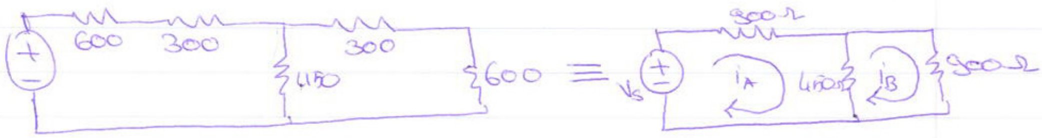
3.72) With Attenuator, calculate the power delivered to the load.

To do that calculate the current on the load




Mesh A: $-V_s + 900\, i_A + 450\, i_A - 450\, i_B = 0$

Mesh B: $-450\, i_A + 450\, i_B + 900\, i_B = 0$

from mesh B

$$1350\, i_B = 450\, i_A \Rightarrow 3 i_B = i_A$$

substitute into mesh A

$$\Rightarrow i_B = V_s/3600$$

Current flow over the 600 Ω load is equal to $i_B$ then we can calculate Power from equation

$$P_1 = i^2 \cdot R = \frac{V_s^2}{(3600)^2} \cdot 600 = \frac{V_s^2}{3600 \cdot 6}$$

Now remove the Attenuator

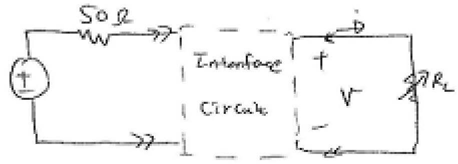

from voltage division

$$V = \frac{600}{(600+600)} V_s = \frac{V_s}{2}$$

$$P_2 = \frac{V^2}{R} = \frac{V_s^2}{600 \cdot 4}$$

then Power ratios in dB $\quad 10 \log(P_2/P_1) = 10 \log(9)$

$\boxed{3.73}$

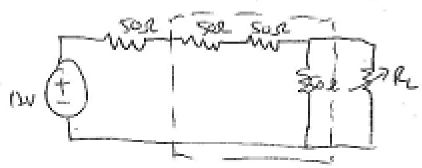


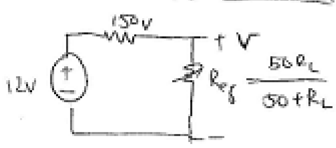

Design I/C such that

$V \le 4V$ & $i \le 100mA$ regardless of $R_L$

$\Rightarrow$ At least 1 of the 50-Ω resistors must be arranged in parallel with $R_L$ in order to meet the design criteria.

Use 1 in parallel and 2 in series:




$$\therefore V = \frac{\left(\frac{50R_L}{50+R_L}\right)}{150+\left(\frac{50R_L}{50+R_L}\right)} \times 12 \quad \leqslant 4 \text{ for any } R_L$$

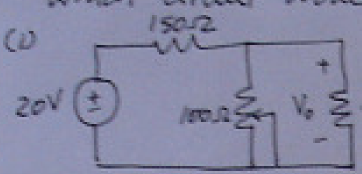
$$i_{max} = \frac{12}{150} = 0.08\,A \le 100mA \qquad \#$$

3.45)

Required to deliver 0 to 5 V to a 100Ω Load from a 20V source rated at 2.5W.

Which circuit would you choose?

(i)

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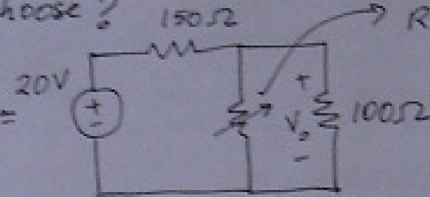
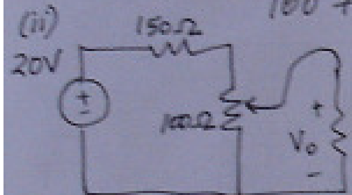



$$V_o = \left( \frac{\frac{100R_V}{100+R_V}}{150 + \frac{100R_V}{100+R_V}} \right) 20V = 20\left( \frac{100R_V}{15000 + 250R_V} \right)$$

