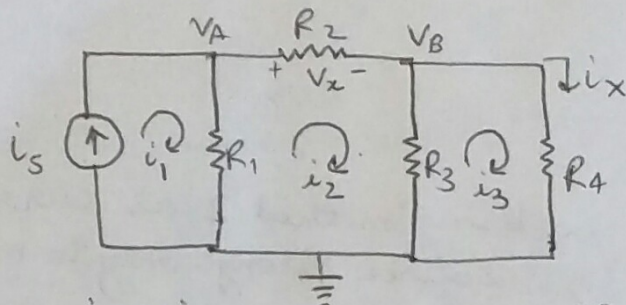


3.2)



$i_1 = i_s$, we use method 2 as current belongs only to one mesh. (mesh 1)

a) KVL in loop 2: $i_2 R_2 + (i_2 - i_3) R_3 - (i_1 - i_2) R_1 = 0$.

KVL in loop 3: $i_3 R_4 - (i_2 - i_3) R_3 = 0$.

in matrix form

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 \\ -R_3 & R_3 + R_4 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} R_1 i_s \\ 0 \end{bmatrix}$$

(b)

$$\begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \frac{1}{(R_1 + R_2 + R_3)(R_3 + R_4) - R_3^2} \begin{bmatrix} R_3 + R_4 & R_3 \\ R_3 & R_1 + R_2 + R_3 \end{bmatrix} \begin{bmatrix} R_1 i_s \\ 0 \end{bmatrix}$$

Optional:

Node

voltage.

$$\begin{cases} V_A = (i_1 - i_2) R_1 \\ = i_s \left(1 - \frac{R_1 (R_3 + R_4)}{(R_1 + R_2)(R_3 + R_4) + R_3 R_4} \right) R_1 = \frac{R_2 (R_3 + R_4) + R_3 R_4}{(R_1 + R_2)(R_3 + R_4) + R_3 R_4} R_1 i_s \\ V_B = i_x R_4 = i_3 R_4 = \frac{i_s R_1 R_3 R_4}{(R_1 + R_2)(R_3 + R_4) + R_3 R_4} \end{cases}$$

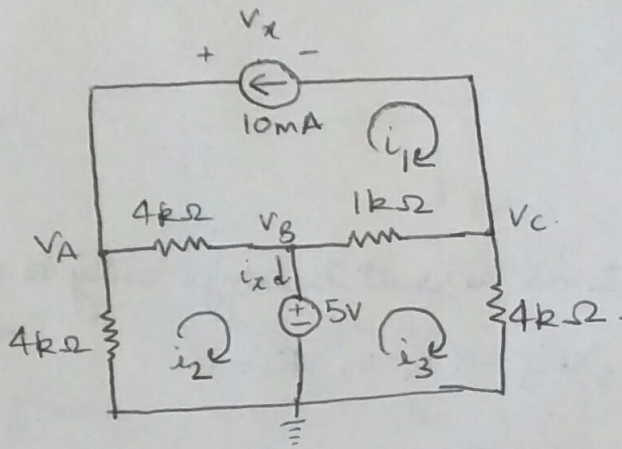
c) $V_x = V_A - V_B = i_s \left[\frac{R_1 R_2 (R_3 + R_4)}{(R_1 + R_2)(R_3 + R_4) + R_3 R_4} \right] \quad \left. \vphantom{\frac{R_1 R_2 (R_3 + R_4)}}{}} \right\} B$

$$i_x = i_3 = \frac{i_s R_1 R_3}{(R_1 + R_2)(R_3 + R_4) + R_3 R_4}$$

or

$$V_x = i_2 R_2 = \frac{i_s R_1 R_2 (R_3 + R_4)}{(R_1 + R_2)(R_3 + R_4) + R_3 R_4}$$

3.5)



Using method 2 as current source belongs only to mesh 1.

a a)

$$i_1 = -10 \text{ mA}$$

~~mesh 1:~~ ~~$V_x + (i_1 - i_3) \times 1000 + (i_1 - i_2) \times 4000 = 0$~~

mesh 2: $4000 i_2 + (i_2 - i_1) \times 4000 + 5 = 0$

mesh 3: $1000 (i_3 - i_1) + 4000 i_3 - 5 = 0$

$$\Rightarrow \begin{bmatrix} 8000 & 0 \\ 0 & 5000 \end{bmatrix} \begin{bmatrix} i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -45 \\ -5 \end{bmatrix}$$

$$\Rightarrow i_2 = -5.625 \text{ mA} \quad i_3 = -1 \text{ mA}$$

b) $V_C = i_3 \times 4000 = -1 \times 10^{-3} \times 4000 = -4 \text{ V}$

$V_A = -i_2 \times 4000 = 5.625 \times 10^{-3} \times 4000 = 22.5 \text{ V}$

} optional Node voltage.

