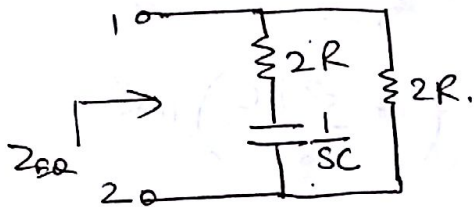


10.3)

a) observing the circuit in s domain



$$\begin{aligned} \therefore Z_{EQ} &= \left( 2R + \frac{1}{sC} \right) \parallel 2R = \left( \frac{2RCs + 1}{sC} \right) \parallel 2R \\ &= \frac{\frac{2RCs + 1}{sC} \times 2R}{\frac{2RCs + 1}{sC} + 2R} = \frac{(2RCs + 1)2R}{2RCs + 1 + 2RCs} \\ &= \frac{(2RCs + 1)2R}{4RCs + 1} \end{aligned}$$

Zeros:  $2RCs + 1 = 0 \Rightarrow s = -\frac{1}{2RC}$  (zero)

Poles:  $4RCs + 1 = 0 \Rightarrow s = -\frac{1}{4RC}$  (pole)

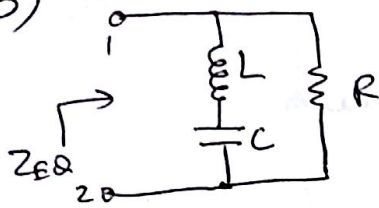
b) select R & C so that pole at  $-640$  rad/s.

$$\therefore -\frac{1}{4RC} = -640 \Rightarrow RC = \frac{1}{4 \times 640}$$

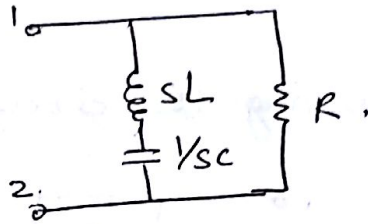
Let  $R = 1k\Omega$ .  $\therefore C = \frac{1}{4 \times 640 \times 10^3} = 0.391 \mu F$

$$\begin{aligned} \therefore \text{Zero} &= -\frac{1}{2RC} = \frac{-1}{2 \times 1k \times 0.391 \mu F} = 2 \times \left( -\frac{1}{4RC} \right) \\ &= 2 \times (-640) = -1280 \text{ rad/s.} \end{aligned}$$

10.5)



$\xrightarrow[\text{s-domain}]{} d$



$$a) Z_{EQ} = \left( sL + \frac{1}{sC} \right) \parallel R = \left( 1 + \frac{s^2 LC}{sC} \right) \parallel R.$$

$$= \frac{\left( \frac{1+s^2 LC}{sC} \right) \times R}{\frac{1+s^2 LC}{sC} + R} = \frac{R + s^2 LCR}{1 + sRC + s^2 LC}$$

Zeros:  $R + s^2 LCR = 0 \Rightarrow s = \pm j \sqrt{\frac{1}{LC}}$  (zeros)

Poles:  $1 + sRC + s^2 LC = 0$

$$\Rightarrow s^2 + s \frac{R}{L} + \frac{1}{LC} = 0 \Rightarrow s = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

$$= \frac{-RC \pm \sqrt{R^2 C^2 - 4LC}}{2LC} \text{ (poles)}$$

b)  $R = 2k\Omega$ ,  $C = 0.1 \mu F$ .

Zeros at  $\pm j 5000 \text{ rad/s}$ .

$$\Rightarrow 5000 = \sqrt{\frac{1}{LC}} \Rightarrow L = \frac{1}{C \times (5000)^2} = \frac{1}{0.1 \times 10^{-6} \times 25 \times 10^6}$$

$$= 0.4 \text{ H}. \quad \therefore \boxed{L = 0.4 \text{ H}}$$

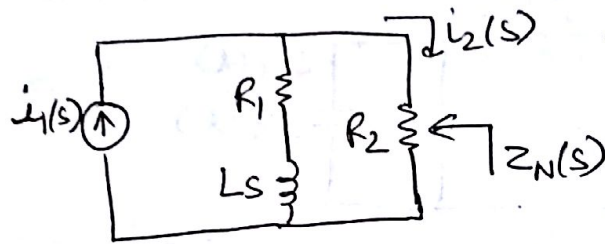
c)  $s = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$

$$= -\frac{2 \times 10^3}{2 \times 0.4} \pm \sqrt{\frac{4 \times 10^6}{4 \times 0.16} - (5000)^2}$$

$$\therefore s = -2.5 \times 10^3 \pm j 4330.1$$

$$\boxed{s = -2500 \pm j 4330 \text{ rad/s}}$$

10.15)

