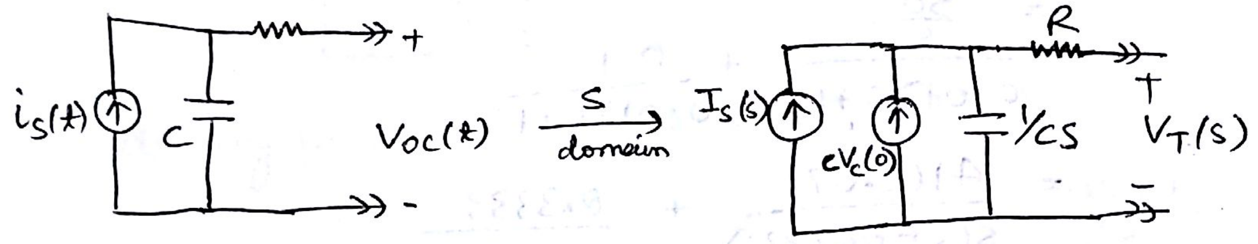
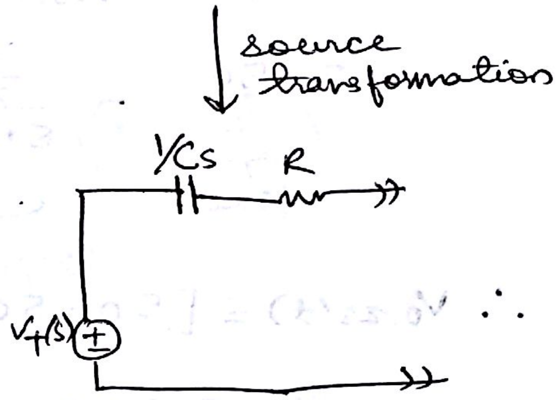
10.35) a) for open-circuit voltage:





$$\therefore V_T(s) = (I_s(s) + CV_c(0)) \cdot \frac{1}{Cs}.$$

$$= \frac{I_s(s) + CV_0}{Cs.}$$

(as $V_c(0) = V_0$ Volts)

$$Z_T(s) = \frac{1}{Cs} + R = \frac{RCs+1}{Cs}$$

b) Voltage delivered to load (by voltage division)

$$V_0(s) = \frac{5R}{5R + Z_T(s)} V_T(s) = \frac{5R(I_s(s) + cV_0)}{6RCs + 1}$$

$V_c(0) = 10V \Rightarrow V_0 = 10$

$i_s(t) = 10u(t)\, mA \Rightarrow I_s(s) = \frac{10 \times 10^{-3}}{s} = \frac{10^{-2}}{s}$

$R = 1k\Omega,\ C = 2\mu F.$

$$\therefore V_0(s) = \frac{5 \times 10^3 \left(\frac{10^{-2}}{s} + 2 \times 10^{-6} \times 10\right)}{6 \times 10^3 \times 2 \times 10^{-6} s + 1}$$

$$= \frac{\frac{50}{s} + 0.1}{0.012s + 1} = \frac{50}{s} + \frac{8.3333 - 50}{s + 83.3333}$$

$$\therefore V_0(s) = \left[50 - 41.667e^{-83.3333t}\right] u(t)$$

c)

$$V_o(s) = \frac{\overset{\rightarrow \text{zero-state}}{5RI_s(s)}}{6RCs+1} + \frac{\overset{\rightarrow \text{zero-input}}{5RCV_o}}{6RCs+1}$$

$$= \frac{\frac{50}{s}}{0.012s+1} + \frac{0.1}{0.012s+1}$$

$$= \frac{4166.67}{s(s+83.3333)} + \frac{8.3333}{s+83.3333}$$

$$= \underbrace{\frac{50}{s}}_{\text{forced}} \underbrace{- \frac{50}{s+83.3333} + \frac{8.3333}{s+83.3333}}_{\text{natural}}$$

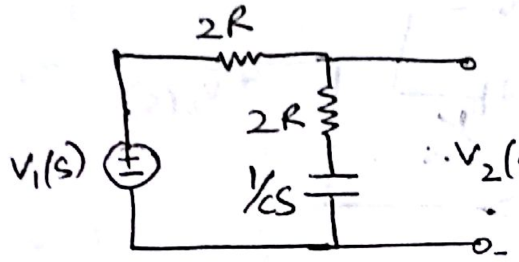
$$\therefore\ V_{o,zs}(t) = \left[50 - 50e^{-83.3333t}\right]u(t)\ V$$

$$V_{o,zi}(t) = 8.3333e^{-83.3333t}u(t)\ V$$

$$V_{o,forced}(t) = 50u(t)V \qquad V_{o,natural}(t) = -41.667e^{-83.3333t}u(t)$$

✓

11.1)



driving point impedance $= 2R + 2R + \frac{1}{Cs}$

$$= 4R + \frac{1}{Cs} = \frac{4RCs+1}{Cs}$$

For voltage transfer function.