c) $V_x = V_A - V_C$

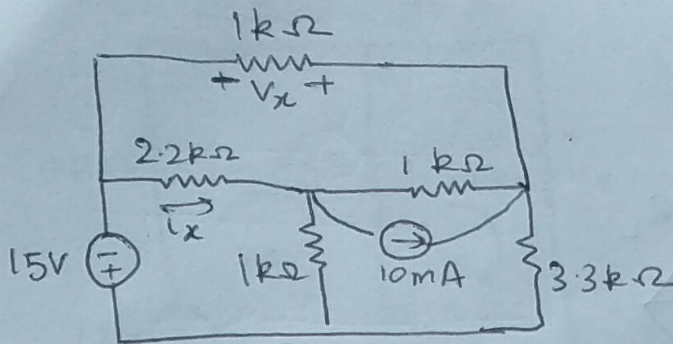
optional: Node Voltage.

$= 22.5 - (-4) = 26.5 \text{ V}$

} or $V_x = 4 \times 10^3 \times (i_2 - i_1) + 1 \times 10^3 \times (i_3 - i_1)$
 $= 26.5 \text{ V}$

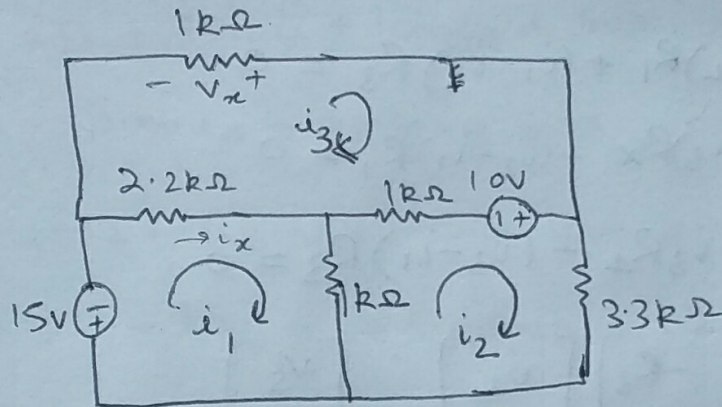
$i_x = i_2 - i_3 = -5.625 - (-1)$
 $= -4.625 \text{ mA}$

3.8)



↓ source transformation. (Method 1)

a)



$$\begin{bmatrix} (2.2+1) \times 10^3 & -1 \times 10^3 & -2.2 \times 10^3 \\ -1 \times 10^3 & (1+3.3) \times 10^3 & -1 \times 10^3 \\ -2.2 \times 10^3 & -1 \times 10^3 & (1+2.2) \times 10^3 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -15 \\ 10 \\ -10 \end{bmatrix}$$

b) $i_1 = -11.29 \text{ mA}$

$i_2 = -1.89 \text{ mA}$

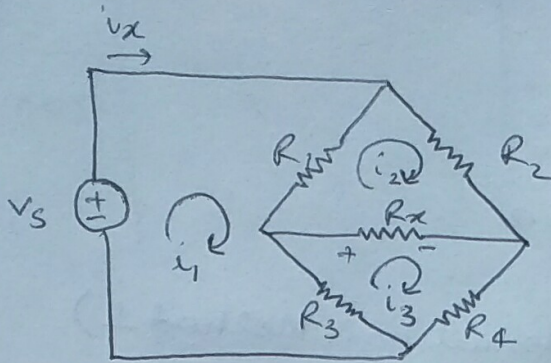
$i_3 = -8.74 \text{ mA}$

$i_x = i_1 - i_3 = -2.55 \text{ mA}$

$V_x = -i_3 \times 1000 = 8.74 \text{ V}$

3.10)

a)



mesh 1: $-v_s + (i_1 - i_2)R_1 + (i_1 - i_3)R_3 = 0$

mesh 2: $i_2 R_2 + (i_2 - i_3)R_x + (i_2 - i_1)R_1 = 0$

mesh 3: $(i_3 - i_2)R_x + i_3 R_4 + (i_3 - i_1)R_3 = 0$

$$\rightarrow \begin{bmatrix} R_1 + R_3 & -R_1 & -R_3 \\ -R_1 & R_2 + R_x + R_1 & -R_x \\ -R_3 & -R_x & R_x + R_3 + R_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}$$

b) $R_1 = R_4 = 2 \text{ k}\Omega$ $R_2 = R_3 = 500 \Omega$, $R_x = 750 \Omega$, $v_s = 15 \text{ V}$

$$\rightarrow \begin{bmatrix} 2500 & -2000 & -500 \\ -2000 & 3250 & -750 \\ -500 & -750 & 3250 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix}$$