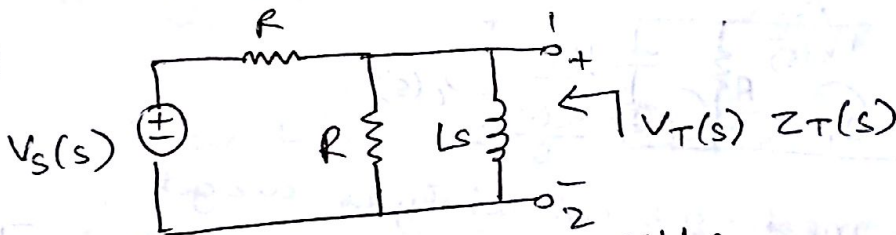


$$a) \frac{i_2(s)}{i_1(s)} = \frac{1/R_2}{1/R_2 + 1/(R_1 + sL)} = \frac{VR_2}{\frac{R_1 + sL + R_2}{R_2(R_1 + sL)}} = \frac{sL + R_1}{R_1 + R_2 + sL}$$

$$\therefore i_2(s) = \frac{sL + R_1}{R_1 + R_2 + sL} \times i_1(s)$$

$$b) Y_n(s) = R_2 \parallel (R_1 + sL) = \frac{R_2(R_1 + sL)}{sL + R_1 + R_2}$$

10.17)



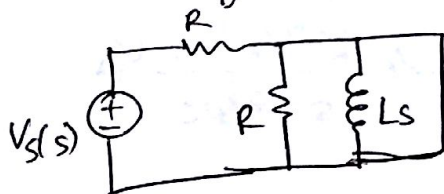
$V_T(s)$  = voltage across  $R \parallel Ls$

$$\therefore V_T(s) = \frac{(R \parallel Ls)}{(R + R \parallel Ls)} \times V_s(s) = \frac{\frac{RLs}{R+Ls}}{\frac{RLs}{R+Ls} + R} \times V_s(s)$$

$$= \frac{Ls}{R+2Ls} V_s(s)$$

~~ZT(s) by turning off sources~~

By short circuiting 1 & 2, we get:



$$I_{sc}(s) = \frac{V_s(s)}{R}$$

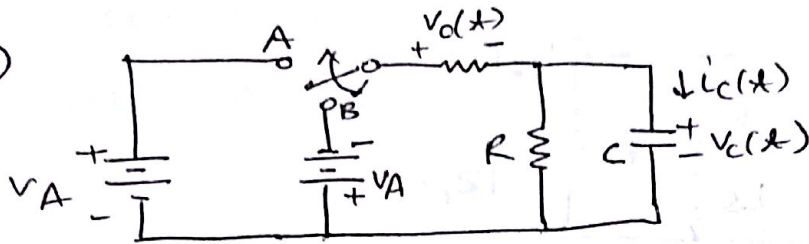
$$\text{pole at } s = -\frac{R}{2L}$$

if pole at  $s = -12 \text{ krad/s}$

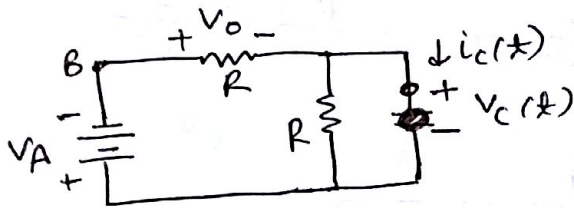
$$\therefore Z_T(s) = \frac{V_T(s)}{I_{sc}(s)} = \frac{RLs}{R+2Ls}$$

choose  $R = 3 \text{ k}\Omega$   
 $L = 125 \text{ mH}$

10.22)



initial condition (switch at B)