(ii)

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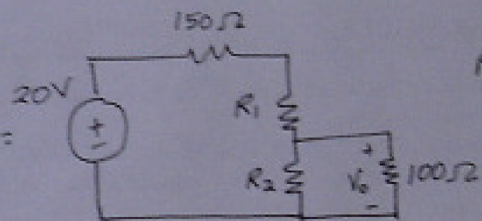



$R_1 + R_2 = 100\Omega$

$$V_o = 20\left( \frac{\frac{100R_2}{100+R_2}}{150 + R_1 + \frac{100R_2}{100+R_2}} \right) = 20\left( \frac{100R_2}{(150+R_1)(100+R_2) + 100R_2} \right)$$

Since $R_1 + R_2 = 100$

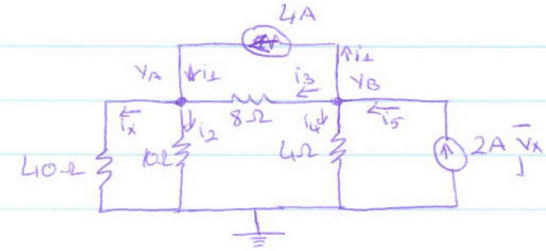
$\Rightarrow R_1 = 100 - R_2$

$$V_o = 20\left( \frac{100R_2}{25000 + 250R_2 - R_2^2} \right)$$

We are required to find a circuit where the output voltage is close to 0. By looking at the equations for $V_o$ for each of the circuit, it can be seen that $V_o$ in the I circuit is more sensitive to changes in Rv. Therefore, it could be difficult to obtain $V_o$ close to 0. On the other hand, $V_o$ in the second circuit is less sensitive to changes in $R_2$. Therefore, circuit 2 should be chosen.

20

3.2)




a) step 1: The reference node, node voltage and element currents are shown in the figure.

step 2:

Node A: $i_1 + i_3 - i_x - i_2 = 0$

Node B: $i_5 - i_1 - i_4 - i_3 = 0$

step 3:

$i_1 = 4A$ $\qquad i_4 = \frac{V_B}{4}$

$i_2 = \frac{V_A}{10}$ $\qquad i_5 = 2A$

$i_3 = \frac{(V_B - V_A)}{8}$ $\qquad i_x = \frac{V_A}{40\Omega}$

Step 4: Substituting the element equation into KCL constraints

Node A: $V_A \frac{1}{4} - V_B \frac{1}{8} = 4 \Rightarrow 2V_A - V_B = 32$

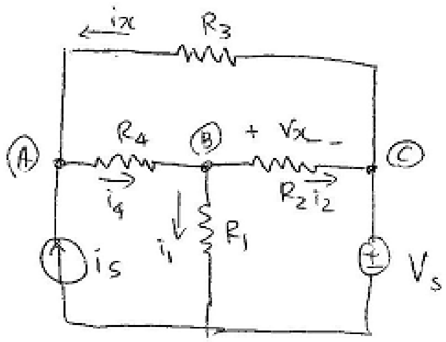
Node B: $V_A \frac{1}{8} - 3V_B \frac{1}{8} = 2 \Rightarrow V_A - 3V_B = 16$

$$\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 32 \\ 16 \end{bmatrix}$$

b) By solving the linear equation, we can find $V_A = 16$ and $V_B = 0$

$i_x = \frac{V_A}{40} \Rightarrow i_x = \frac{4}{10} A$ and $V_x = V_B = 0$

## 3.5




a) Formulate node-voltage equations

KCL @ A

$$i_s + i_x - i_4 = 0 \quad \cdots (1)$$

KCL @ B

$$i_4 - i_1 - i_2 = 0 \quad \cdots (2)$$

$$i_1 = \frac{V_B}{R_1}, \quad i_2 = \frac{V_B - V_C}{R_2}, \quad i_4 = \frac{V_A - V_B}{R_4}, \quad i_x = \frac{V_C - V_A}{R_3}, \quad V_C = V_s$$

