_2)(R_3+R_5)} i_S = 3V$$

[+1 point]

Part IV

By superposition, the open circuit voltage as seen from terminals A and B is the sum of the answers of Part II (current source off) and Part III (voltage source off).

[+0.5 point]

Therefore,

$$V_T = V_{OC} = (V_{AB})_1 + (V_{AB})_2 =$$

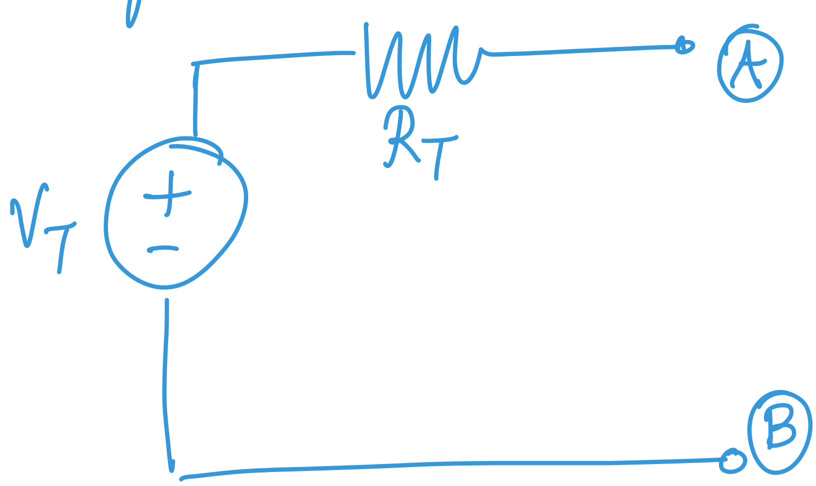
$$= \frac{R_2 R_5}{(R_3+R_5)(R_1+R_2)+R_1R_2} \cdot V_S - \frac{\left(R_1R_2 + R_3(R_1+R_2)\right)R_5}{R_1R_2+(R_1+R_2)(R_3+R_5)}\, i_S$$

$$= R_5 \cdot \frac{R_2 V_S - \left(R_1R_2 + R_3(R_1+R_2)\right) i_S}{R_1R_2 + (R_1+R_2)(R_3+R_5)} = 5V$$

From Part I,

$$R_T = R_{EQ} = \frac{R_5 \cdot \left(R_1R_2 + R_3(R_1+R_2)\right)}{(R_1+R_2)(R_3+R_5)+R_1R_2} = 3\Omega$$

Therefore, the Thevenin equivalent is



[+0.5 point]

Part V

We compute numerical values for the Thevenin equivalent (in case students have not already computed them above)

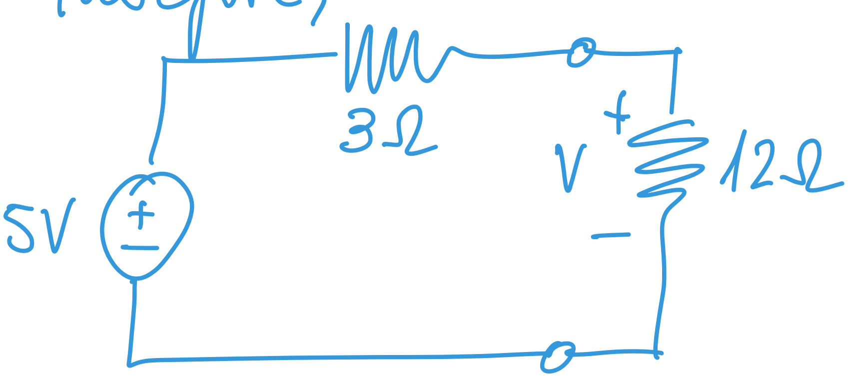
$$V_T = R_5 \cdot \frac{R_2 V_S - (R_1R_2 + R_3(R_1+R_2))\, i_S}{R_1R_2 + (R_1+R_2)(R_3+R_5)} =$$

$$= 5\, \frac{5 \cdot 10 - (25+50)(-1)}{25+100} = 5V$$

$$R_T = \frac{R_5 \cdot (R_1R_2 + R_3(R_1+R_2))}{(R_1+R_2)(R_3+R_5) + R_1R_2} =$$

$$= \frac{5\,(25 + 5(10))}{100+25} = 3\,\Omega$$

Therefore, we have



$$V = \frac{12}{12+3} \cdot 5 = 0.8 \cdot 5$$
$$= 4V$$

[+1 extra point]

The power provided to the resistor is then

$$P = \frac{1}{R} V^2 = \frac{1}{12} \cdot 4^2 = 1.33\,W$$

This means that the minimum power rating of the 12Ω-resistor has to be 1.33W.

[+1 extra point]

3.- The way to read the specification of the real-world power supply (rwps) is: provide 24V up to 1A current. So, for $R = 100\Omega$, we have

$$i = \frac{24}{100} = 0.24A < 1A,$$ which within spec.

[+0.5 points]

For $R = 10\Omega$, we have

$$i = \frac{24}{10} = 2.4A > 1A,$$ out of spec [+0.5 points]

For $R = 1\Omega$, we have

$$i = \frac{24}{1} = 24A > 1A,$$ out of spec [+0.5 points]

So the table of output voltage from the rwps looks like

[+0.5 points each]

R	voltage output
100Ω	24V
10Ω	less than 24V, 0V, or broken
1Ω	less than 24V, 0V, or broken