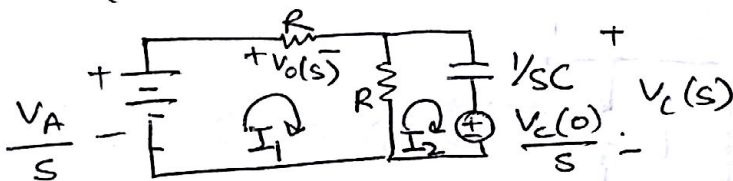
For  $t < 0$ , circuit in DC steady state condition & capacitor acts as open circuit.

$$\therefore i_C(0) = 0$$

$$v_C(0) = v_R(t) = \frac{R}{R+R} (-V_A) = -\frac{1}{2} V_A$$

$$v_0(0) = -\frac{1}{2} V_A$$

Transform the circuit in s-domain for  $t > 0$  (switch at A)



Using mesh-current analysis, we get

$$\underbrace{\begin{bmatrix} R+R & -R \\ -R & R+\frac{1}{sC} \end{bmatrix}}_A \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_X = \underbrace{\begin{bmatrix} V_A/s \\ -\frac{v_C(0)}{s} \end{bmatrix}}_b = \underbrace{\begin{bmatrix} V_A/s \\ V_A/2s \end{bmatrix}}_b$$

$$X = A^{-1}b = \frac{sC}{R(2+sRC)} \begin{bmatrix} \frac{V_A}{s} \left( \frac{1+sRC}{sC} + \frac{R}{2} \right) \\ \frac{V_A}{s} (R+R) \end{bmatrix}$$

$$\Rightarrow I_1(s) = \frac{sC}{R(2+sRC)} \times \frac{V_A}{s} \times \frac{1+1.5sRC}{sC} = \frac{1+1.5sRC}{sR(2+sRC)} V_A$$

$$I_2(s) = \frac{sC}{R(2+sRC)} \times \frac{V_A}{s} \times 2R = \frac{2C}{2+sRC} V_A = \frac{2/R}{s+2/RC} V_A$$

$$= \frac{2}{R} \left( \frac{1}{s + \frac{2}{RC}} \right) V_A$$

10.22) continued

$$\therefore V_o(s) = R I_1(s) = \boxed{\frac{1 + 1.5sRC}{s(2 + sRC)} V_A}$$

$$V_o(t) = \mathcal{L}^{-1}\{V_o(s)\} = \mathcal{L}^{-1}\left\{\left(\frac{k_1}{s} + \frac{k_2}{s + \frac{2}{RC}}\right) V_A\right\}$$

$$k_1 = s V_o(s) \Big|_{s=0} = \frac{1}{2}$$

$$k_2 = \left(s + \frac{2}{RC}\right) V_o(s) \Big|_{s = -\frac{2}{RC}} = 1$$

$$\therefore V_o(t) = \mathcal{L}^{-1}\left\{\left(\frac{1}{2s} + \frac{1}{s + \frac{2}{RC}}\right) V_A\right\}$$

$$\Rightarrow \boxed{V_o(t) = \left(\frac{1}{2} + e^{-2t/RC}\right) V_A u(t)}$$

$$V_c(s) = \frac{1}{sC} I_2(s) + \frac{V_c(0)}{s} = \frac{2}{RC} \left(\frac{1}{s(s + \frac{2}{RC})}\right) V_A - \frac{1}{2s} V_A$$

$$= \frac{2}{RC} \left[ \frac{k_1}{s} + \frac{k_2}{s + \frac{2}{RC}} \right] V_A - \frac{1}{2s} V_A$$