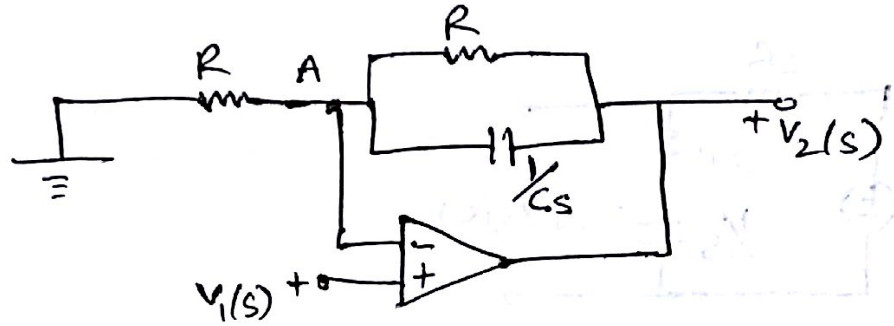
$V_2(s) =$ voltage across $\left(2R + \frac{1}{Cs}\right)$

$$\therefore V_2(s) = \frac{\left(2R+\frac{1}{Cs}\right)}{\left(\frac{4RCs+1}{Cs}\right)} \times V_1(s).$$

$$= \frac{2RCs+1}{4RCs+1} \times V_1(s)$$

$$\therefore T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{2RCs+1}{4RCs+1}$$

11.11)



Using nodal analysis at A : ($V_A(s) = V_1(s)$).

$$\frac{V_A(s) - 0}{R} = (V_2(s) - V_A(s)) \times \left(\frac{1}{R} + Cs\right)$$

$$\Rightarrow \frac{V_A(s)}{R} = (V_2(s) - V_A(s)) \frac{(1 + RCs)}{R}.$$

$$\Rightarrow V_1(s) + V_1(s)\left(\frac{1 + RCs}{\cancel{R}}\right) = V_2(s)(1 + RCs)$$

$$\Rightarrow \frac{V_2(s)}{V_1(s)} = T_V(s) = \frac{2 + RCs}{1 + RCs}.$$

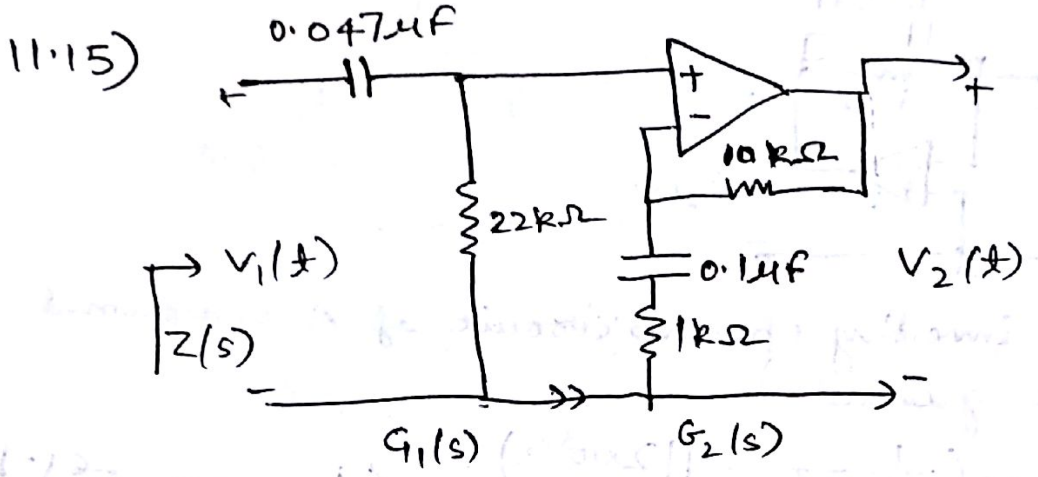
alternately it is a non-inverting op-amp.

$$\therefore \frac{V_2(s)}{V_1(s)} = \frac{R + R \| \frac{1}{Cs}}{R} = \frac{2 + RCs}{1 + RCs}.$$

Pole at $s = -\frac{1}{RC}$ we want $\frac{1}{RC} = 100\text{k rad/s} = 10^5$ rad/s.

$$\therefore \boxed{R = 1\,k\Omega \;\&\; C = 10\,pf}$$

11.15)



This is a cascade of voltage divider & non-inverting op amp.