Eqn (1) becomes

$$\frac{V_s - V_A}{R_3} - \frac{V_A - V_B}{R_4} = -i_s \quad \cdots (3)$$

Eqn (2) becomes

$$\frac{V_A - V_B}{R_4} - \frac{V_B}{R_1} - \frac{V_B - V_s}{R_2} = 0 \quad \cdots (4)$$

#

b) $R_1 = R_2 = R_3 = R_4 = 10\,k\Omega$

$V_s = 20\,V$

$i_s = 2\,mA$

Solve for $V_x$ and $i_x$

Eqn ③: $\frac{V_s}{R_3} - \frac{V_A}{R_3} - \frac{V_A}{R_4} + \frac{V_B}{R_4} = -i_s$

$$-\left(\frac{1}{R_3} + \frac{1}{R_4}\right)V_A + \frac{V_B}{R_4} = -i_s - \frac{V_s}{R_3} \quad \cdots ⑤$$

Eqn ④: $\frac{V_A}{R_4} - \frac{V_B}{R_4} - \frac{V_B}{R_1} - \frac{V_B}{R_2} + \frac{V_s}{R_2} = 0$

$$\frac{V_A}{R_4} - \left(\frac{1}{R_4} + \frac{1}{R_1} + \frac{1}{R_2}\right)V_B = -\frac{V_s}{R_2} \quad \cdots ⑥$$

From Eqn ⑥,

$$V_A = \left(\left(\frac{1}{R_4} + \frac{1}{R_1} + \frac{1}{R_2}\right)V_B - \frac{V_s}{R_2}\right)R_4$$

Plug into Eqn ⑤.

$$-\left(\frac{1}{R_3} + \frac{1}{R_4}\right)\left(\left(\frac{1}{R_4} + \frac{1}{R_1} + \frac{1}{R_2}\right)V_B - \frac{V_s}{R_2}\right)R_4 + \frac{V_B}{R_4} = -i_s - \frac{V_s}{R_3}$$

$$V_B\left[-\left(\frac{1}{R_3} + \frac{1}{R_4}\right)\left(\frac{1}{R_4} + \frac{1}{R_1} + \frac{1}{R_2}\right)R_4 + \frac{1}{R_4}\right] + R_4\left(\frac{1}{R_3} + \frac{1}{R_4}\right)\frac{1}{R_2}V_s = -i_s - \frac{V_s}{R_3}$$

$$-\frac{1}{2000}V_B + \frac{1}{500}(20) = -2\times10^{-3} - \frac{20}{10000}$$

$$-\frac{1}{2000}V_B = -\frac{1}{125}$$

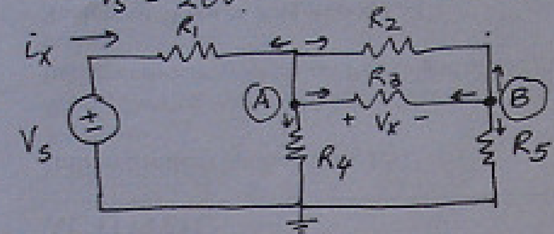
$$V_B = 16\,V$$

$$V_A = 28\,V$$

$$\boxed{\begin{aligned} \therefore V_x &= V_B - V_C = 16 - 20 = -4V \\ \therefore I_x &= \frac{V_C - V_A}{R_3} = \frac{20-28}{10000} = -0.0008\,A = -0.8\,mA \end{aligned}}$$

3.6) a.) Formulate node voltage eqns for the circuit.

b) Solve for $V_x$ and $i_x$ when $R_1 = R_2 = R_3 = R_4 = R_5 = 10k\Omega$, and $V_s = 20V$.