$i_1 = 15.5 \text{ mA}$ $i_2 = 10.6 \text{ mA}$, $i_3 = 4.8 \text{ mA}$

$i_x = i_1 = 15.5 \text{ mA}$

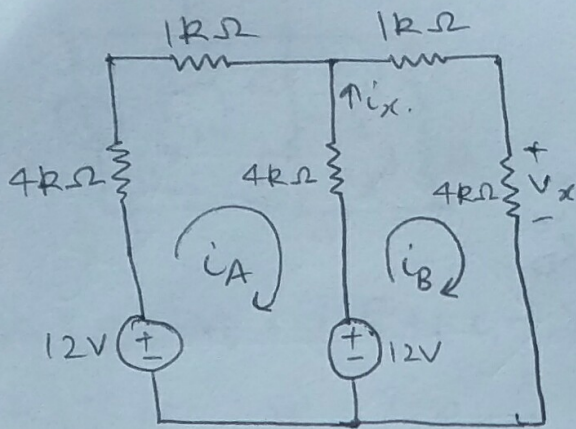
$v_x = (i_3 - i_2)R_x = (4.8 - 10.6) \times 10^{-3} \times 750 = -4.35 \text{ V}$

c) Voltage across $R_x = 0 \Rightarrow i_2 = i_3$ so R_4 ?

$v_{R_x} = 0$

$$\begin{bmatrix} 2500 & -2000 & -500 \\ -2000 & 3250 & -750 \\ -500 & -750 & 1250 + R_4 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ 0 \end{bmatrix} \quad \therefore R_4 = 125 \Omega$$

3.15) a)



$$\begin{aligned} \text{mesh A: } -12 + 4000i_A + 1000i_A + 4000(i_A - i_B) + 12 &= 0 \\ &= 9000i_A - 4000i_B = 0 \end{aligned}$$

$$\begin{aligned} \text{mesh B: } -12 + (i_B - i_A) \times 4000 + 1000i_B + 4000i_B &= 0 \\ \Rightarrow -4000i_A + 9000i_B &= 12 \end{aligned}$$

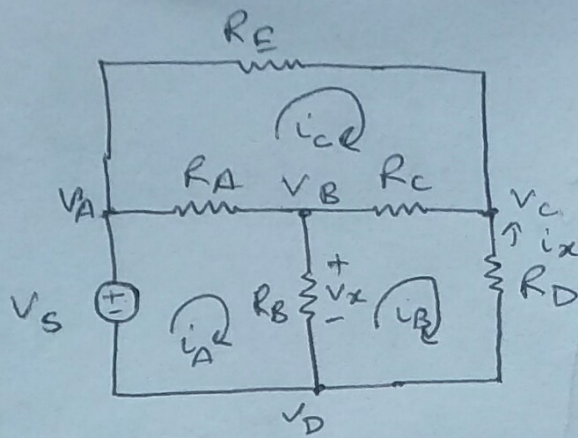
$$b) \Rightarrow \begin{bmatrix} 9000 & -4000 \\ -4000 & 9000 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \end{bmatrix}$$

$$i_A = 0.7 \text{ mA} \quad i_B = 1.7 \text{ mA}$$

$$c) \quad V_x = i_B \times 4000 = 1.7 \times 10^{-3} \times 4000 = 6.8 \text{ V}$$

$$i_x = i_B - i_A = (1.7 - 0.7) \text{ mA} = 1 \text{ mA}$$

3.17)



a) mesh A: $-v_s + (i_A - i_C)R_A + (i_A - i_B)R_B = 0$

mesh B: $(i_B - i_A)R_B + (i_B - i_C)R_C + i_B R_D = 0$

mesh C: $(i_C - i_B)R_C + (i_C - i_A)R_A + i_C R_E = 0$

$$\begin{bmatrix} R_A + R_B & -R_B & -R_A \\ -R_B & R_B + R_C + R_D & -R_C \\ -R_A & -R_C & R_A + R_C + R_E \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}$$

c) Nodal easier to solve, if v_D is ground & $v_A = v_s$.
Hence, only two unknowns instead of 3 as here.