It is non-loading hence $T_V(s) = G_1(s) \cdot G_2(s)$
(because of infinite input impedance of ideal op-amp)

$$G_1(s) = \frac{22k\Omega}{22k\Omega + \frac{1}{47\times10^{-9}F.s}} = \frac{22\times10^3\times47\times10^{-9}s}{22\times10^3\times47\times10^{-9}s+1}$$

$$= \frac{1.034\times10^{-3}s}{1+1.034\times10^{-3}s} = \frac{s}{s+967.12}$$

$$G_2(s) = 1+\frac{10k\Omega}{1k\Omega+\frac{1}{0.1\times10^{-6}F.s}} = 1+\frac{10^{-3}s}{1+10^{-4}s}$$

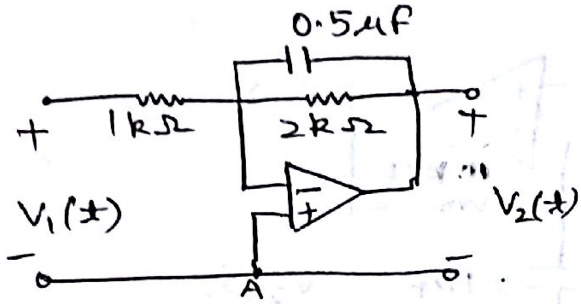
$$= 1+\frac{10s}{10000+s} = 11\left(\frac{s+909.1}{s+10000}\right)$$

$$\therefore T_V(s) = G_1(s).G_2(s) = 11.\frac{s(s+909.1)}{(s+967.12)(s+10000)}$$

Zeros = $\{-909.1, 0\}$

poles = $\{-967.12, -10000\}$

11.35)



This is inverting op amp circuit if A is assumed to be grounded.

$$\therefore \frac{V_2(s)}{V_1(s)} = -\frac{\left(\frac{1}{0.5\times10^{-6}\,F.s} \,||\, 2\times10^3\,\Omega\right)}{1 \times 10^3\,\Omega} = -\frac{\left(\frac{1}{2\times10^3} + 0.5\times10^{-6}s\right)^{-1}}{1\times10^3}$$

$$\Rightarrow T_v(s) = \frac{V_2(s)}{V_1(s)} = \frac{-2}{1+10^{-3}s} = \frac{-2000}{s+1000} \quad \therefore \text{pole} = -1000 \text{ rad/s}$$

For $v(t) = 10\cos\omega t$, use $T_v(j\omega)$ to find $V_{2ss}(t)$

$$T_v(j\omega) = \frac{-2000}{j\omega+1000} = \frac{-2000(-j\omega+1000)}{\omega^2+10^6} = \frac{-2\times10^6+2000\omega j}{\omega^2+10^6}$$

$$\Rightarrow V_{2ss}(t) = 10\,|T_v(j\omega)| \cdot \cos(\omega t + \measuredangle T_v(j\omega))$$

($\omega = 500$ rad/s)

$\therefore |T_v(j\omega)| = 1.789$, $\measuredangle T_v(j\omega) = 2.678$ rad.

$V_{2ss}(t) = 17.89\cos(500t + 2.678)$ V.

$= \cancel{16\cos\omega t}\,(-16\cos 500t - 8\sin 500t)$ V

($\omega = 1$ krad/s)

$|T_v(j\omega)| = 1.414$ $\measuredangle T_v(j\omega) = 2.356$ rad.

$V_{2ss}(t) = 14.14\cos(1000t + 2.356)$ V $= -10(\cos 1000t + \sin 1000t)$ V

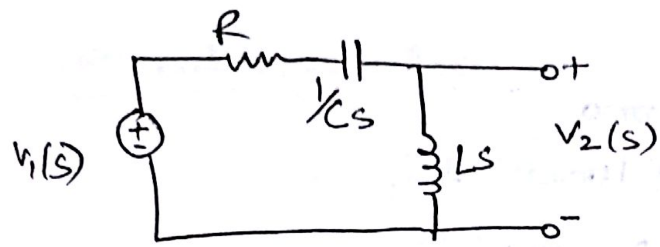
($\omega = 10$k rad/s)

$|T_v(j\omega)| = 0.1990$, $\measuredangle T_v(j\omega) = 1.670$ rad

$V_{2ss}(t) = 1.99\cos(10000t + 1.670)$ V

$= (-0.198\cos 10000t - 1.98\sin 10000t)$ V

110.36)



$R = 10\Omega$
$C = 50\mu F$
$L = 5mH$

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{Ls}{R + \frac{1}{Cs} + Ls} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{s^2}{s^2 + 2\times10^3 s + 4\times10^6}$$

$\therefore$ poles at $\{-1000 \pm 1732j\}$, zeros at $\{0, 0\}$

when $v_1(t) = 25\cos(2000t)$, $\omega = 2000$, $T(j\omega) = j$

$|T(j\omega)| = 1$, $\angle T(j\omega) = 1.57$)