$V_x = V_A - V_B$

$i_x = \dfrac{V_s - V_A}{R_1} = \dfrac{20 - V_A}{10k}$

a.) Node-voltage eqn

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} & \frac{-1}{R_2} - \frac{1}{R_3} \\ \frac{-1}{R_2} - \frac{1}{R_3} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} \cancel{i_x} \\ 0 \end{bmatrix} \quad \frac{V_s}{R_1}$$

KCL @ Node A:

$$\frac{V_A}{R_4} + \frac{V_A - V_B}{R_3} + \frac{V_A - V_B}{R_2} + \frac{V_A - V_s}{R_1} = 0$$

$$V_A\left(\frac{1}{R_4} + \frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_1}\right) + V_B\left(\frac{-1}{R_3} - \frac{1}{R_2}\right) = \frac{V_s}{R_1}$$

KCL @ Node B:

$$\frac{V_B}{R_5} + \frac{V_B - V_A}{R_3} + \frac{V_B - V_A}{R_2} = 0$$

$$V_B\left(\frac{1}{R_5} + \frac{1}{R_3} + \frac{1}{R_2}\right) + V_A\left(\frac{-1}{R_3} - \frac{1}{R_2}\right) = 0 \quad \text{——} (*)$$

b.) $V_A\left(\frac{1}{10k} + \frac{1}{10k} + \frac{1}{10k} + \frac{1}{10k}\right) + V_B\left(\frac{-1}{10k} - \frac{1}{10k}\right) = \frac{20V}{10k}$

$V_A\left(\frac{4}{10k}\right) - V_B\left(\frac{2}{10k}\right) = 2mA$

$4V_A - 2V_B = 20$ —— (1)

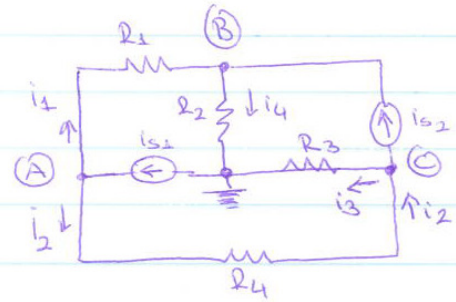
from (*) $\quad 3V_B - 2V_A = 0 \Rightarrow V_B = \frac{2V_A}{3}$

from (1) $\quad 4V_A - 2\left(\frac{2V_A}{3}\right) = 20$

$12V_A - 4V_A = 60 \Rightarrow 8V_A = 60 \Rightarrow V_A = 7.5V \quad \therefore V_B = 5V$

$\therefore \boxed{V_x = 7.5 - 5\,V = 2.5V} \quad \boxed{i_x = 1.25mA}$

3.7)




a) Step 1: The reference node, node voltage and element currents are shown in the figure

Step 2:

Node A: $i_{s1} - i_1 - i_2 = 0$

Node B: $i_{s2} + i_1 - i_4 = 0$

Node C: $i_2 - i_3 - i_{s2} = 0$

Step 3:

$$i_1 = \frac{(V_A - V_B)}{R_1}$$

$$i_2 = \frac{(V_A - V_C)}{R_4} \qquad i_4 = \frac{V_B}{R_2}$$

$$i_3 = \frac{V_C}{R_3}$$

Step 4: Substituting the element eqn. into KCL constraints

Node A: $\left(\frac{1}{R_1} + \frac{1}{R_4}\right) V_A - \frac{V_B}{R_1} - \frac{V_C}{R_4} = i_{s1}$

Node B: $\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_B - \frac{V_A}{R_1} = i_{s2}$

Node C: $\frac{V_A}{R_4} - \left(\frac{1}{R_4} + \frac{1}{R_3}\right) V_C = i_{s2}$

3.7) cont.)