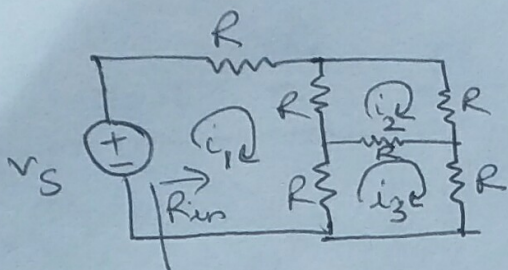
d)
$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \frac{1}{\det(A)} \begin{bmatrix} v_s (R_A(R_B + R_C + R_D) + R_C(R_B + R_D) + R_E(R_B + R_D)) \\ v_s (R_A(R_B + R_C) + R_B(R_C + R_E)) \\ v_s (R_A(R_B + R_C + R_D) + R_B R_C) \end{bmatrix}$$

$$\det(A) = R_A(R_B R_D + R_B R_E + R_C R_D + R_C R_E + R_D R_E) + R_B(R_C R_D + R_C R_E + R_D R_E)$$

$$i_x = -i_B = - \frac{R_A(R_B + R_C) + R_B(R_C + R_E)}{\det(A)} \cdot v_s$$

$$v_x = (i_A - i_B)R_B = \frac{R_A R_D + R_C(R_D + R_E) + R_E R_D}{\det(A)} \cdot R_B v_s$$

3.23)



$$\text{a) mesh 1: } -v_s + i_1 R + (i_1 - i_2) R + (i_1 - i_3) R = 0.$$

$$\text{mesh 2: } (i_2 - i_1) R + i_2 R + (i_2 - i_3) R = 0$$

$$\text{mesh 3: } (i_3 - i_1) R + (i_3 - i_2) R + i_3 R = 0.$$

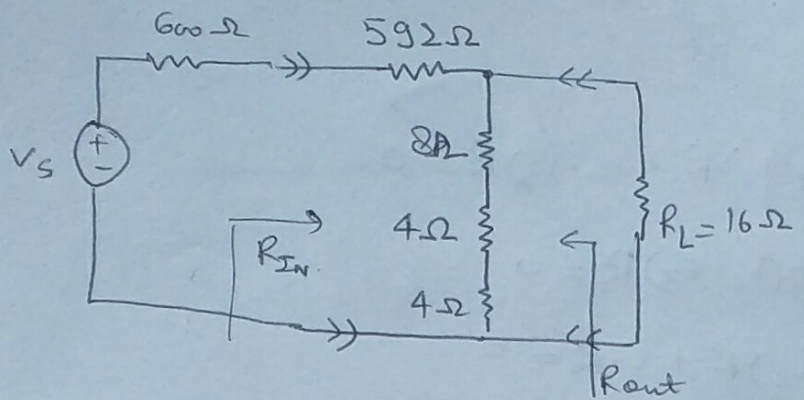
$$\Rightarrow \begin{bmatrix} 3R & -R & -R \\ -R & 3R & -R \\ -R & -R & 3R \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}$$

$$R_{in} = \frac{v_s}{i_1} \quad \text{on solving above matrix}$$

$$i_1 = \frac{v_s}{2R}.$$

$$\boxed{R_{in} = 2R}$$

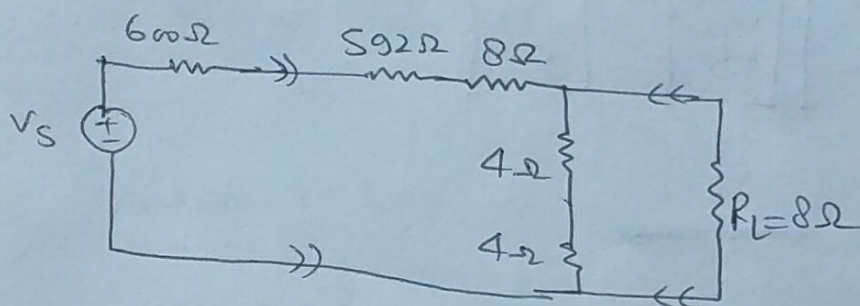
3.91) when speakers $R_L = 16\Omega$.