$\therefore v_{2ss}(t) = 25\cos(2000t + 1.57)$ V

For $v_1(t) = 25\cos(10^4 t)$, $\omega = 10^4$

$|T(j\omega)| = 1.02$ $\angle T(j\omega) = 0.205$)

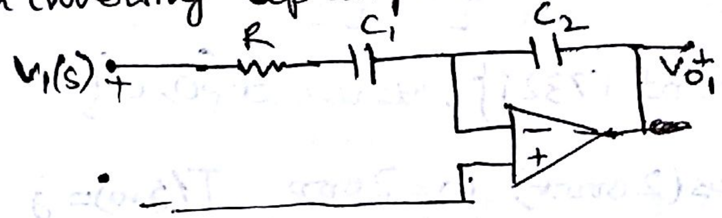
$\Rightarrow v_{2ss}(t) = 25.5\cos(10^4 t + 0.205)$ V

11.63) $T_v(s) = \dfrac{20000}{s+1000}$

$T_v(s) = (-1) \cdot \left(-\dfrac{20000}{s+1000} \right).$

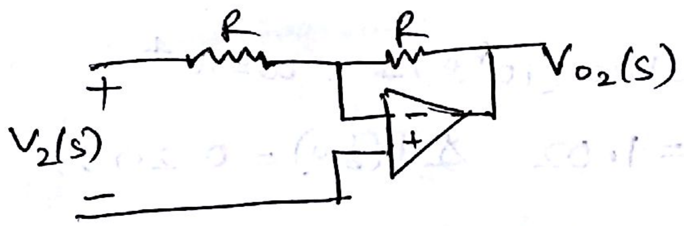
inverting inverting.

For an inverting ~~op~~ amp.



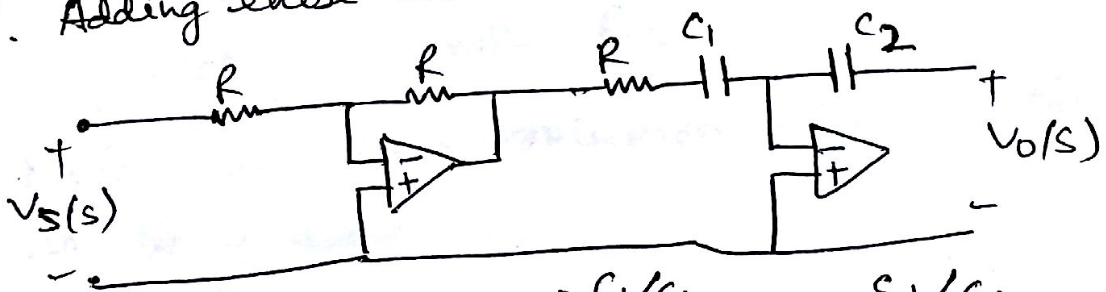
$$\frac{V_{o}(s)}{V_1(s)} = -\frac{1/sC_2}{R+\frac{1}{sC_1}} = -\frac{C_1/C_2}{1+RC_1 s}.$$

& for



$$\frac{V_{o_2}(s)}{V_2(s)} = -\frac{R}{R} = -1.$$

∴ Adding these circuits to multiply gains.



∴ gain $K_v = -1 \times -\dfrac{C_1/C_2}{1+RC_1 s} = \dfrac{C_1/C_2}{1+RC_1 s}.$

choose $R_1 = 1\,k\Omega$, $C_1 = 1\,\mu F$.

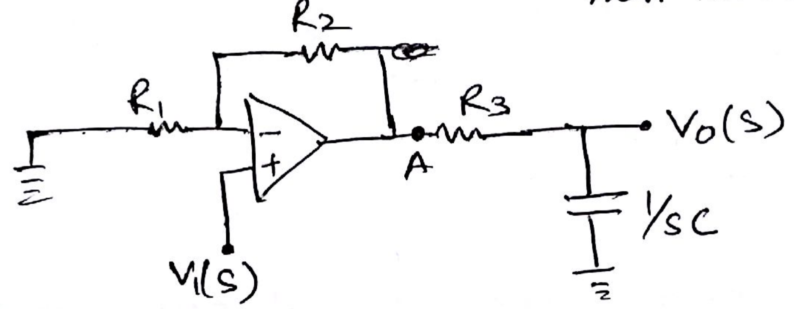
$C_2 = 0.05\,\mu F$

11.63) <u>Alternate solution</u>

$$T_V(S) = \frac{20000}{S+1000} = (20) * \left(\frac{1000}{S+1000}\right)$$

voltage divider.

non-inverting amplifier.



$$\therefore \frac{V_A(S)}{V_1(S)} = \frac{R_1+R_2}{R_1}$$

choose R_1 & R_2 such that $\frac{R_1+R_2}{R_1} = 20$.

