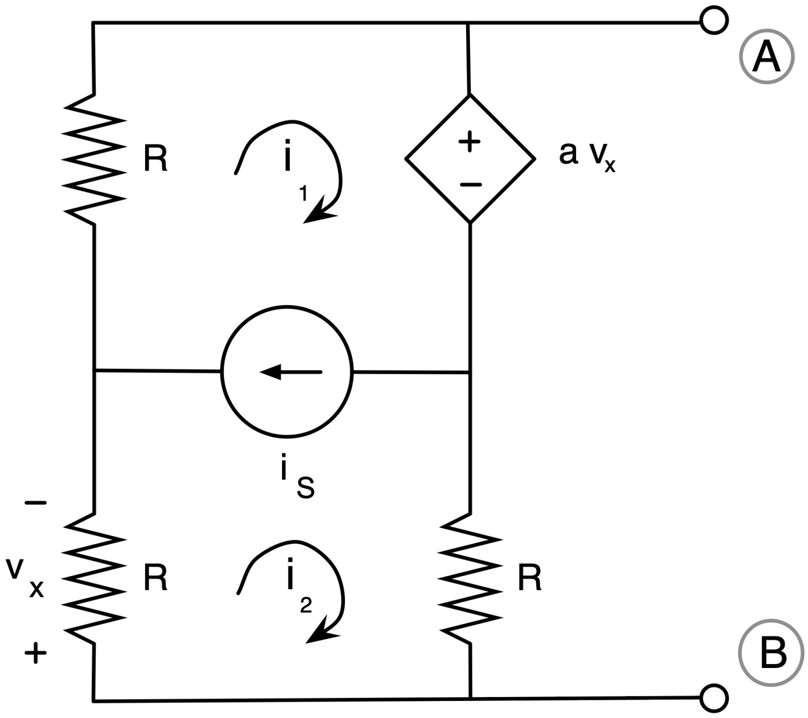
1.- Part I



Looking at the circuit, we observe the presence of 1 current source, which is a problem we need to deal with to use mesh current analysis.
The current source belongs to two meshes, so we use a supermesh to deal with it [method 3].
The supermesh equation is

$$i_1 - i_2 = i_S \qquad [+1 \text{ point}]$$

KCL for the supermesh reads like

$$a v_x + R i_2 + R i_2 + R i_1 = 0 \qquad [+1 \text{ point}]$$

We also need to account for the presence of the dependent source.

Looking at the circuit, we see that

$$V_x = R i_2$$ [+1 point]

This discussion leads to a total of 3 eqs in 3 unknowns i_1, i_2, V_x. We now solve them to find the unknowns.

$$i_1 = i_2 + i_S$$

$$aR i_2 + 2R i_2 + R(i_2 + i_S) = 0$$

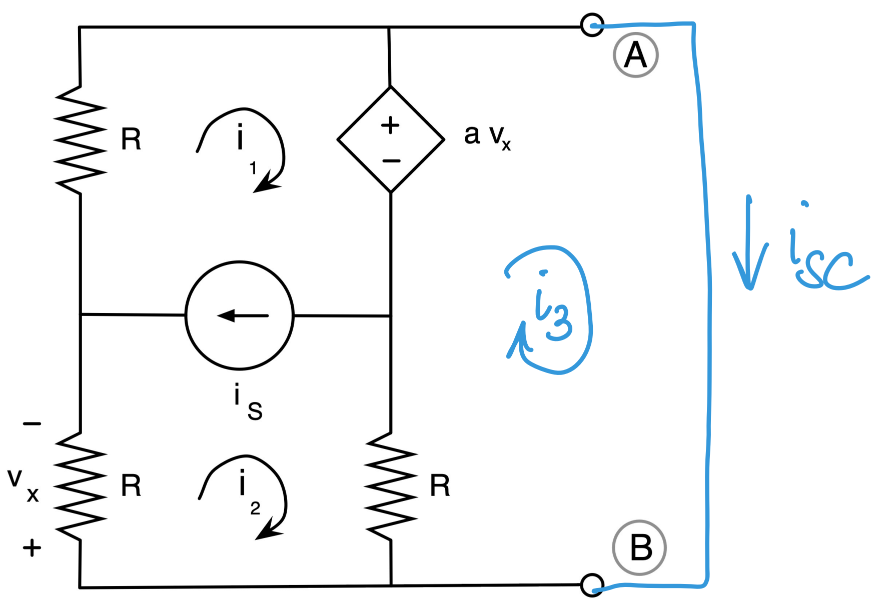
$$(aR + 3R)\, i_2 = -R i_S \Rightarrow i_2 = -\frac{1}{3+a} i_S .$$