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_4} & \frac{-1}{R_1} & \frac{-1}{R_4} \\ -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2} & 0 \\ \frac{1}{R_4} & 0 & \frac{-1}{R_4}-\frac{1}{R_3} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_{s1} \\ i_{s2} \\ i_{s2} \end{bmatrix}$$

b) $R_1 = 1k\Omega$, $R_2 = 2k\Omega$, $R_3 = 4k\Omega$, $R_4 = 2k\Omega$ and $i_{s1} = i_{s2} = 2mA$

Then

Node A: $\frac{3}{2}V_A - V_B - \frac{V_C}{2} = 2$ (I)

Node B: $\frac{3}{2}V_B - V_A = 2 \Rightarrow V_B = \frac{2}{3}V_A + \frac{4}{3}$ (II)

Node C: $\frac{V_A}{2} - \frac{3}{4}V_C = 2 \Rightarrow V_C = \frac{2}{3}V_A - \frac{8}{3}$ (III)

Substitute eqn (II) and (III) in to eqn (I)

and find $\underline{\underline{V_A = 4\,V}}$ (units $k\Omega \sim mA$ gives Volt)

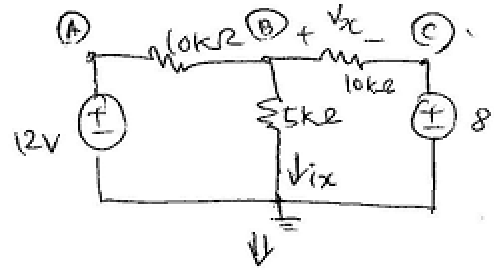
By using eqn (II) and $V_A = 4V$

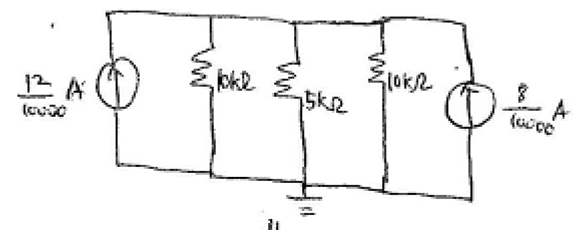
$$V_B = \frac{2}{3}\cdot 4 + \frac{4}{3} = \frac{12}{3} = \underline{\underline{4\,V}} = V_B$$

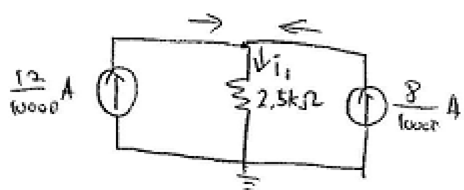
and finally eqn (III)

$$V_C = \frac{2}{3}\cdot 4 - \frac{8}{3} = \underline{\underline{0}} = V_C$$

3.9




Nodal Analysis:

$$\frac{12}{10000} + \frac{8}{10000} = i_1$$

$$i_1 = 2\times10^{-3}\ A$$

$$V_B = i_1(2.5k\Omega) = 5V$$

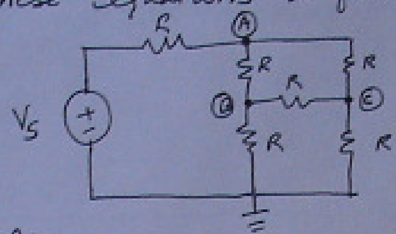
$$V_B = 8V + V_x = 5V$$

$$\therefore V_x = 5-8 = \underline{\underline{-3V}}$$

$$\therefore i_x = \frac{V_B}{5k\Omega} = \frac{5}{5000} = \underline{\underline{1mA}}$$

3.16 a) Formulate node-voltage eqns for the circuit

b) Use these equations to find the input resistance.




a) KCL @ node A:

$$\frac{V_A - V_S}{R} + \frac{V_A - V_B}{R} + \frac{V_A - V_C}{R} = 0$$

$$V_A\left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right) - V_B\left(\frac{1}{R}\right) - V_C\left(\frac{1}{R}\right) = \frac{V_S}{R} \Rightarrow 3V_A - V_B - V_C = V_S \quad \text{—(1)}$$