Now, using voltage division..

$$\frac{V_o(S)}{V_A(S)} = \frac{1/sC}{1/sC + R_3} = \frac{1}{R_3Cs+1} = \frac{1/R_3C}{s + 1/R_3C}$$

choose R_3, C such that

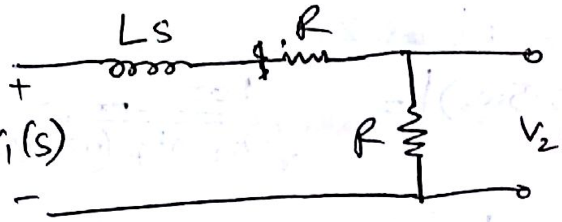
$$1/R_3C = 1000.$$

Let $R_3 = 1k\Omega$ $C = 1\mu F$

Let $R_1 = 100\Omega$ $R_2 = 1.9k\Omega$.

Note: in this solution only one op-amp is being used.

12.4)



$R = 100\,\Omega$
$L = 100\,mH$

$$T_V = \frac{V_2(s)}{V_1(s)} = \frac{R}{R+Ls+R} = \frac{100}{100+100+100\times10^{-3}s}$$

$$= \frac{100}{200+0.1s} = \frac{1000}{s+2000}$$

DC gain $= \lim_{s\to 0} T_V(s) = \frac{1000}{2000} = 0.5 = \frac{1}{2}$

Infinite frequency gain $\Rightarrow \lim_{s\to\infty} T_V(s) = 0$

cutoff frequency ω_c where $|T_V(j\omega_c)| = \frac{1}{\sqrt{2}} T_{V\,max}$.

$$\Rightarrow \frac{1000}{\sqrt{(2000)^2+\omega^2}} = \frac{1}{\sqrt{2}} \times \frac{1}{2}.$$

$$\Rightarrow (1000)^2 \times (2\sqrt{2})^2 = (2000)^2 + \omega^2$$

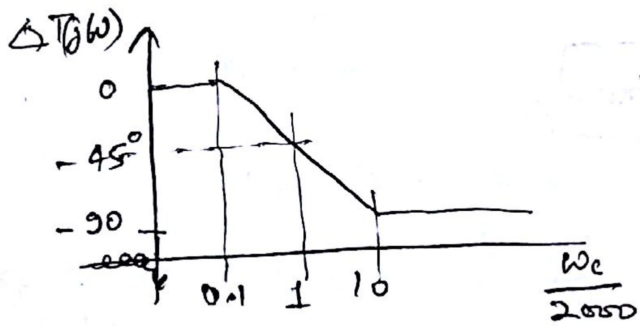
$$\Rightarrow \omega^2 = 4\times10^6$$

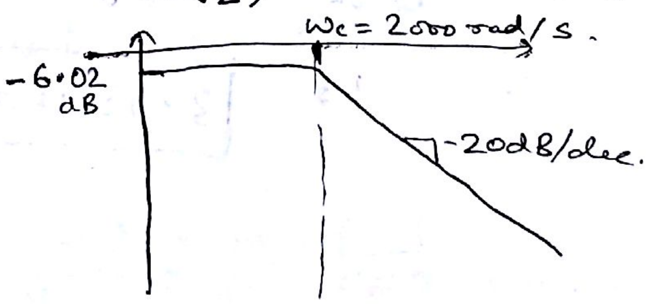
$$\Rightarrow \boxed{\omega_c = 2000\ rad/s}$$ low-pass filter

b) $|T(j\omega)|_{dB} = 20\log_{10}|T(j\omega)|$

gain at $\omega = 0 = 20\log_{10}(0.5) = -6.02\,dB$.

gain at $\omega_c = 20\log_{10}\left(\frac{1}{2\sqrt{2}}\right) = -9.03\,dB$.





c) gain at $\omega = 0.5\omega_c$, ω_c & $2\omega_c$.

$$|T_v(j0.5\omega_c)| = \frac{1000}{\sqrt{(2000)^2 + (0.5\times 2000)^2}} = 0.4472.$$

$$|T_v(j\omega_c)| = \frac{1}{\sqrt{2}} \times \frac{1}{2} = 0.3536.$$

$$|T_v(2\omega_c j)| = \frac{1000}{\sqrt{(2000)^2 + (2\times 2000)^2}} = 0.2236.$$

d) From matlab plot.

