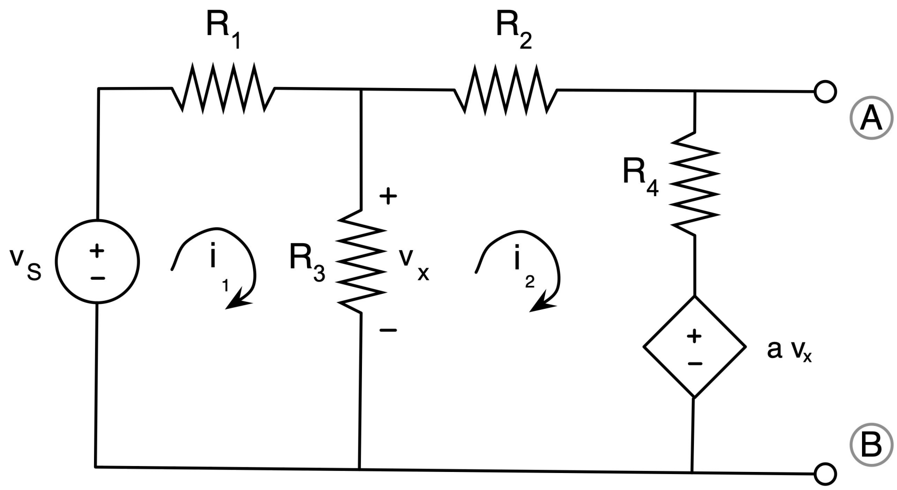
1.– Part I



There are no current sources in the circuit, so we can just directly set up mesh current equations. We do it by inspection,

$$\begin{pmatrix} R_1+R_3 & -R_3 \\ -R_3 & R_2+R_3+R_4 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} V_S \\ -aV_x \end{pmatrix}$$

[+2 points]

We need to account for the presence of the dependent source (in fact, the 2 equations above have 3 unknowns, i_1, i_2, V_x).

Looking at the circuit, we see that

$$V_x = R_3 (i_1 - i_2)$$ [+1 point]

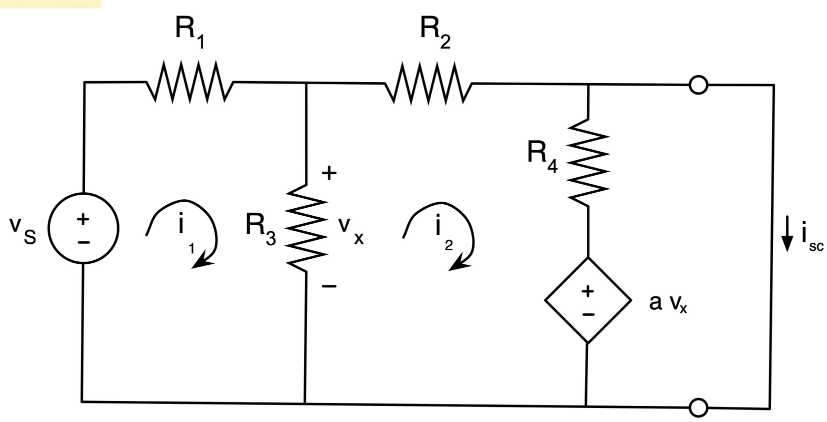
This gives a total of 3 eqs. in 3 unknowns.

In matrix form,

$$\begin{pmatrix} R_1+R_3 & -R_3 & 0 \\ -R_3 & R_2+R_3+R_4 & a \\ R_3 & -R_3 & -1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ V_x \end{pmatrix} = \begin{pmatrix} V_S \\ 0 \\ 0 \end{pmatrix}$$

(One could also substitute V_x in first matrix system, and write 2 eqs in 2 unknowns i_1, i_2) [+1 point]

Part II



With the terminals Ⓐ & Ⓑ connected, we now have three meshes. We write mesh-current equations again by inspection

$$\begin{pmatrix} R_1+R_3 & -R_3 & 0 \\ -R_3 & R_2+R_3+R_4 & -R_4 \\ 0 & -R_4 & R_4 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_{sc} \end{pmatrix} = \begin{pmatrix} V_S \\ -aV_x \\ aV_x \end{pmatrix}$$

[+3 points]

We again have to accommodate the presence of the dependent source,

$$V_x = R_3(i_1 - i_2)$$

[+1 point]

This yields a total of 4 eqs in 4 unknowns,

$$\begin{pmatrix} R_1+R_3 & -R_3 & 0 & 0 \\ -R_3 & R_2+R_3+R_4 & -R_4 & a \\ 0 & -R_4 & R_4 & -a \\ R_3 & -R_3 & 0 & -1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_{sc} \\ V_x \end{pmatrix} = \begin{pmatrix} V_S \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