The open-circuit voltage as seen from terminals A-B is

$$V_{AB} = a v_x + R i_2 = (aR + R)\left(-\frac{1}{3+a}\right) i_S =$$

$$= -R\frac{1+a}{3+a} i_S$$ [+1 point]

Part II



As instructed, we have connected the terminals A and B.

The short-circuit current is $i_3 = i_{SC}$. We use mesh-current analysis to find i_1, i_2, i_3 (and v_x).

We deal with the current source i_s with a supermesh,

$$i_1 - i_2 = i_S$$ [+1 point]

KCL for the supermesh now reads

$$av_x + R(i_2 - i_3) + Ri_2 + Ri_1 = 0$$ [+1 point]

KCL for the other mesh is

$$R(i_3 - i_2) - av_x = 0$$ [+1 point]

We account for the presence of the dependent source with

$$V_x = Ri_2$$ [+1 point]

So we have 4 eqs in 4 unknowns (i_1, i_2, i_3, V_x). Solving,

$$i_2 = -i_1 \Rightarrow -2i_2 = i_S \Rightarrow i_2 = -\frac{1}{2} i_S$$

$$V_x = -\frac{Ri_S}{2}$$

$$Ri_3 = aV_x + Ri_2 = -\frac{aR}{2} i_S - \frac{R}{2} i_S = -\frac{1+a}{2} Ri_S \Rightarrow i_3 = -\frac{1+a}{2} i_S$$

Therefore

$$i_{SC} = i_3 = -\frac{1+a}{2} i_S$$ [+1 point]

Part III

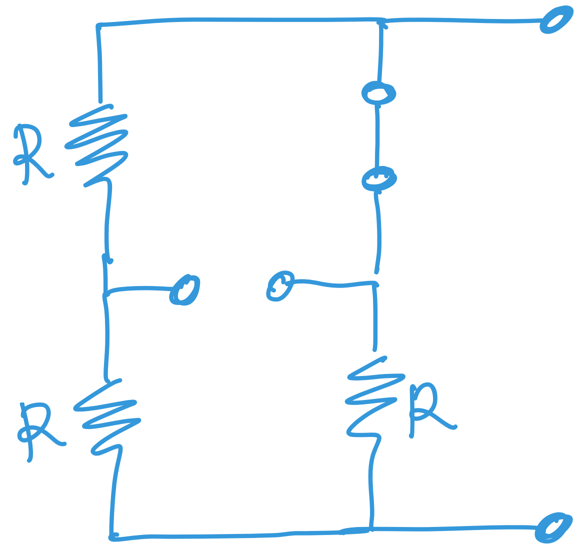
With the answers to Parts I & II, we have

$$V_T = V_{AB} = -R\frac{1+a}{3+a} i_S$$ [+0.5 point]

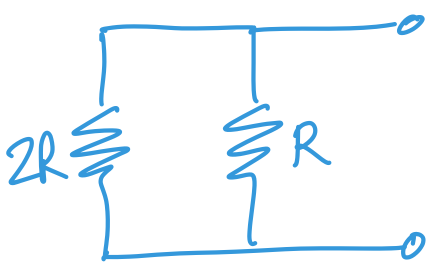
$$R_T = \frac{V_T}{i_{SC}} = +R\frac{\cancel{1+a}}{3+a}\cancel{i_S} \cdot \frac{+2}{\cancel{1+a}} \frac{1}{\cancel{i_S}} = \frac{2R}{3+a}$$ [+0.5 point]