$|T_v(j0.5\omega_c)|_{dB} = -7$

$\Rightarrow |T_v(j0.5\omega_c)| = 0.4467.$

$|T_v(j\omega_c)|_{dB} = -9.03$

$\Rightarrow |T_v(j\omega_c)| = 0.3536.$

$|T_v(j2\omega_c)|_{dB} = -13$

$\Rightarrow |T_v(j2\omega_c)| = 0.2239.$

Hence we see that the values calculated in c) are almost the same as from the bode plot in matlab.

e) one octave past the cut-off

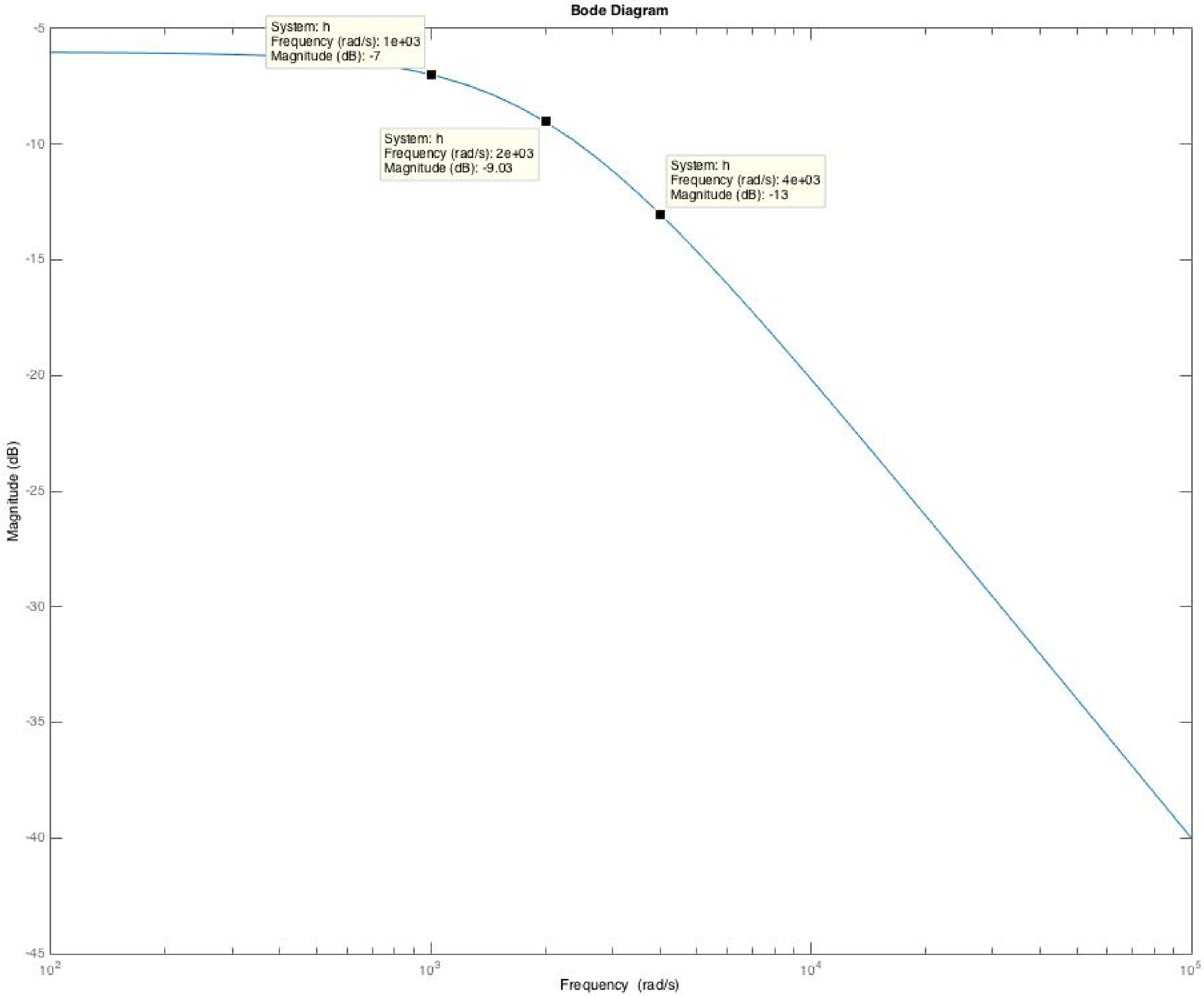
$\omega = 2\omega_c$ Gain $|T_v(2j\omega_c)| = 0.2236$

Gain in passband = dC gain $= \frac{1}{2}$.