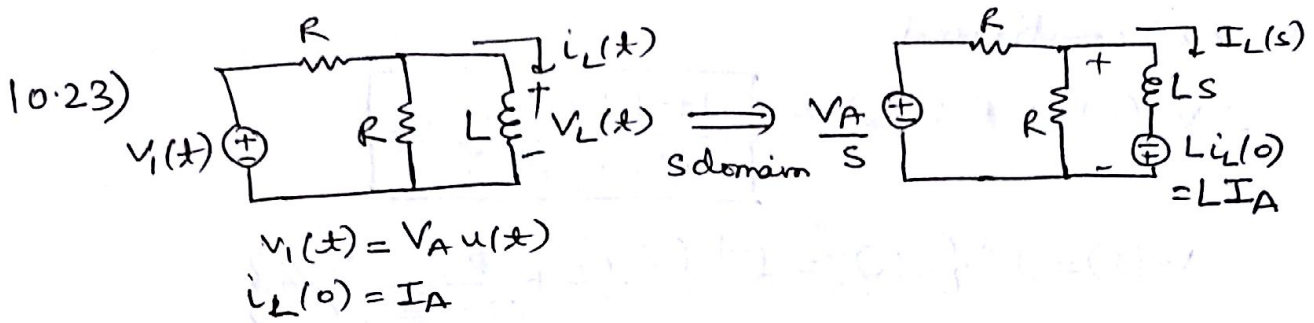
$$k_1 = \frac{1}{s \left( \frac{2}{RC} \right)} \Big|_{s=0} = \frac{RC}{2}$$

$$k_2 = \frac{1}{s} \Big|_{s = -\frac{2}{RC}} = -\frac{RC}{2}$$

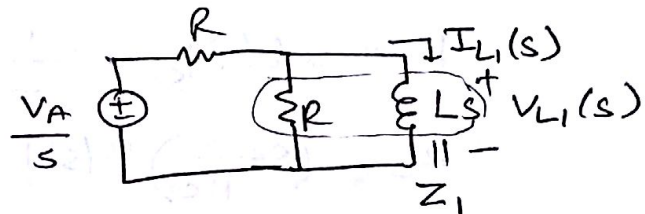
$$\therefore V_c(s) = \left[ \frac{1}{s} - \frac{1}{s + \frac{2}{RC}} \right] V_A - \frac{1}{2s} V_A = \left( \frac{1}{2s} - \frac{1}{s + \frac{2}{RC}} \right) V_A$$

$$\therefore V_c(t) = \mathcal{L}^{-1}\{V_c(s)\}$$

$$\Rightarrow \boxed{V_c(t) = \frac{V_A}{2} (1 - 2e^{-2t/RC}) u(t)}$$



Using superposition.  
 (i) turn  $L I_A$  off.



$$I_{L1}(s) = \frac{V_{L1}(s)}{L/s}$$

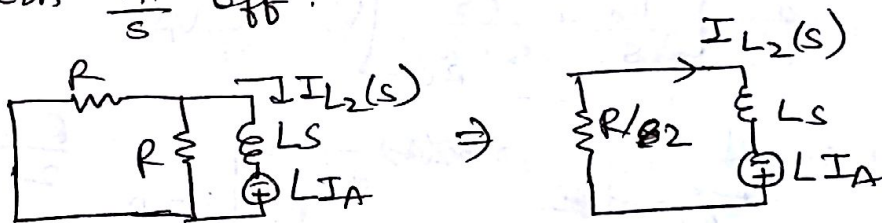
voltage division

$$V_{L1}(s) = \frac{Z_1 \times \frac{V_A}{s}}{R + Z_1}$$

$$Z_1 = \frac{L R s}{R + L s}$$

$$\Rightarrow I_{L1}(s) = \frac{V_A}{s(R + 2Ls)} = \frac{V_A}{2L} \left( \frac{1}{s(s + \frac{R}{2L})} \right)$$

(ii) turn  $\frac{V_A}{s}$  off.



$$I_{L2}(s) = \frac{L I_A}{\frac{R}{2} + Ls} = \frac{I_A}{s + \frac{R}{2L}}$$

$$\begin{aligned} \therefore I_L(s) &= I_{L1}(s) + I_{L2}(s) = \frac{V_A}{2L} \left( \frac{1}{s(s + \frac{R}{2L})} \right) + \frac{I_A}{s + \frac{R}{2L}} \\ &= \frac{V_A}{R} \frac{1}{s} + \frac{I_A - \frac{V_A}{R}}{s + \frac{R}{2L}} \end{aligned}$$