(One could also substitute V_x and write 3 eqs in 3 unknowns)

Part III

From Part I, we know that the open-circuit voltage for the circuit in Figure 1 is

$$V_T = V_{oc} = V_{AB} = R_4 i_2 + a v_x =$$

$$= \frac{(1-a)R_3 R_4 V_S + a(R_2+R_4)R_3 V_S}{R_1R_2 + (1-a)R_1R_3 + R_2R_3 + R_1R_4 + R_3R_4} =$$

[+1 point]

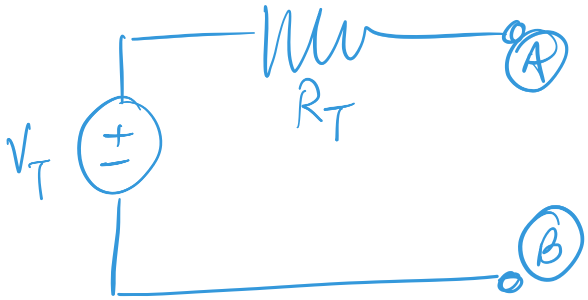
$$= \frac{(R_4 + aR_2)R_3 V_S}{R_1R_2 + (1-a)R_1R_3 + R_2R_3 + R_1R_4 + R_3R_4}$$

From Part II, we know the short-circuit current, so

$$R_T = \frac{V_{oc}}{i_{SC}} = \frac{(R_1R_2 + R_1R_3 + R_2R_3)R_4}{R_1R_2 + (1-a)R_1R_3 + R_2R_3 + R_1R_4 + R_3R_4}$$

[+1 point]

This yields the Thévenin equivalent



Part IV

From Part III, we have

$$V_{AB} = \frac{(R_4 + aR_2)\, R_3\, V_S}{R_1R_2 + (1-a)\, R_1R_3 + R_2R_3 + R_1R_4 + R_3R_4}$$

The dominant term in the numerator is the one with aR_3. Same thing for the denominator. Therefore

extra [+1 point]

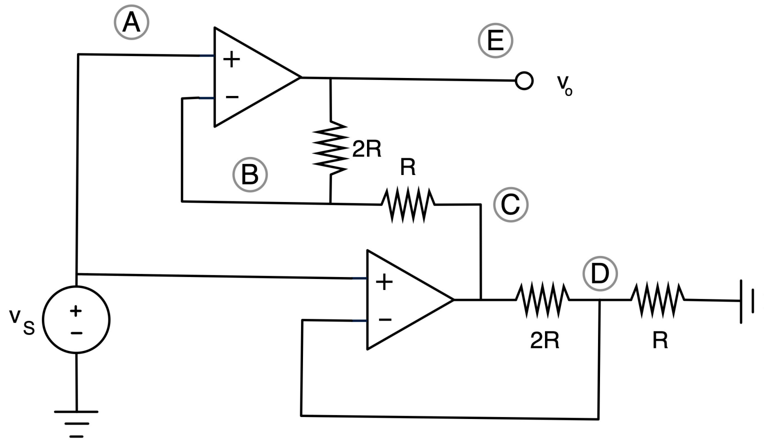
$$V_{AB} \approx \frac{aR_2R_3}{-aR_1R_3} V_S = -\frac{R_2}{R_1} V_S$$

This is a model for an <u>inverting op-amp.</u>

extra [+1 point]

2.-

Part I



As instructed, we use nodal analysis to figure out the output voltage. Note $V_O = V_E$

We know $V_A = V_S$. [+1 point]

KCL at node B yields

$$\frac{1}{2R}(V_B - V_E) + \frac{1}{R}(V_B - V_C) = 0 \quad [+1\ \text{point}]$$

(here, we have used op-amp ideal conditions, in particular $i_p = i_N = 0$)

KCL at node D yields

$$\frac{1}{2R}(V_D - V_C) + \frac{1}{R}(V_D - 0) = 0 \quad [+1\ \text{point}]$$

We don't write KCL eqs for the output nodes C and E. But we resort to ideal conditions to state

$$V_A = V_B \quad \& \quad V_A = V_D$$ [+2 points]

Solving for V_C in KCL @ D, we get

$$V_D - V_C + 2V_D = 0 \Rightarrow V_C = 3V_S$$

Solving for V_E in KCL @ B, we get

$$V_B - V_E + 2V_B - 2V_C = 0 \Rightarrow$$

$$V_E = 3V_B - 2V_C = 3V_S - 6V_S = -3V_S$$

Therefore,

$$\boxed{V_O = -3V_S}$$ [+1 point]

Part II

With $V_{CC} = \pm 25V$ for op-amp at the top and $V_{CC} = \pm 50V$ for op-amp at the bottom, we have

$$V_C = 3V_S = 3 \cdot 9 = 27V < 50V$$

However,

$$-3V_S = -27V < -25V$$

So the op-amp at the top gets saturated and hence $V_o = -25V$. [+1 point]

