MAE40 - Linear Circuits - Fall 25
Final Exam, December 11, 2025

Instructions

(i) You can use 3 two-sided 1-page handwritten cheatsheets.
(ii) The exam has 4 questions, for a total of 40 points and 5 bonus points.
(iii) You have from 3:00pm to 6:00pm to do the exam – but should require less time!
(iv) You can use a calculator with no communication capabilities.
(v) In your responses, clearly *articulate your reasoning* and *properly justify the steps.*
(vi) **Important:** start each part below on a separate page, and write your name & PID at the top of each page.

Good luck!

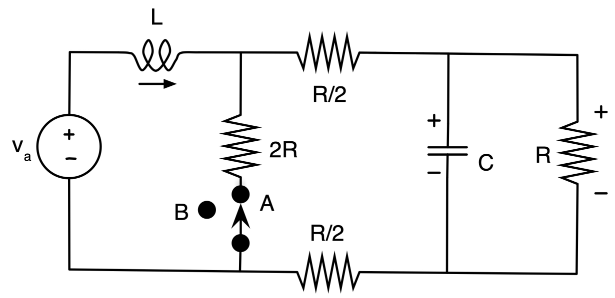

Figure 1: Circuit for Question 1.

1. **s-Domain Analysis**

Consider the circuit depicted in Figure 1. The value v_a of the voltage source is constant. The switch is kept in position **A** for a very long time. At time $t = 0$, it is moved to position **B**.

Part I: [3 points] Find the initial condition $v_C(0)$ for the capacitor and $i_L(0)$ for the inductor.

Part II [4 points] Transform the circuit in Figure 1 to the s-domain, using voltage sources to account for the initial conditions.

Part III [3 points] Set up mesh-current equations for the transformed circuit obtained in Part II (write them in matrix form). No need to solve any equations!

Part IV [1 bonus point] Express the transform $V(s)$ of the R-resistor voltage in terms of the mesh currents. What element in the transformed circuit obtained in Part II has voltage drop $V(s) - v_C(0)/s$?

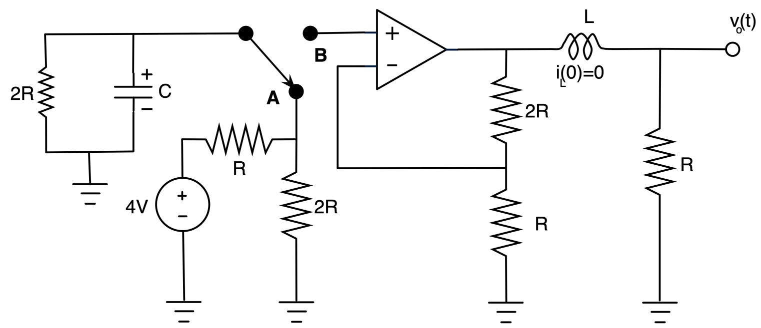

Figure 2: Circuit for Question 2.

2. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 2. The voltage source is constant. The switch is kept in position **A** for a very long time. At $t = 0$ it is moved to position **B**. Show that the initial capacitor voltage is given by

$$v_C(0^-) = 2V.$$

[Show your work]

Part II: [3 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the capacitor in which the initial condition appears as a voltage source. Show that the transfer function of the circuit is

$$T(s) = \frac{6R^2Cs}{(Ls + R)(2RCs + 1)}.$$

[Show your work]

Part III: [5 points] Use domain circuit analysis and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $C = \frac{1}{6}F$, $L = 1H$, and $R = 3\Omega$ is

$$v_o(t) = 9(e^{-t} - e^{-3t})u(t).$$

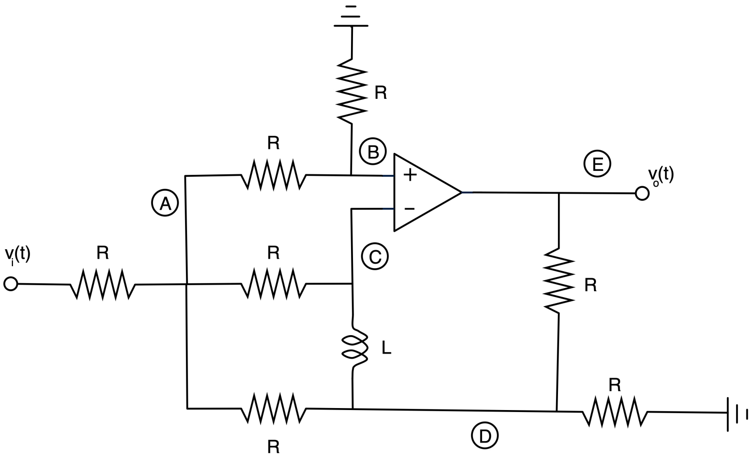

Figure 3: Circuit for Question 3.