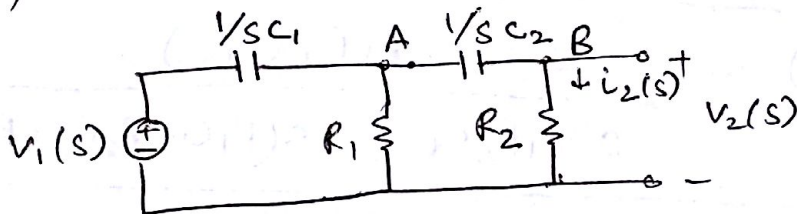
$$\therefore i_L(t) = \mathcal{L}^{-1}\{I_L(s)\} = \left[ \frac{V_A}{R} + \left( I_A - \frac{V_A}{R} \right) e^{-\frac{R}{2L}t} \right] u(t)$$

10.23) continued.

$$\begin{aligned}
 V_L(s) &= (LS)I_L(s) - LIA \\
 &= LS \left[ \frac{V_A}{R} + \frac{IA - \frac{V_A}{R}}{s + R/2L} \right] - LIA \\
 &= \frac{\frac{1}{2}(V_A - RIA)}{s + \frac{R}{2L}}
 \end{aligned}$$

$$\therefore V_L(t) = \mathcal{L}^{-1}\{V_L(s)\} = \frac{1}{2}(V_A - RIA) e^{-\frac{R}{2L}t} u(t)$$

10.43) a) Circuit in s-domain.



By observation, node voltage equations at A & B are.

$$\begin{aligned}
 \text{A: } & \left( \frac{1}{1/sC_1} + \frac{1}{1/sC_2} + \frac{1}{R_1} \right) V_A - \left( \frac{1}{1/sC_2} \right) V_B = \frac{1}{1/sC_1} \times V_1(s) \\
 \Rightarrow & (sC_1 + sC_2 + \frac{1}{R_1}) V_A - sC_2 V_B = sC_1 V_1(s) \quad \text{--- (i)}
 \end{aligned}$$

$$\begin{aligned}
 \text{B: } & \left( -\frac{1}{1/sC_2} \right) V_A + \left( \frac{1}{1/sC_2} + \frac{1}{R_2} \right) V_B = 0 \\
 \Rightarrow & -sC_2 V_A + \left( sC_2 + \frac{1}{R_2} \right) V_B = 0 \quad \text{--- (ii)}
 \end{aligned}$$

b) Solving for  $V_2(s)$ . ( $V_2(s) = V_B$ )

$$\text{in eqn (ii) } V_2(s) = V_B = \frac{sC_2 V_A}{sC_2 + \frac{1}{R_2}}$$

$$V_A = \frac{sC_1 V_1(s) + sC_2 V_2(s)}{sC_1 + sC_2 + \frac{1}{R_1}}$$

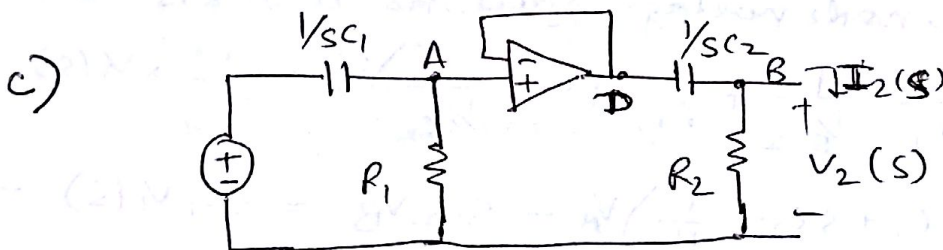
Substituting this in (i), we get .

$$-sC_2 \times \frac{sC_1 V_1(s) + sC_2 V_2(s)}{sC_1 + sC_2 + \frac{1}{R_1}} + \left(sC_2 + \frac{1}{R_2}\right) V_2(s) = 0.$$

$$\Rightarrow \frac{-s^2 C_1 C_2 R_1 R_2 V_1(s) - s^2 C_2^2 R_1 R_2 V_2(s) + (sR_2 C_2 + 1)(sR_1 C_1 + sR_1 C_2 + 1) V_2(s)}{(R_1 C_1 s + R_1 C_2 s + 1) R_2} = 0$$

$$\Rightarrow -s^2 C_1 C_2 R_1 R_2 V_1(s) - s^2 C_2^2 R_1 R_2 V_2(s) + (s^2 R_1 R_2 C_1 C_2 + s^2 R_1 R_2 C_2^2 + sR_2 C_2 + sR_1 C_1 + sR_1 C_2 + 1) V_2(s) = 0.$$

$$\Rightarrow \boxed{V_2(s) = \frac{s^2 C_1 C_2 R_1 R_2 V_1(s)}{s^2 R_1 R_2 C_1 C_2 + s(R_1 C_1 + R_2 C_2 + R_1 C_2) + 1}}$$



as the output impedance of opamp is very low.