Part IV

If we turn off the current source, we end up with the following circuit



We combine the 2 resistors R in series to get



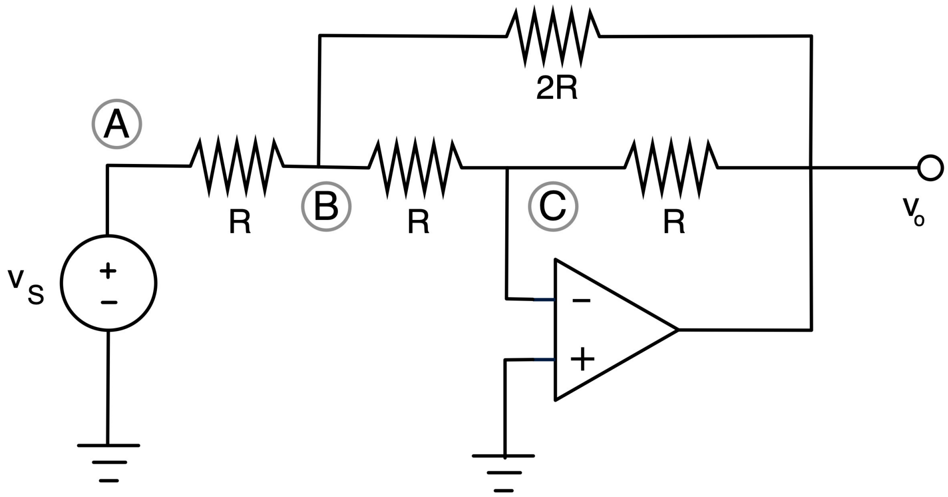
And now the resistors in parallel,

$$2R \| R = \frac{2R^{\cancel{2}}}{3\cancel{R}} = \frac{2R}{3}$$

[+0.5 extra point]

This is not the same as R_T. This is because when we turn off the current source, the dependent source also gets turned off, and its effect is not taken into account. [+0.5 extra point]

2.-



Part I

As instructed, we use nodal analysis to figure out the output voltage.

We know $V_A = V_S$. [+1 point]

KCL at node B gives us

$$\frac{1}{R}(V_B - V_S) + \frac{1}{R}(V_B - V_C) + \frac{1}{2R}(V_B - V_o) = 0$$

[+1 point]

KCL at node C (with $i_n = 0$),

$$\frac{1}{R}(V_C - V_B) + \frac{1}{R}(V_C - V_o) = 0$$ [+1 point]

Ideal conditions mean that

$$V_C = 0$$ [+1 point]

We have 3 eqs. in 3 unknowns V_B, V_C, V_O, so we can solve. From the 2nd and 3rd eqs,

$$V_O = -V_B$$

Substituting into the 1st equation,

$$\frac{1}{R}(V_B - V_S) + \frac{1}{R}V_B + \frac{1}{2R}(V_B + V_B) = 0$$

$$\frac{3}{R}V_B = \frac{V_S}{R} \Rightarrow V_B = \frac{V_S}{3}$$

Therefore $V_O = -\frac{V_S}{3}$,

so the correct answer is the second one. [+1 point]

Part II

[+1 point]

No, the output voltage will not change under ideal Op-Amp conditions because the op-amp has zero output resistance.

[+1 point]

Part III

With $V_S = 9V$, we would have $V_O = -\frac{V_S}{3} = -3V$.

