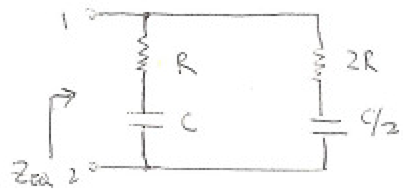
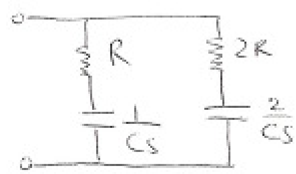
10.2 ✓

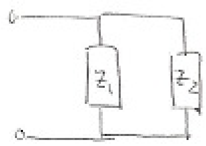
Find $Z_{EQ}(s)$. Express $Z_{EQ}(s)$ as a rational fraction & locate its poles & zeros



⇒



⇓



⇓

$$\Rightarrow \quad Z_1 = R + \frac{1}{Cs} = \frac{RCs+1}{Cs}$$

$$Z_2 = 2R + \frac{2}{Cs} = \frac{2(RCs+1)}{Cs}$$

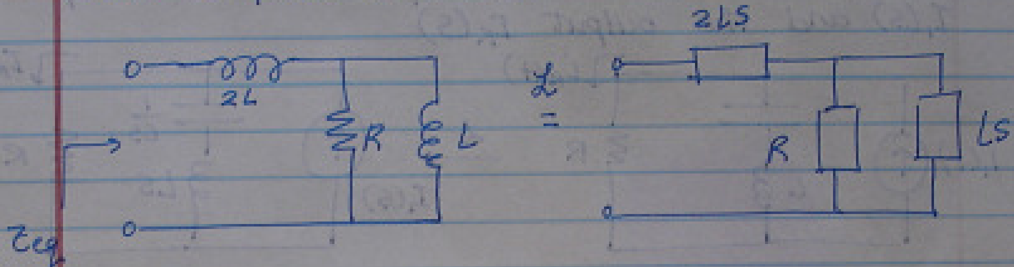
$$Z_{EQ} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{\frac{RCs+1}{Cs} \cdot \frac{2(RCs+1)}{Cs}}{\frac{RCs+1+2RCs+2}{\cancel{Cs}}}$$

$$= \frac{2(RCs+1)\cancel{(RCs+1)}}{3\cancel{(RCs+1)}Cs}$$

$$= \boxed{\frac{2}{3}\frac{RCs+1}{Cs}}$$

$\therefore$ poles : 0 , zeros = $-\frac{1}{RC}$ //

10.5) Find $Z_{eq}(s)$. Express $Z_{eq}(s)$ as a rational function and locate its poles and zeros.



$$Z_{eq} = 2Ls + \frac{1}{\frac{1}{R} + \frac{1}{Ls}} = 2Ls + \frac{1}{\frac{Ls+R}{RLs}} = 2Ls + \frac{RLs}{Ls+R}$$

$$Z_{eq}(s) = \frac{2Ls(Ls+R) + RLs}{Ls+R}$$

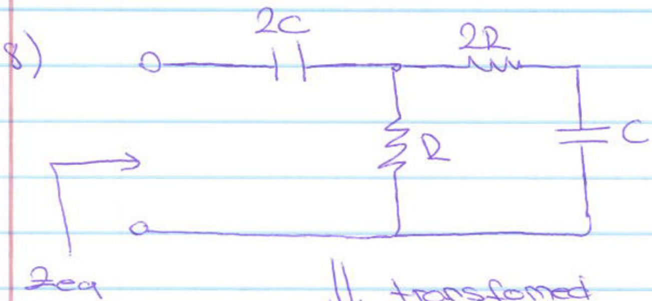
$$= \frac{s^2(2L^2) + 2RLs + RLs}{L(s+\frac{R}{L})}$$

$$= \frac{s^2(2L^2) + 3RLs}{L(s+\frac{R}{L})} = \frac{s(2Ls+3R)}{(s+\frac{R}{L})}$$

poles : $-R/L$

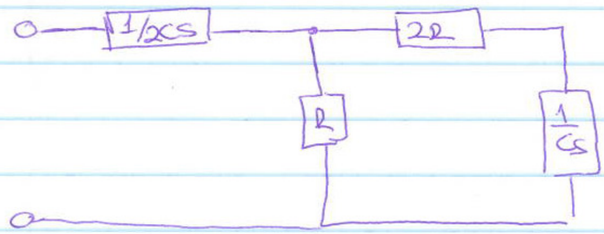
zeros : 0 , $\frac{-3R}{2L}$

10.8)