3. **Frequency Response Analysis**

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 3 into the s-domain.

Part II: [3 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ is given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{-3Ls}{5R + Ls}.$$

[Show your work]

Part III [4 points] Let $R = 100$ mΩ, $L = 10$ mH. Compute the gain and phase functions of $T(s)$. What are the DC gain and the ∞-freq gain? What is the cut-off frequency ω_c? Use these values to sketch the magnitude of the frequency response. Is the circuit a low-pass, high-pass, or band-pass filter? [Explain your answer]

Part IV [2 points] Using what you know about frequency response, compute the steady state response $v_o^{SS}(t)$ of this circuit when $v_i(t) = -2\cos(50t - \frac{\pi}{4})$ using the same values of R and L as in Part III.

Part V: [2 bonus points] Design an inverting OpAmp circuit that has transfer function $T(s)$. What design would you recommend, your design or the one in Figure 3? Why?

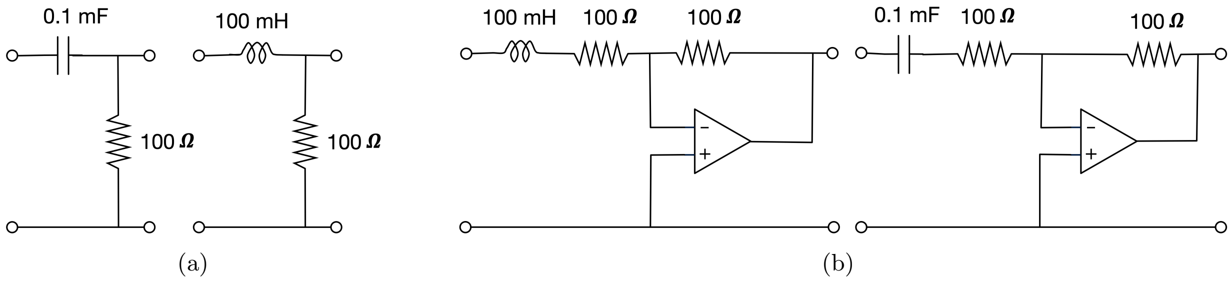

Figure 4: Circuits for Question 4.

4. **Loading and the Chain Rule**

A former instructor of MAE140 was given the task of designing a circuit with the following transfer function

$$T(s) = \frac{1000s}{s^2 + 1100s + 10^5}$$

Part I: [2 points] The instructor first decomposed the transfer function as follows

$$T(s) = \left(\frac{s}{s+100}\right)\left(\frac{1000}{s+1000}\right)$$

and came up with a design that combines in series two voltage dividers, see Figure 4(a). Compute the transfer function of each voltage divider and show that their product is equal to $T(s)$.

Part II: [3 points] When the instructor connected the two stages in series and used the input $v_i(t) = \cos(500t)$, they were surprised to observe that the steady-state output was not $v_o^{SS}(t) = \sqrt{\frac{10}{13}}\cos(500t - 0.2662)$, which is what the instructor was expecting. Can you explain why the instructor was expecting that response and why they did not get it? Justify your answer.

Part III: [1 point] Could you fix the design provided by the instructor, still employing the two voltage dividers and possibly using one op-amp, so that the instructor gets the steady-state output they were aiming for? Explain how.

Part IV: [3 points] The instructor could not figure out why the design with voltage dividers was not working, so they abandoned it. The instructor decomposed again the transfer function, this time as

$$T(s) = \left(\frac{-1000}{s+1000}\right)\left(\frac{-s}{s+100}\right)$$

and came up with a design that combines in series the two stages in Figure 4(b). Compute the transfer function of each stage and show that their product is equal to $T(s)$.

Part V: [1 point] If the instructor connects in series the two stages in Figure 4(b) and uses the input $v_i(t) = \cos(500t)$, would the steady-state output be what they were expecting in Part II? Why? Justify your answer.

Part VI: [2 bonus points] If the instructor were to connect a source circuit whose Thévenin equivalent is $v_T(t) = v_i(t)$ and $R_T = 100$Ohms to the design with op-amps in Part IV, should the instructor expect to get the steady-state output they were looking for too?