$$V_A = V_D$$

at Node A:  $\left(\frac{1}{R_1} + sC_1\right) V_A = V_1(s) \times sC_1$

at Node B:  $(-sC_2) V_A + \left(sC_2 + \frac{1}{R_2}\right) V_2(s) = 0.$

Solving them we get .

$$(-sC_2) \left(\frac{V_1(s) sC_1}{sC_1 + \frac{1}{R_1}}\right) + \left(sC_2 + \frac{1}{R_2}\right) V_2(s) = 0$$

$$\Rightarrow V_2(s) = \frac{s^2 C_1 C_2 R_1 R_2 V_1(s)}{(sC_1 R_1 + 1)(sC_2 R_2 + 1)} = \frac{s^2 V_1(s)}{\left(s + \frac{1}{C_1 R_1}\right) \left(s + \frac{1}{C_2 R_2}\right)}$$



The buffer changes the denominator as. now at Node A, current through the op-amp would be zero & hence the current from source doesn't pass through op-amp & routes through  $R_1$

d) we need first poles at  $10^3 \text{ rad/s}$  & second at  $10^5 \text{ rad/s}$ .

In case with op-amp buffer.

$$\text{poles are } s = -\frac{1}{C_1 R_1}, -\frac{1}{C_2 R_2}$$

$$\therefore \text{for first pole } \frac{1}{C_1 R_1} = 10^3$$

$$\Rightarrow C_1 R_1 = 10^{-4}$$

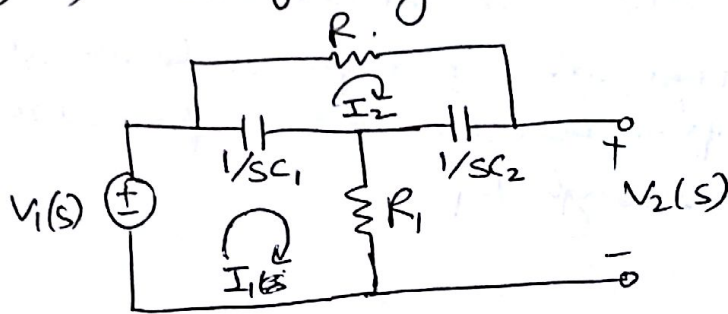
$$R_1 = 1 \text{ k}\Omega \Rightarrow C_1 = 0.01 \mu\text{F}$$

$$\text{for second pole } \frac{1}{C_2 R_2} = 10^5$$

$$\Rightarrow C_2 R_2 = 10^{-5}$$

$$R_2 = 1 \text{ k}\Omega \Rightarrow C_2 = 0.1 \mu\text{F}$$

10.45) a) Transforming the circuit in s domain.



in mesh 1:  $-V_1(s) + \frac{I_1}{sC_1} - \frac{I_2}{sC_1} + I_1 R_1 = 0$

mesh 2:  $I_2 R + \frac{I_2}{sC_2} + \frac{I_2 - I_1}{sC_1} = 0$

∴ the equations are,

$$\left(R_1 + \frac{1}{sC_1}\right) I_1 - \left(\frac{1}{sC_1}\right) I_2 = V_1(s)$$

$$-\left(\frac{1}{sC_1}\right) I_1 + \left(R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}\right) I_2 = 0$$

b) Solving for  $I_1$  &  $I_2$ . using matlab we get.

~~$$\begin{pmatrix} R_1 + \frac{1}{sC_1} & -\frac{1}{sC_1} \\ -\frac{1}{sC_1} & R_2 + \frac{1}{sC_1} + \frac{1}{sC_2} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1(s) \\ 0 \end{pmatrix}$$~~

~~$$\Rightarrow R_1 I_1 + \left(R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}\right) I_2 - \frac{1}{sC_1} I_2 = V_1(s)$$~~

~~$$\Rightarrow R_1 I_1 + R_2 I_2 + \frac{1}{sC_2} I_2 = V_1(s)$$~~

~~$$\Rightarrow R_1 I_1 + \left(R_2 + \frac{1}{sC_2}\right) I_2 = V_1(s)$$~~

~~$$I_1 = \frac{V_1(s) - \left(R_2 + \frac{1}{sC_2}\right) I_2}{R_1}$$~~

using in second equation.