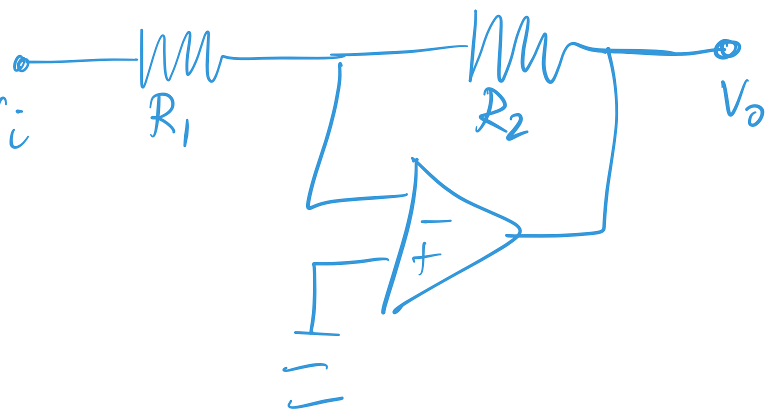
The power consumed by the load resistor is then

$$P_L = \frac{1}{R_L} V_o^2 = \frac{25^2}{100} = 6.25W$$

So the engineer was surprised b/c the saturation lowered the power the engineer was expecting, $\frac{27^2}{100} = 7.29W$. [+1 point]

Part III

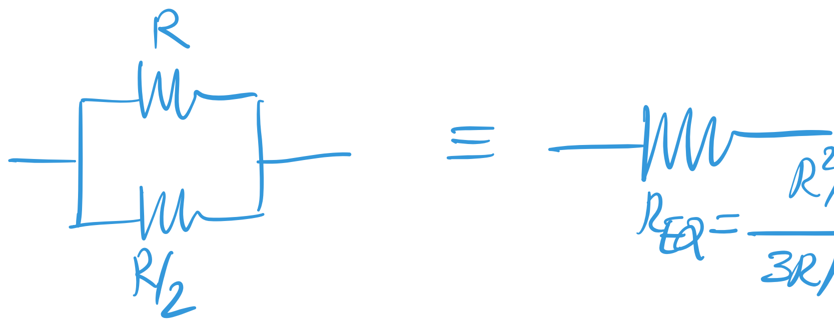
Since we want to generate $V_o = -3V_s$, we use an inverting op-amp.



$$V_o = -\frac{R_2}{R_1} V_i$$

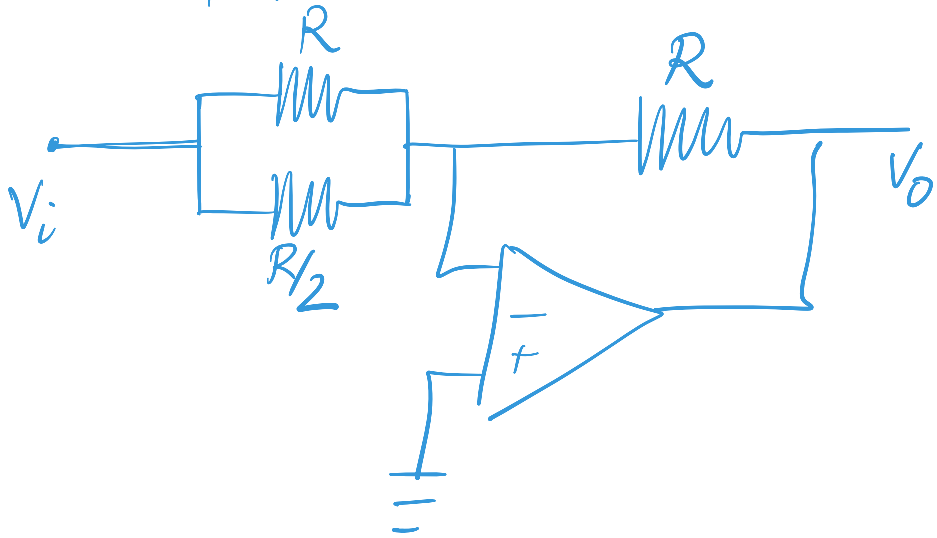
We only have 3 resistors: 2 of value R and 1 of value $R/2$ to make $\frac{R_2}{R_1} = 3$.

If we put R and $R/2$ in parallel, we obtain



$$R_{EQ} = \frac{R^2/2}{3R/2} = \frac{R}{3}$$

So our proposed design is



So that
$$V_o = \frac{-R}{R/3} V_i = -3V_i$$

[+1 point for using inverting op-amp, +1 point for correct design]