(t domain)

$\Downarrow$ transformed to s domain



(s domain)

$$Z_{eq}(s) = \left(\left(2R+\frac{1}{Cs}\right) // R\right) + \frac{1}{2Cs}$$

$$Z_{eq}(s) = \frac{R\left(2R+\frac{1}{Cs}\right)}{3R+\frac{1}{Cs}} + \frac{1}{2Cs}$$

$$= \frac{2R^2Cs+R}{3RCs+1} + \frac{1}{2Cs} = \frac{4R^2C^2s^2+5RCs+1}{6RC^2s^2+2Cs}$$

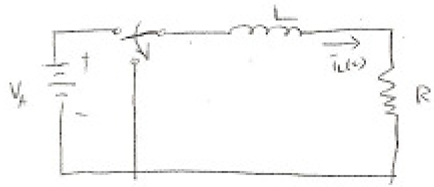
zeros : $\frac{-5RC \mp \sqrt{25R^2C^2-16R^2C^2}}{8R^2C^2} \Rightarrow s = \frac{-1}{4RC}$ and $s = \frac{1}{RC}$

poles : $s(6RC^2s+2C) \Rightarrow S=0$ and $= \frac{-1}{3R^2C}$

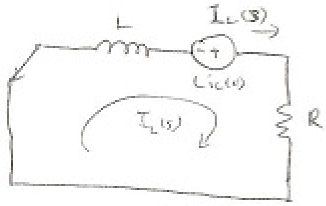
10.11 ✓

Switch moved to position B at t=0

Transform the circuit into s domain & solve for $I_L(s)$ and $I_L(t)$



⇒ For $t > 0$, $\mathcal{L}$ yields



$$i_L(0) = \frac{V_A}{R}$$

$$LsI_L(s) - Li_L(0) + RI_L(s) = 0$$

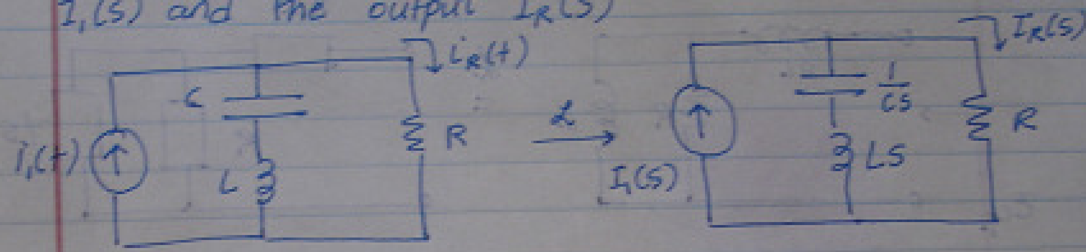
$$I_L(s)(Ls + R) = L\frac{V_A}{R}$$

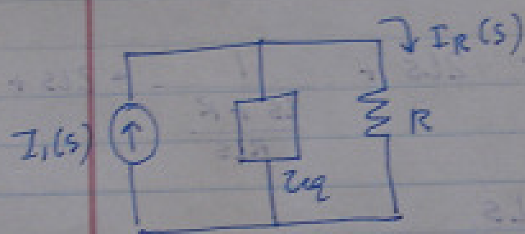
$$I_L(s) = \frac{LV_A}{R(Ls+R)}$$

$$= \boxed{\frac{V_A}{R}\,\frac{1}{s+R/L}}$$

$$\boxed{\therefore\ I_L(t) = \mathcal{L}^{-1}\{I_L(s)\} = \frac{V_A}{R}e^{-R/L\,t}\,u(t)}$$