If we connect a 10Ω load resistor, then the delivered power would be

$$P_{10\Omega} = \frac{1}{R}v^2 = \frac{1}{10} \cdot 9 = 0.9\,W \quad \text{(as claimed)}$$

If, instead, $V_S = 15V$, then (assuming the op-amp behaves linearly), $V_0 = -\frac{15}{3} = -5V$, and the power should be

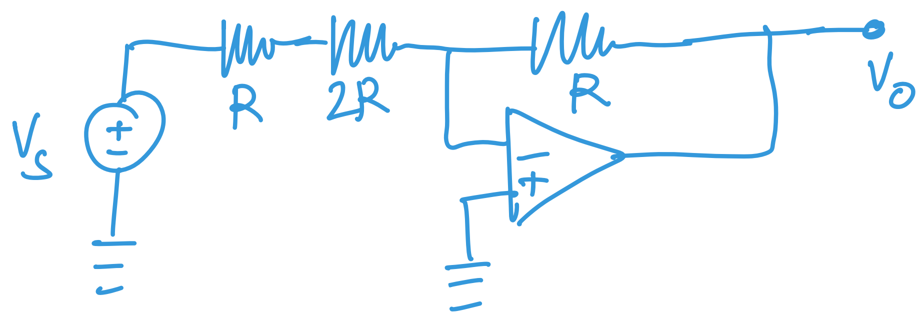
$$P_{10\Omega} = \frac{1}{10} 25 = 2.5W$$

However, the engineer measured the smaller power 1.6W. This must then be because the op-amp was saturated when we used $V_S = 15V$. Note that $P_{10\Omega} = 1.6W$ corresponds to a voltage across the load of magnitude 4V.
Therefore, it must be the case that $V_0 = -4V$ (instead of $V_0 = -5V$), so we deduce $-V_{CC} = -4V$.

[+2 points]

Part IV

The relationship $V_0 = -\frac{V_S}{3}$ can be easily realized with an inverting op amp, as follows



extra
[+0.5 point for using inv-op amp; extra +0.5 point for using correct values]

Note that we can write

$$V_0 = -\frac{R}{R+2R} V_S = -\frac{V_S}{3}$$

3.- **Part I**

The configuration is an inverting op. amp.

[+1 point]

Therefore

$$V_o = -\frac{R}{R_{FSR}} V_d$$

For $V_d = -5V$, we have

$$V_o = +\frac{5R}{R_{FSR}}$$

[+1 point]

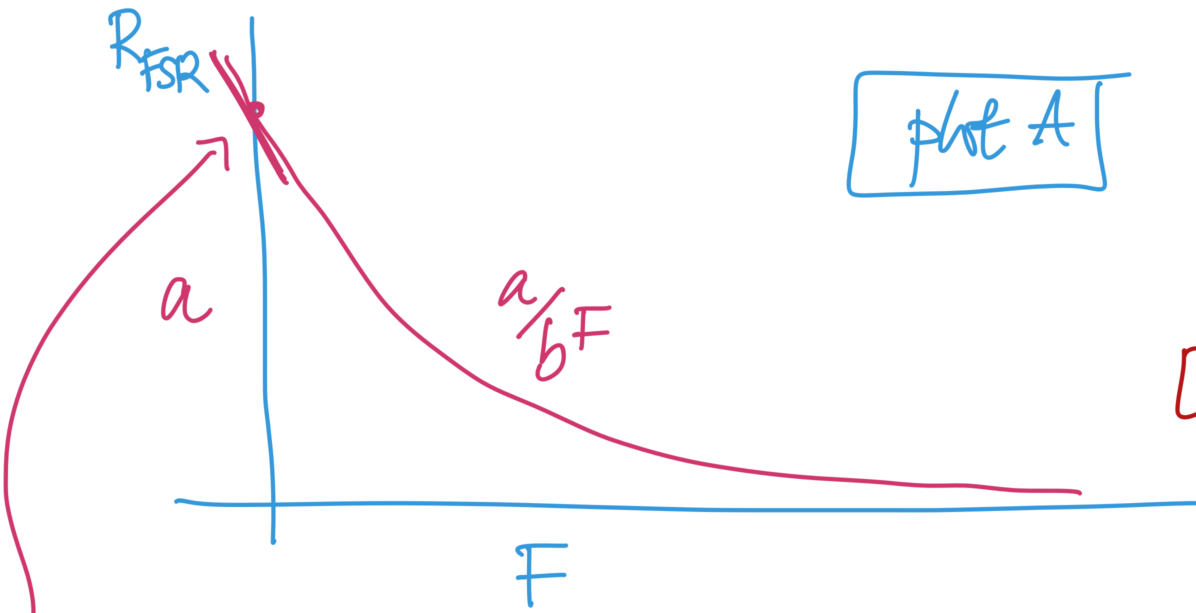
Part II

[+1 point]

If $V_d = 0$, then $V_o = 0$ too. This would not be a good of measuring the force on the FSR. In fact, no matter what value of the force, we will always measure $V_o = 0$, which does not allow us to distinguish one force value from another.