$$R_{in} = 592 + ((8+4+4) \parallel 16) \\ = 600\Omega$$

$$R_{out} = (600 + 592) \parallel (8+4+4) \\ = 15.79\Omega$$

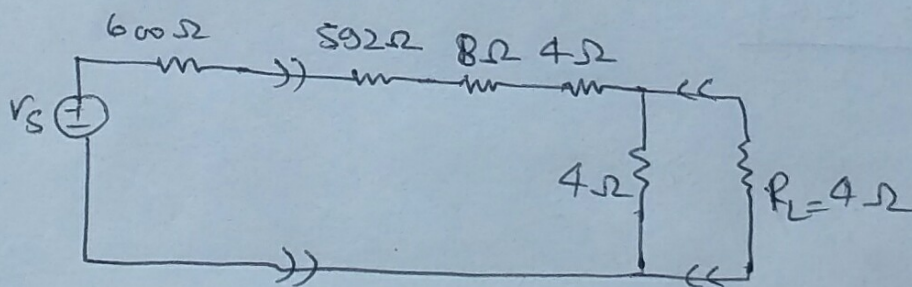
when $R_L = 8\Omega$.



$$R_{in} = 592 + 8 + (4+4) \parallel 8 \\ = 604\Omega$$

$$R_{out} = (600 + 592 + 8) \parallel (4+4) \\ = 7.95\Omega$$

when $R_L = 4\Omega$.



$$R_{in} = (600 + 592 + 8 + 4) \\ + 4 \parallel 4 \\ = 606\Omega$$

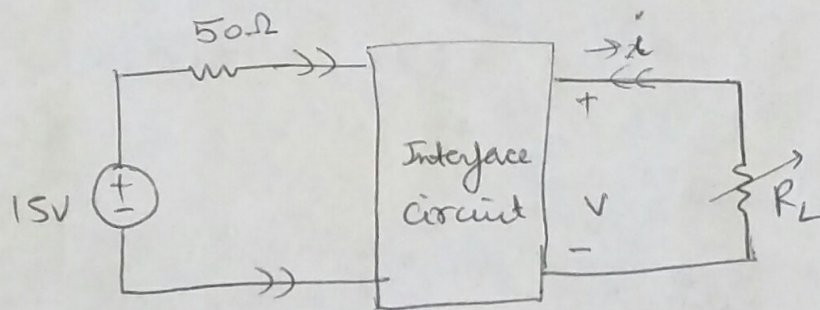
$$R_{out} = (600 + 592 + 8 + 4) \parallel 4 \\ = 3.99\Omega$$

Since all R_{in} are ^{in between} ~~are~~ $600\Omega \pm 2\%$.

& all R_{out} are in between $16, 8, 4, \pm 2\%$.

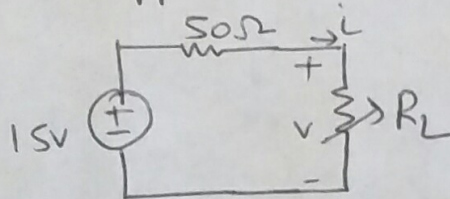
Hence, the claim is true.

3.93)



given, $V \leq 5V$ & $i \leq 100mA$.

Suppose the ~~source~~ interface circuit is simple.

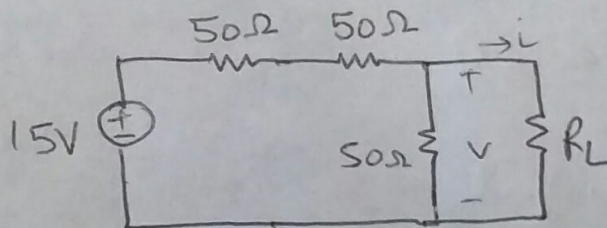


$$i = \frac{15}{R_L + 50} \quad \& \quad V = \frac{R_L}{R_L + 50} \times 15.$$

if $R_L = 0$, $i = 0.3A$ & $V = 0$

if $R_L = \infty$, $i = 0A$ & $V = 15V$.