10.19) The circuit in the figure below is in the zero state. Find the s-domain relationship between the input $I_1(s)$ and the output $I_R(s)$





$$Z_{eq} = \frac{1}{Cs} + Ls$$

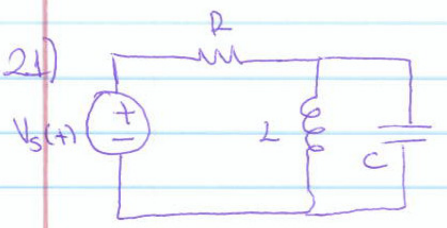
$$= \frac{1 + LCs^2}{Cs}$$

To find $I_R(s)$, use current division

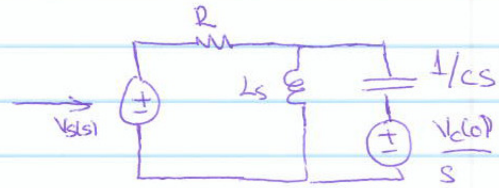
$$I_R(s) = \frac{Z_{eq}}{Z_{eq} + R} I_1(s) = \frac{\frac{1 + LCs^2}{Cs}}{R + \frac{1 + LCs^2}{Cs}} I_1(s)$$

$$\boxed{I_R(s) = \frac{1 + LCs^2}{RCs + 1 + LCs^2} I_1(s)}$$

10.21)

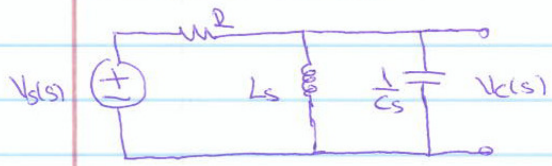


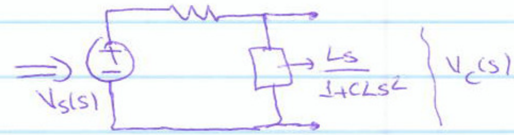
time domain



s domain $(i_L(0)=0)$

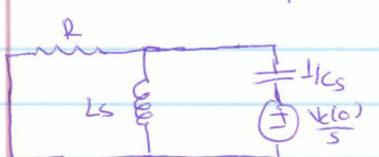
For zero state

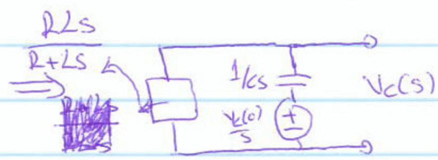




By voltage division $\quad V_c(s) = \dfrac{Ls}{RCLs^2+Ls+R} V_s(s)$

For zero input



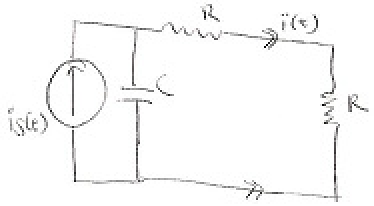


By using voltage division

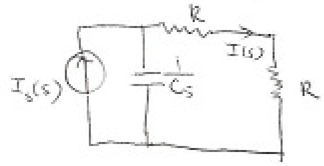
$$V_c(s) = \frac{\left(\frac{RLs}{R+Ls}\right)}{\left(\frac{1}{Cs}+\frac{RLs}{R+Ls}\right)} \frac{V_c(0)}{s} = \frac{V_c RCLs}{(CRLs^2+Ls+R)}$$

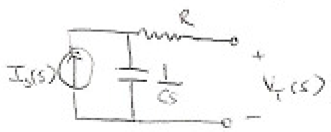
10.25 V

Find the zero-state Thevenin Equivalent & use it to find the relationship between $I_s(s)$ & $I(s)$.

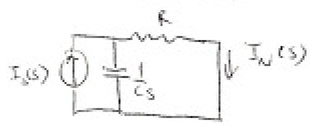


↓ ℒ



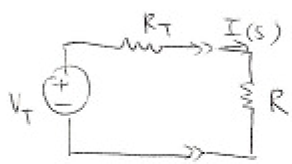


$V_T = \frac{I_s}{Cs}$



$I_N(s) = \frac{1/R}{1/R + Cs} I_s(s) = \frac{I_s(s)}{RCs+1}$

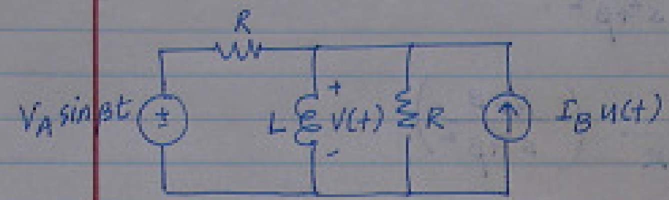
$R_T = \frac{V_T}{I_N} = \frac{\cancel{I_s}}{Cs} \cdot \frac{RCs+1}{\cancel{I_s}} = \frac{RCs+1}{Cs}$