the filter magnitude down the the passband, at one octave past the cut-off is

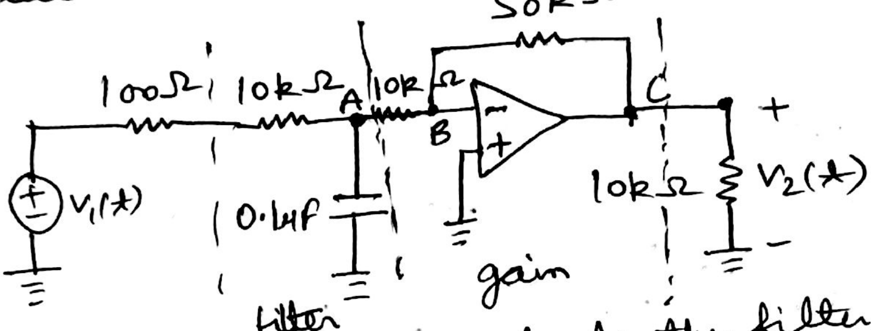
$$= 20\log\left(\frac{0.2236}{0.5}\right)$$

$$= -6.99 \text{ dB lower.}$$



Bode Diagram
System: h
Frequency (rad/s): 1e+03
Magnitude (dB): -7
System: h
Frequency (rad/s): 2e+03
Magnitude (dB): -9.03
System: h
Frequency (rad/s): 4e+03
Magnitude (dB): -13
Magnitude (dB)
-5
-10
-15
-20
-25
-30
-35
-40
-45
10^2
10^3
10^4
10^5
Frequency (rad/s)

12.9) Let us look first at configuration when filter stage is added between source & gain stage & gain stage is added to load:



Here the gain stage loads the filter stage.

in s transform domain.

at node A.

$$\frac{V_1(s) - V_A(s)}{10k\Omega + 100\Omega} = \frac{V_A(s) - 0}{(0.1\mu F . s)^{-1}} + \frac{V_A(s) - 0}{10k\Omega}.$$

$$\Rightarrow \frac{V_1(s)}{10.1k\Omega} = \left(\frac{1}{10k\Omega} + \frac{1}{10.1k\Omega} + 0.1\mu F s\right) V_A(s)$$

$$\Rightarrow \frac{V_A(s)}{V_1(s)} = \frac{1/10.1k\Omega}{\left(\frac{1}{10k\Omega} + \frac{1}{10.1k\Omega} + 0.1\mu F . s\right)}$$

$$\therefore T_F(s) = \frac{V_A(s)}{V_1(s)} = \frac{1}{\left(\frac{10.1\times10^3}{10\times10^3} + \frac{10.1\times10^3}{10.1\times10^3} + 0.1\times10^{-6}\times10.1\times10^3\right)}$$

$$= \frac{1}{2.01 + 0.00101 s}$$

for gain stage.

$$V_2(s) = -\frac{50k\Omega}{10k\Omega} V_A(s) = -5 V_A(s)$$

$$\Rightarrow T_g(s) = -5.$$

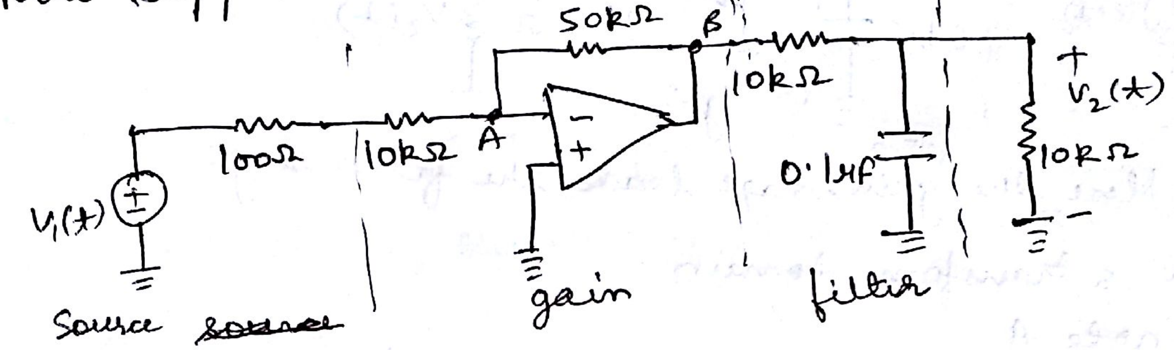
$$\therefore T_v(s) = \frac{V_2(s)}{V_1(s)} = \frac{V_A(s)}{V_1(s)} \times \frac{V_2(s)}{V_A(s)} = \left(\frac{1}{2.01 + 0.00101 s}\right) \times (-5)$$

For this $\omega_c = 1990$ rad/s.

& DC gain $= -2.488$.

hence we do not get the desired cutoff frequency of 1 krad/s & gain of -5.

