MAE140 - Linear Circuits - Winter 09
Midterm, February 5

Instructions

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 70 minutes

(iii) Do not forget to write your name, student number, and instructor

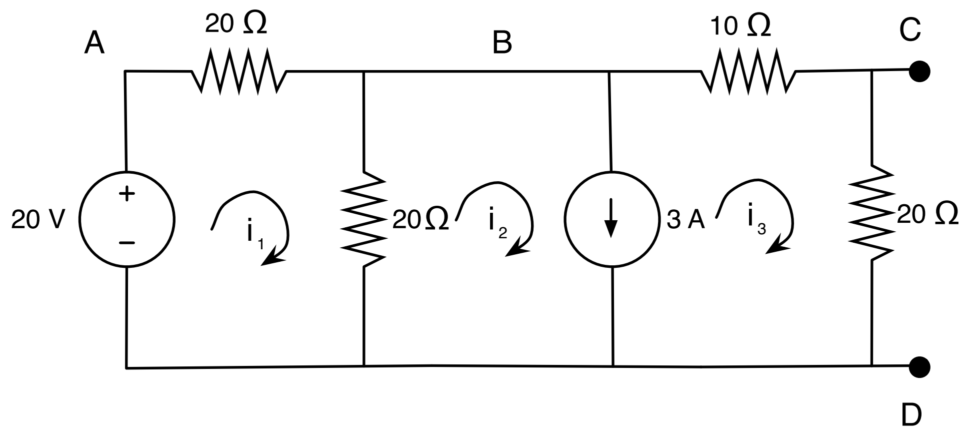


Figure 1: Circuit for questions 1-4.

1. **Equivalent circuits**

Part I: [2 points] Turn off all the sources in the circuit of Figure 1 and find the equivalent resistance as seen from terminals C and D.

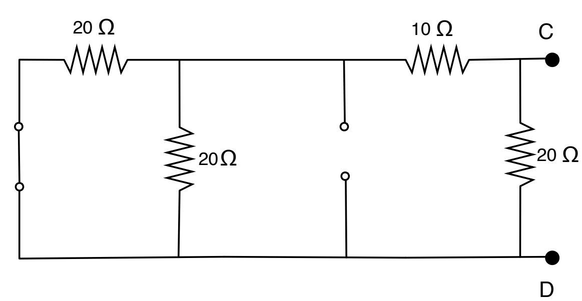
Part II: [3 points] Find the Thévenin equivalent as seen from terminals C and D.
Hint: If you want, you can use the result obtained in Part I

Part III: [1 point] Find the power absorbed by a $9\,\Omega$ resistor that is connected to terminals C and D.

Solution: Part I: We begin by turning all sources off.

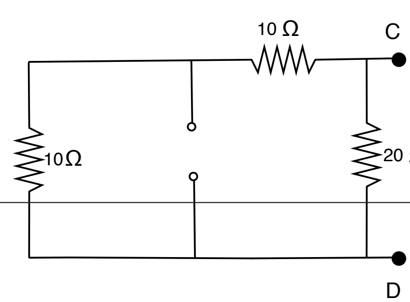
We substitute the voltage source by a short circuit, and the current source by an open circuit. Then, we get the circuit on the right

[1 point]



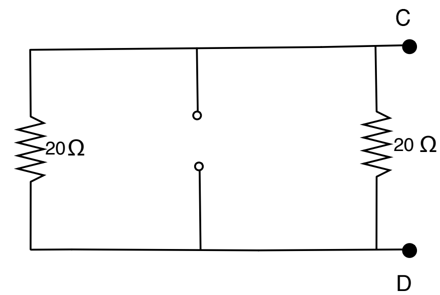
Now, we combine the two resistances in parallel to get the circuit on the right

[.33 point]



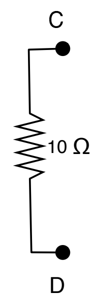
Now we sum the resistances in series to get the circuit on the right

[.33 point]



Finally, combine the two resistances in parallel to get the equivalent resistance

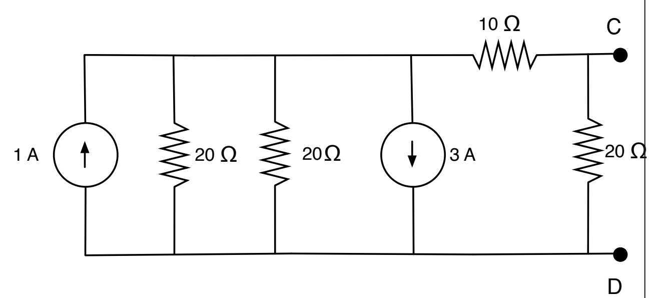
[.33 point]



Part II: We have already computed in Part I the Thévenin resistance, $R_T = 10\Omega$ (alternatively, we could recompute it here by computing the open circuit voltage and the short circuit current). Let us compute v_T by computing the open circuit voltage.

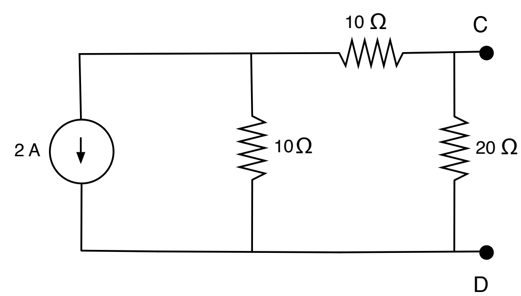
We begin by doing a source transformation to get two resistances in parallel and two current sources in parallel.