~~$$\left(-\frac{1}{sC_1}\right) \times \frac{V_1(s) - \left(R_2 + \frac{1}{sC_2}\right) I_2}{R_1}$$~~

$$I_1 = \frac{s(C_1 + C_2 + C_1 C_2 R_2 s) V_1(s)}{C_1 R_1 s + C_2 R_1 s + C_2 R_2 s + C_1 C_2 R_1 R_2 s^2 + 1}$$

$$I_2 = \frac{s C_2 V_1(s)}{C_1 R_1 s + C_2 R_1 s + C_2 R_2 s + C_1 C_2 R_1 R_2 s^2 + 1}$$

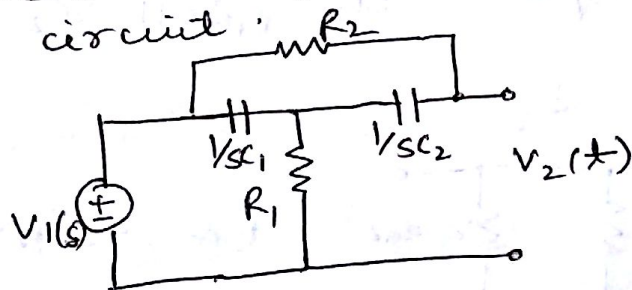
Now

$$-I_1 R_1 - I_2 \times \frac{1}{s C_2} + V_2(s) = 0$$

$$\Rightarrow V_2(s) = I_1 R_1 + I_2 \times \frac{1}{s C_2}$$

$$= \frac{[s(C_1 + C_2 + C_1 C_2 R_2 s) R_1 + 1] V_1(s)}{C_1 R_1 s + C_2 R_1 s + C_2 R_2 s + C_1 C_2 R_1 R_2 s^2 + 1}$$

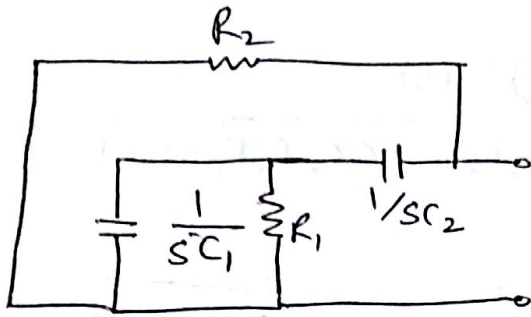
10.47) We need to find the Thevenin equivalent of the circuit.



as calculated in 10.45)

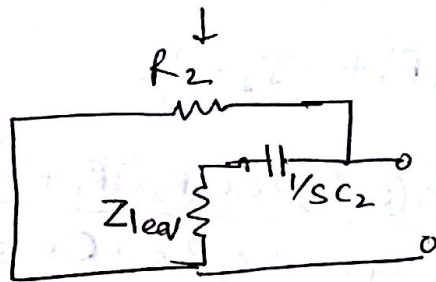
$$V_{oc}(s) = V_2(s) = \frac{[s(C_1 + C_2 + C_1 C_2 R_2 s) R_1 + 1] V_1(s)}{C_1 R_1 s + C_2 R_1 s + C_2 R_2 s + C_1 C_2 R_1 R_2 s^2 + 1}$$

to find  $Z_T$ , we switch off the circuit & redraw it as:



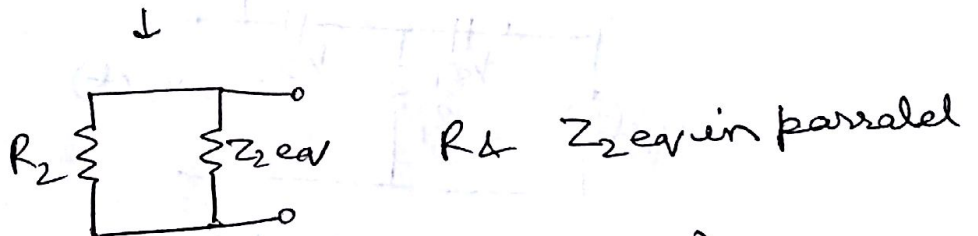
$\frac{1}{sC_1}$  &  $R_1$  are in parallel.

$$Z_{1eq} = \frac{R_1 \times \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{C_1 R_1 s + 1}$$



$Z_{1eq}$  &  $\frac{1}{sC_2}$  are in series.