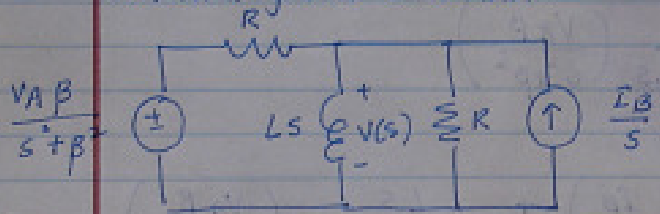
$I(s) = \frac{V_T}{R_T + R} = \frac{I_s(s)}{Cs} \cdot \frac{1}{\frac{RCs+1}{Cs} + R}$

$$\boxed{\therefore\ I(s) = \frac{I_s(s)}{2RCs+1}}$$

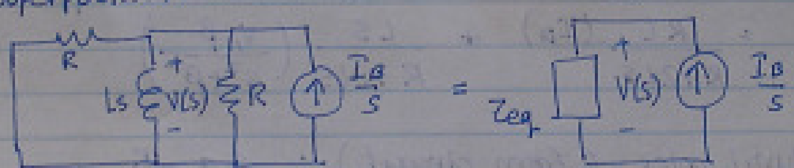
10.28) There is no initial energy stored in the circuit below. Transform the circuit into the s-domain and use superposition to find V(s). Identify the forced and natural poles in V(s)



Transforming into s-domain:



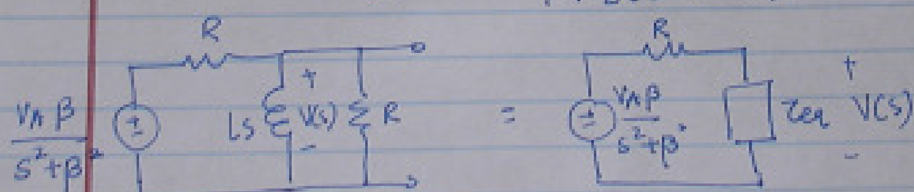
Superposition



$$\frac{1}{Z_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{Ls} = \frac{2Ls + R}{RLs}$$

$$Z_{eq} = \frac{RLs}{2Ls + R}$$

$$V(s) = Z_{eq}\frac{I_B}{s} = \frac{RLs}{1 + 2Ls}\left(\frac{I_B}{s}\right)$$



$$Z_{eq} = \frac{RLs}{R + Ls}$$

Using voltage division

$$V(s) = \frac{Z_{eq}}{Z_{eq}+R} \frac{V_A\beta}{s^2+\beta^2}$$

$$= \frac{\frac{RLs}{R+Ls}}{\frac{RLs}{R+Ls}+R}\left(\frac{V_A\beta}{s^2+\beta^2}\right)$$

$$= \frac{RLs}{RLs+R^2+RLs}\left(\frac{V_A\beta}{s^2+\beta^2}\right)$$

Total

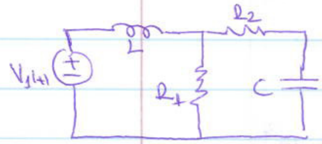
$$V(s) = \frac{RLs}{R+2Ls}\left(\frac{I_B}{s}\right) + \frac{Ls}{Ls+R+Ls}\left(\frac{V_A\beta}{s^2+\beta^2}\right)$$

$$= \frac{RL}{R+2Ls}(I_B) + \frac{Ls}{R+2Ls}\left(\frac{V_A\beta}{s^2+\beta^2}\right)$$

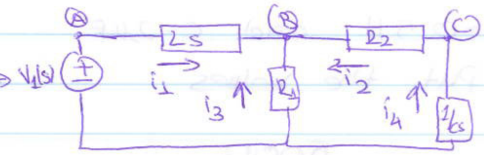
Natural poles (from circuit) : $s = \frac{-R}{2L}$

Forced poles (from input) : $s = \pm j\beta$

10.32) There is no initial energy stored!