KCL @ node B:

$$\frac{V_B - V_A}{R} + \frac{V_B - V_C}{R} + \frac{V_B}{R} = 0$$

$$V_B\left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right) - V_A\left(\frac{1}{R}\right) - V_C\left(\frac{1}{R}\right) = 0 \Rightarrow -V_A + 3V_B - V_C = 0 \quad \text{—(2)}$$

KCL @ node C:

$$\frac{V_C - V_A}{R} + \frac{V_C - V_B}{R} + \frac{V_C}{R} = 0$$

$$V_C\left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right) - V_A\left(\frac{1}{R}\right) - V_B\left(\frac{1}{R}\right) = 0 \Rightarrow -V_A - V_B + 3V_C = 0 \quad \text{—(3)}$$

Node-voltage equations:

$$\begin{bmatrix} 1/R + 1/R + 1/R & -1/R & -1/R \\ -1/R & 1/R + 1/R + 1/R & -1/R \\ -1/R & -1/R & 1/R + 1/R + 1/R \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} V_S/R \\ 0 \\ 0 \end{bmatrix}$$

b) From eqn (2): $V_C = -V_A + 3V_B$ —(4)

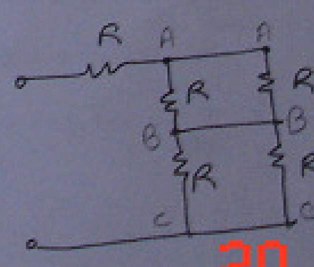
substituting (4) in (3) $\quad -V_A - V_B - 3V_A + 9V_B = 0$

$$-4V_A + 8V_B = 0$$

$$V_A = 2V_B \quad \text{—(5)}$$

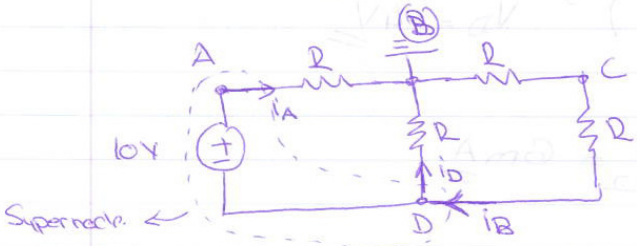
substituting (5) in (4) $\quad V_C = -2V_B + 3V_B$

$$V_C = V_B$$




$$\boxed{R_{IN} = R + \frac{R.R}{R+R} + \frac{R.R}{R+R} = 2R}$$

3.17) Nodal Analysis:




Here, we need to use Supernode approach. The Voltage source is ungrounded

Write the eqn for supernode

$$i_A + i_D - i_B = 0$$

We can write $i_A$, $i_D$ and $i_B$ as (Remember $V_B = 0$ (ground))

$i_A = V_A \cdot G$
$i_B = (V_C - V_D) G$ The $V_C$ is given $V_C = -2$
$i_D = V_D\, G$

Then first eqn

$$V_A G + V_D G - (-2 - V_D) G = 0$$
$$\Rightarrow \quad V_A + 2V_D = -2 \quad (1^{st}\ \text{Eqn})$$

We have two unknowns, we need $2^{nd}$ equation
Applying the fundamental property of the node voltage inside the supernode we can write

$$V_A - V_D = 10\,V \quad (2^{nd}\ \text{Eqn})$$

$$\left.\begin{array}{l} V_A + 2V_D = -2 \\ V_A - V_D = 10 \end{array}\right\} \Rightarrow \begin{array}{l} V_A = 6V \\ V_D = -4V \end{array}$$

$$i_A = V_A \cdot \frac{1}{R} = \frac{6V}{1000\Omega} = 6mA$$

$$i_B = (V_C - V_D) \cdot \frac{1}{R} = \frac{-2+4}{1000} = 2mA$$