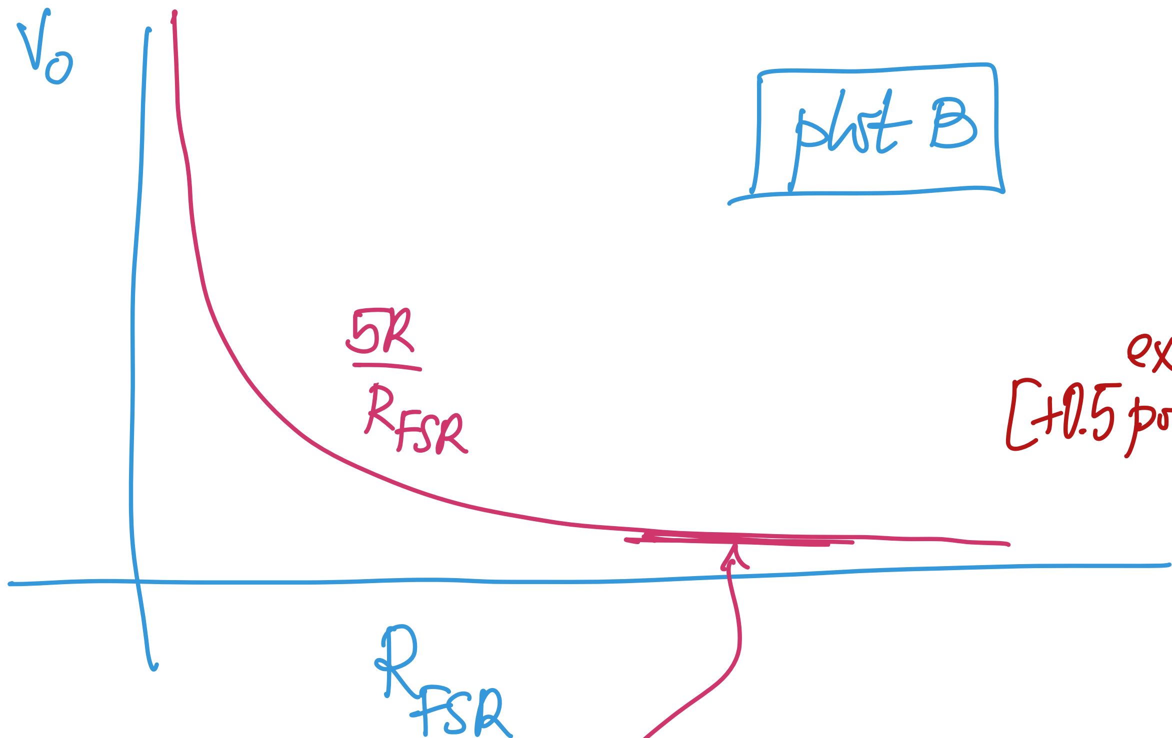
[+1 point]

Part III So, in general, we have $R_{FSR} = \frac{a}{b^F}$



extra [+0.5 point]

small change in F leads to a large change in R_{FSR}



extra [+0.5 point]

small change in R_{FSR} leads to small change in V_o.

The effect of using the Op-amp circuit is to smooth out / linearize the relationship between the output and the applied force. This can be seen by observing plots A & B.

W/o op-amp circuit : in plot A, we see that, when the force is small, a small change in the applied force results in a large change in the FSR.

W/ op-amp circuit: in plot B, we see that when the force is small ($\equiv$ large value of R_{FSR}), changes in the FSR still induce small changes in the output voltage.

extra
[+1 point]