Hence, since we want the voltage across load to be less than equal to 5V, we can connect 50Ω in parallel with R_L to reduce its equivalent resistance & add a ~~series~~ 50Ω in series with the 50Ω resistance in circuit so we get rectified voltage division



$$V = \frac{(50 \parallel R_L)}{50 + 50 + 50 \parallel R_L} \times 15.$$

$$= \frac{50 \times R_L}{50 + R_L} \times \frac{15}{100 + \frac{50 \times R_L}{50 + R_L}}$$

for $R_L = 0$, $V = 0$, $i = \frac{15}{50+50} = 150mA$

$R_L = \infty$, $V = 5V$, $i =$

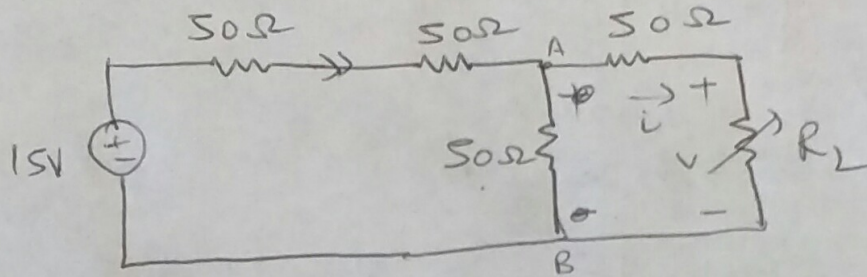
hence, voltage condition is met.

~~to~~ for $R_L = 0$, no current flows in the 50Ω resistor parallel to it. (short circuit)

$$i = \frac{15}{50+50} = 150mA$$

& as R_L increases, i would decrease.

so to decrease current through R_L , let's increase its resistance by adding 50Ω resistor so that when $R_L = 0$, there is a lesser current flowing through load side.



$$\begin{aligned}
 \text{for } R_L = 0, i &= \frac{15}{50 + 50 + 50 \parallel 50} \times \frac{1}{2} \quad (\text{current division}) \\
 &= \frac{15}{125} \times \frac{1}{2} \quad (\text{at A as two } 50\Omega \text{ resistors in parallel}) \\
 &= \frac{15}{250} = 60 \text{ mA}
 \end{aligned}$$

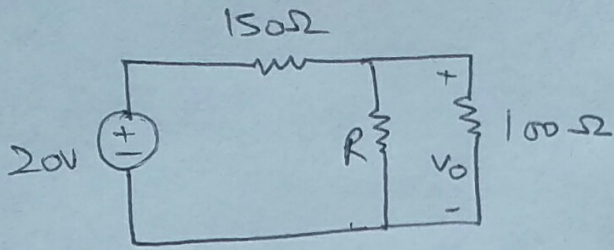
\therefore for $R_L \geq 0$, $i \leq 60 \text{ mA}$.

~~for $R_L \geq 0$ & $v \leq 5 \text{ V}$~~

hence conditions are satisfied.

3.95)

Circuit 1:



$$0 \leq R \leq 100 \Omega$$

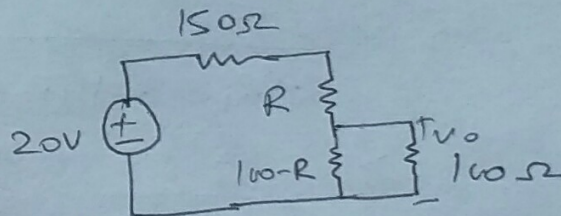
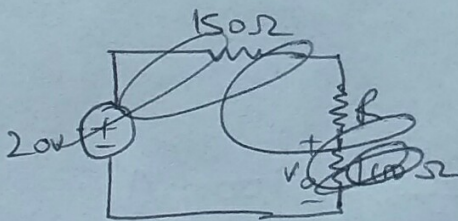
$$V_0 = \frac{(100 \parallel R)}{(100 \parallel R) + 150} \times 20 \quad \therefore 0 \leq V_0 \leq 5 \text{ V}$$

$$\text{Power delivered from source } P = \frac{(20)^2}{150 + (100 \parallel R)}$$

$$\therefore 2 \leq P \leq 2.67$$

but $P = 2.5 \text{ W}$ hence, circuit 1 fails as there can be a value of R where power rating is not satisfied.

Circuit 2:



$$V_0 = \frac{100}{150 + 100 + R} \times 20$$

$$\Rightarrow 5 \text{ V} \leq V_0$$

$$\therefore V_0 = \frac{(100 - R) \parallel 100}{150 + R + (100 - R) \parallel 100} \times 20$$

when $R = 0 \Omega$ $V_0 = 5 \text{ V}$

when $R = 100 \Omega$ $V_0 = 0 \text{ V}$

Hence, $0 \leq V_0 \leq 5 \text{ V}$

$$\text{Power delivered by source } P = \frac{(20)^2}{150 + R + (100 \parallel (100 - R))}$$

$R = 0 \Omega$, $P = 2 \text{ W}$

$R = 100 \Omega$, $P \leq 1.6 \text{ W}$
($< 2.5 \text{ W}$)

Hence, requirements are met in circuit 2.