t domain



s domain

a) Node Voltage analysis

Ⓐ $V_A = V_1(s)$ and Ⓑ $i_1 + i_3 + i_2 = 0$ Ⓒ $i_4 - i_2 = 0$

$\Rightarrow i_4 = i_2$

Node B: $0 = \dfrac{V_A - V_B}{Ls} + \dfrac{-V_B}{R_1} + \dfrac{V_C - V_B}{R_2}$

$$0 = \frac{V_A}{Ls} - V_B\left(\frac{1}{Ls} + \frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{V_C}{R_2} \quad (V_A = V_1(s))$$

$$\Rightarrow \quad V_B\left(\frac{1}{Ls} - \frac{1}{R_1} - \frac{1}{R_2}\right) - \frac{1}{R_2}V_C = \frac{V_1(s)}{Ls} \quad 1^{st}\text{ Eqn}$$

Node C: $-\dfrac{V_C}{1/Cs} - \dfrac{V_C - V_B}{R_2} = 0 \Rightarrow -V_C\left(Cs + \dfrac{1}{R_2}\right) + V_B\dfrac{1}{R_2} = 0$ 2^{nd} eqn

b) $V_2(s) = V_C(s)$ solve for $V_C(s)$

$V_B = V_C(R_2Cs + 1)$ from 2^{nd} eqn substitute to 1^{st} eqn.

$$V_C\left(\frac{(R_1R_2 + R_2Ls + R_1Ls)(R_2Cs+1)}{R_1R_2Ls} - \frac{1}{R_2}\right) = \frac{V_1(s)}{Ls}$$

$$V_C = \frac{R_1R_2V_1(s)}{(R_1R_2 + R_2Ls + R_1Ls)(R_2Cs+1) - R_1Ls} = \frac{V_1(s)R_1}{((R_1+R_2)LC)s^2 + (R_1R_2C + L)s + R_1}$$

c) Find $v_2(t)$ for $V_1(t) = 10u(t)$ V, $R_1 = R_2 = 500\,\Omega$

$L = 0.5$ H and $C = 2\mu F$

Put the values

$$V_c(s) = \frac{5000}{s(10^{-3}s^2 + s + 500)} = \frac{5000\times10^3}{s(s^2 + 1000s + 500\times10^3)}$$

poles are $s_1 = 0$, $s_2 = -500 + j500$, $s_3 = -500 - j500$

This has complex poles then we can write

$$V_c(s) = \frac{k_1}{s} + \frac{k_2}{s+500-j500} + \frac{k_2^*}{s+500+j500}$$

$$k_1 = \lim_{s\to0} \frac{5000\times10^3}{s^2 + 1000s + 500\times10^3} = 10$$

$$k_2 = \lim_{s\to -500+j500} \frac{5000\times10^3}{s(s+500+j500)} = -\frac{10}{1+j} = -5+5j$$

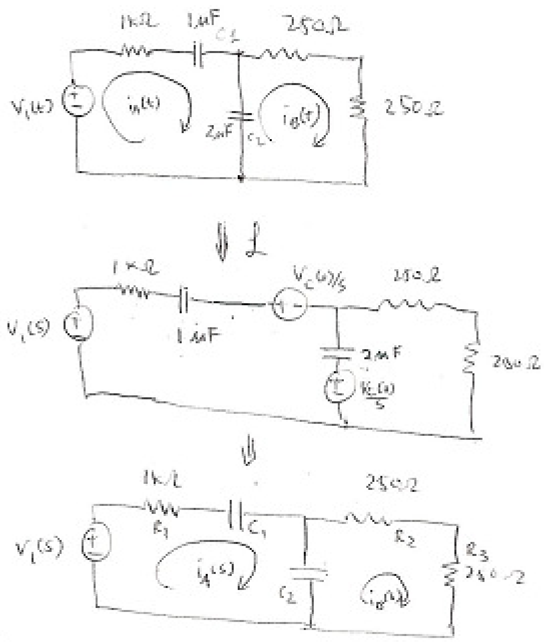
$$V_2(s) = \frac{10}{s} + \frac{(-5+5j)}{s+500-j500} + \frac{(-5-5j)}{s+500+j500}$$

$$V_2(t) = \left[10 + (-5+j5)e^{(-500+j500)t} + (-5-j5)e^{(-500-j500)t}\right]u(t)$$

$$V_2(t) = \left[10 + 10\sqrt{2}\,e^{-500t}\cos(500t + 3\pi/4)\right]u(t)$$

10.37

There is no internal energy stored. Find zero-state mesh currents $i_A(t)$ & $i_B(t)$ when $V_1(t) = 100u(t)$ V.