[1 point]



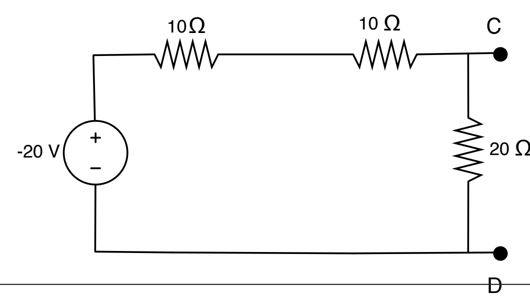
We combine the resistances in parallel and the current sources in parallel.

[.5 point]



We now do one more source transformation to get all resistances in series.

[1 point]

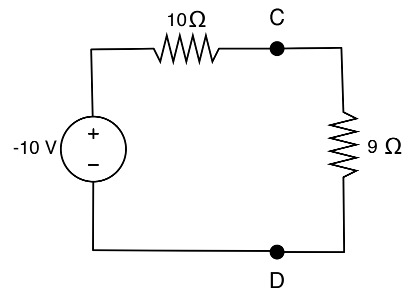


Finally, we use voltage division to find the voltage across the 20Ω resistance as

$$v_T = \frac{20}{20+10+10}(-20) = -10V$$

[.5 point]

Part III: Since we have computed the Thévenin equivalent of the circuit in Part II, we can simply represent the circuit with the 9Ω load as



Using voltage division, we find that the voltage across the 9Ω resistance is

$$\frac{9}{19}(-10) = -4.74\,V$$

Therefore, the power absorbed by the resistance is

$$P = VI = GV^2 = \frac{1}{9}4.74^2 = 2.493\,W \qquad \textbf{[1 point]}$$

2. **Nodal voltage analysis**

   [6 points] Assuming that the node labeled D is the ground node (reference), formulate node-voltage equations for the circuit in Figure 1. Use the node labels provided in the figure and clearly indicate how you handle the presence of a voltage source, the final equations, and the unknowns they must be solved for. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution:

We begin by associating a voltage variable to each node A-C (remember that D is ground). In principle, the independent voltage source poses a problem to write the nodal equations. However, because the node D is grounded, we deduce that

$$v_A = 20V \qquad \textbf{[2 points]}$$

and that takes care of the problem. We can then write the KCL equations for nodes B and C.

KCL for node B looks like

$$\frac{1}{20}(v_B - v_A) + \frac{1}{20}v_B + \frac{1}{10}(v_B - v_C) + 3 = 0 \qquad \textbf{[1 point]}$$

KCL for node C looks like

$$\frac{1}{10}(v_C - v_B) + \frac{1}{20}v_C = 0 \qquad \textbf{[1 point]}$$

Substituting the value of v_A and re-arranging the terms, we get the linear equation

$$\begin{pmatrix} \frac{1}{5} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{3}{20} \end{pmatrix} \begin{pmatrix} v_B \\ v_C \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \qquad \textbf{[2 points]}$$

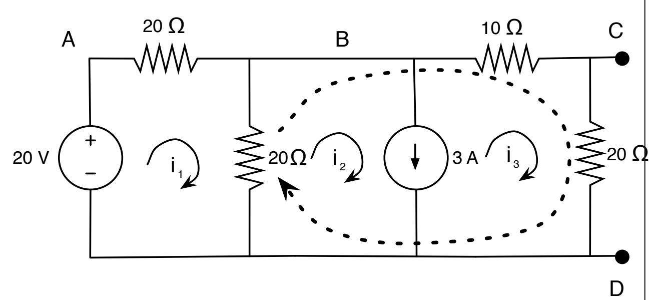
3. **Mesh current analysis**

[6 points] Formulate mesh-current equations for the circuit in Figure 1. Use the mesh currents shown in the figure and clearly indicate how you handle the presence of a current source, the final equations, and the unknowns they must be solved for. **Do not modify the circuit or the labels**. **Do not use any source transformation**. No need to solve any equations! *Hint: Use a supermesh*

Solution: The current source is owned by more than one mesh, so we can not use method #2. The statement of the problem says that we cannot use source transformation – which would work because the current source is in parallel with a resistance (method #1). Therefore, we are only left with method #3, the supermesh.

We create a supermesh as indicated in the figure on the right.

[2 points]



KVL for mesh 1 reads

$$20i_1 + 20(i_1 - i_2) = 20 \qquad \textbf{[1 point]}$$

KVL for the supermesh reads

$$20(i_2 - i_1) + 10i_3 + 20i_3 = 0 \qquad \textbf{[1 point]}$$

Finally, we get the following equation because of the way we chose the supermesh

$$i_2 - i_3 = 3 \qquad \textbf{[1 point]}$$

This gives us 3 equations with 3 unknowns. We can therefore write the linear system

$$\begin{pmatrix} 40 & -20 & 0 \\ -20 & 20 & 30 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 20 \\ 0 \\ 0 \end{pmatrix} \qquad \textbf{[1 point]}$$

4. **Bonus question**

   [1 point] If you were allowed to use source transformations in the circuit of Figure 1, describe what would you do in order to avoid having to use a supermesh in Question 3? **Do not write or solve any equations**!

Solution: If were allowed to do source transformations in the circuit, then we will use method $\#1$, i.e., we would transform the current source in parallel with the $20\,\Omega$ resistance into a voltage source in series with a $20\,\Omega$ resistance. Setting up mesh current analysis after this would be easy because of the independent sources would be voltage sources (and on top of that, we would have one mesh less). **[1 point]**