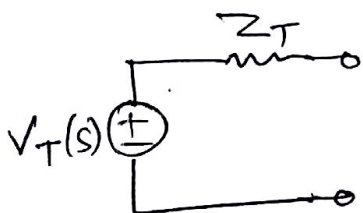
$$Z_{2eq} = Z_1 + \frac{1}{sC_2} = \frac{R_1}{C_1 R_1 s + 1} + \frac{1}{sC_2} = \frac{sR_1 C_2 + C_1 R_1 s + 1}{(C_1 R_1 s + 1)sC_2}$$



$$\therefore Z_T = R_2 \parallel Z_{2eq} = \frac{R_2 \times (sR_1 C_2 + C_1 R_1 s + 1)}{(C_1 R_1 s + 1)sC_2}$$

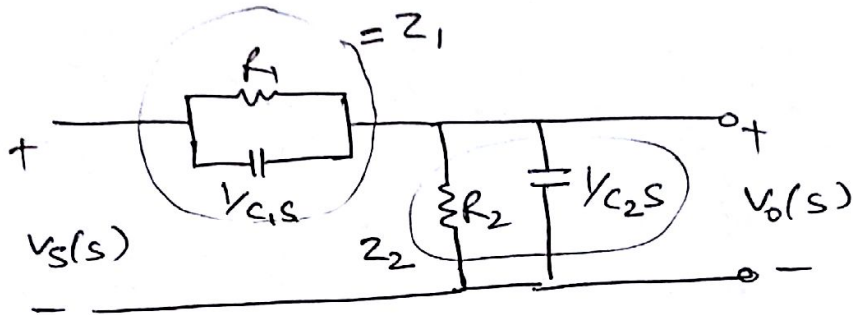
$$R_2 + \frac{sR_1 C_2 + C_1 R_1 s + 1}{(C_1 R_1 s + 1)sC_2}$$

$$= \frac{R_2 ((R_1 C_1 + R_1 C_2)s + 1)}{sR_1 C_2 + sC_1 R_1 + sC_2 R_2 + s^2 C_1 C_2 R_1 R_2 + 1}$$



10.67)

a)



$$Z_1 = R_1 \parallel \frac{1}{C_1 s} = \frac{R_1}{R_1 C_1 s + 1}$$

$$Z_2 = R_2 \parallel \frac{1}{C_2 s} = \frac{R_2}{R_2 C_2 s + 1}$$

Using voltage division.

$$V_o(s) = \frac{Z_2}{Z_1 + Z_2} V_s(s) = \frac{R_2 (R_1 C_1 s + 1) V_s(s)}{R_1 (C_2 R_2 s + 1) + R_2 (C_1 R_1 s + 1)}$$

$$\Rightarrow \frac{V_o(s)}{V_s(s)} = \frac{R_1 R_2 C_1 s + R_2}{s (R_1 R_2 C_2 + R_1 R_2 C_1) + R_1 + R_2}$$

b)  $R_2 = 15 \text{ M}\Omega$   $C_2 = 3 \text{ pF}$

$$\frac{V_o(s)}{V_s(s)} = \frac{1}{2} \Rightarrow \frac{R_1 R_2 C_1 s + R_2}{s R_1 R_2 (C_1 + C_2) + R_1 + R_2} = \frac{1}{2}$$

$$\Rightarrow 2R_1 R_2 C_1 s + 2R_2 = s R_1 R_2 (C_1 + C_2) + R_1 + R_2$$

$$\Rightarrow s R_1 R_2 C_1 + R_2 = s R_1 R_2 C_2 + R_1$$

For  $R_1 = R_2$  &  $C_1 = C_2$ , the eq<sup>n</sup> is true.

we get  $R_1 = 15 \text{ M}\Omega$  &  $C_2 = 3 \text{ pF}$ .