Mesh A

$$-V_1(s) + \left(R_1 + \frac{1}{C_1 s} \right) i_A(s) + \frac{1}{C_2 s} \left(i_A(s) - i_B(s) \right) = 0$$

Mesh B

$$\left(i_B(s) - i_A(s) \right) \frac{1}{C_2 s} + (R_2 + R_3)\, i_B(s) = 0$$

3

10.37 cont'd

Rearranging these eqns:

Mesh A: $\left(R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right) i_A(s) - \frac{1}{C_2 s} i_B(s) = \frac{100}{s}$ ... (1)

Mesh B: $-\frac{1}{C_2 s} i_A(s) + \left(\frac{1}{C_2 s} + R_2 + R_3\right) i_B(s) = 0$ ... (2)

$$\begin{bmatrix} R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s} & -\frac{1}{C_2 s} \\ -\frac{1}{C_2 s} & \frac{1}{C_2 s} + R_2 + R_3 \end{bmatrix} \begin{bmatrix} i_A(s) \\ i_B(s) \end{bmatrix} = \begin{bmatrix} \frac{100}{s} \\ 0 \end{bmatrix}$$

From (2),

$$i_A(s) = (1 + C_2 R_2 s + C_2 R_3 s)\, i_B(s) \quad \text{... (3)}$$

plug (2) into (1)

$$\left(R_1 + \frac{1}{C_1 s} + \frac{1}{C_2 s}\right)(1 + C_2 R_2 s + C_2 R_3 s)\, i_B(s) - \frac{1}{C_2 s} i_B(s) = \frac{100}{s}$$

$$\left[R_1 + C_2 R_1 R_2 s + C_2 R_1 R_3 s + \frac{1}{C_1 s} + \frac{C_2 R_2}{C_1} + \frac{C_2 R_3}{C_1} + \cancel{\frac{1}{C_2 s}} + R_2 + R_3 - \cancel{\frac{1}{C_2 s}}\right] i_B(s) = \frac{100}{s}$$

$$\left[1000 + 2\times10^{-6}(1000)(250)s + 2\times10^{-6}(1000)(250)s + \frac{1}{10^{-6} s} + 2(250) + 2(250) + 500\right] i_B(s) = \frac{100}{s}$$

$$\left[1000 + \frac{1}{2}s + \frac{1}{2}s + \frac{10^6}{s} + 1500\right] i_B(s) = \frac{100}{s}$$

$$i_B(s) = \frac{100}{2500 s + s^2 + 10^6} = \frac{100}{s^2 + 2500 s + 10^6}$$

$$i_A(s) = (1 + C_2R_2s + C_2R_3s)\, i_B(s)$$

$$= \left(1 + 2\times10^{-6}(250)s + 2\times10^{-6}(250)s\right) i_B(s)$$

$$= \left(1 + \frac{s}{2000} + \frac{s}{2000}\right) i_B(s)$$

$$= \left(1 + \frac{s}{1000}\right) i_B(s)$$

$$= \left(1 + \frac{s}{1000}\right)\left(\frac{100}{s^2 + 2500s + 10^6}\right)$$

$$= \frac{1000 + s}{10(s^2 + 2500s + 10^6)}$$

$$\Downarrow$$

$$i_A(s) = \frac{100}{3}\left[\frac{1}{s+500} + \frac{2}{s+2000}\right]$$

$$\mathcal{L}^{-1} \Downarrow$$

$$\underline{\underline{i_A(t) = \frac{100}{3}\left[e^{-500t} + 2e^{-2000t}\right]}}$$

$$i_B(s) = \frac{100}{s^2 + 2500s + 10^6} \Rightarrow 2\times\frac{100}{3}\left[\frac{1}{s+500} - \frac{1}{s+2000}\right]$$

$$\mathcal{L}^{-1} \Downarrow$$

$$\underline{\underline{i_B(t) = \frac{200}{3}\left[e^{-500t} - e^{-2000t}\right]}}$$