Now suppose we add the filter stage after gain stage



here, the gain stage loads the source.

in s-domain at Node A,
$$\frac{V_1(s) - V_A(s)}{100\Omega + 10k\Omega} = \frac{V_A(s) - V_B(s)}{50k\Omega}$$

$V_A(s) = 0$

can use $\left(\frac{V_B(s)}{V_1(s)} = -\frac{R_2}{R_1}\right)$

$R_2 = 50k\Omega$

$R_1 = 10k\Omega + 100\Omega$

$$\Rightarrow \frac{V_1(s)}{10100} = -\frac{V_B(s)}{50000}$$

$$\Rightarrow \frac{V_B(s)}{V_1(s)} = -\frac{50000}{10100} = -4.95.$$

$$V_2(s) = \frac{Z_2}{Z_1+Z_2} V_B(s)$$

$$\Rightarrow \frac{V_2(s)}{V_B(s)} = \frac{\left(\frac{1}{10k\Omega} + 0.1\mu F.s\right)^{-1}}{10k\Omega + \left(0.1\mu F s + \frac{1}{10k\Omega}\right)^{-1}} = \frac{1000}{s+2000}.$$

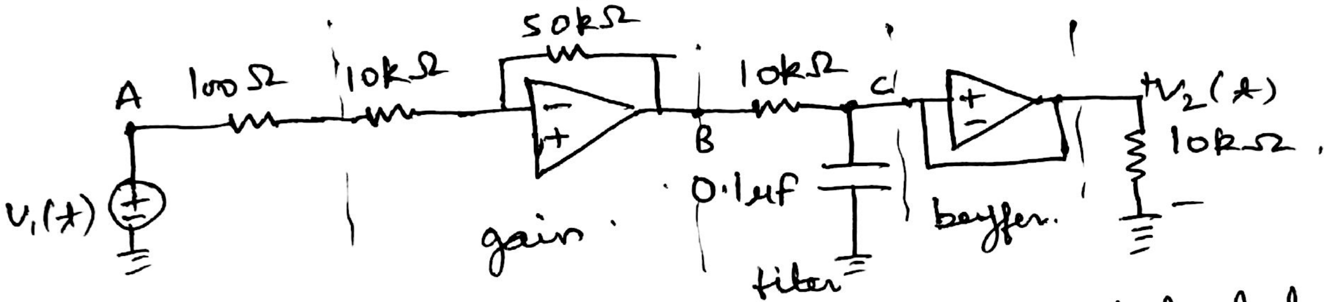
~~we can multiply the gains~~

$$\therefore V_2(s) = \frac{1000}{s+2000} \times V_B(s) = \frac{1000}{s+2000} \times (-4.95).$$

$\omega_c = 2000$, DC gain $= -4.95 \times \frac{1000}{2000} = -2.475.$

Hence we do not get the desired output.

Now, to fix this, we have the source connected to gain stage, which is then connected to the filter. Now we add a buffer between filter & the load.
So the circuit diagram looks like.



Using the buffer, the voltage divider is not loaded with load.

$$\therefore \frac{V_B(s)}{V_A(s)} = -\frac{50\times10^3\,\Omega}{10\times10^3\,\Omega + 100\,\Omega} = -4.95$$

$$\frac{V_C(s)}{V_B(s)} = \frac{\frac{1}{0.1\times10^{-6}\times s}}{\frac{1}{0.1\times10^{-6}s} + 10\times10^3} = \frac{10^7/s}{10^7/s + 10^4}$$

$$= \frac{10^3/s}{10^3/s + 1} = \frac{10^3}{s+10^3} = \frac{1000}{s+1000}.$$

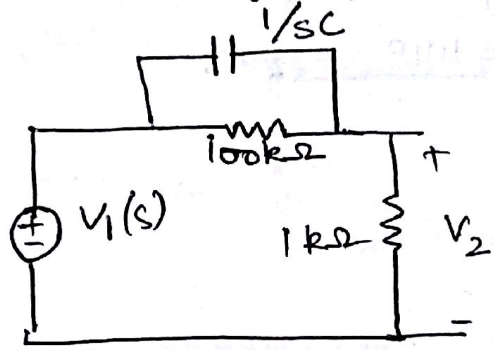
$$\therefore V_2(s) = V_C(s)$$

$$\therefore T_V(s) = \frac{V_2(s)}{V_C(s)} \times \frac{V_C(s)}{V_B(s)} \times \frac{V_B(s)}{V_A(s)} = \frac{-4.95\times1000}{s+1000}.$$

$\therefore$ DC gain $= -4.95 \simeq -5$ (desired)
cutoff freq $= 1000$ rad/s (desired).

~~12.0 We need a low-pass filter with $\omega_c = 1$ krad/s & gain of -5. The filter must fit.~~

12.43)



$C = 1\mu F$.

$$\therefore \frac{V_2(s)}{V_1(s)} = T_v(s) = \frac{1\times 10^3}{1\times 10^3 + \left(100\times 10^3 \parallel \frac{1}{s\times 1\times 10^{-6}}\right)}$$

$$= \frac{10^3}{10^3 + \dfrac{10^5 \times \frac{1}{s\times 10^{-6}}}{10^5 + \frac{1}{s\times 10^{-6}}}}$$

$$= \frac{10^3}{10^3 + \dfrac{10^5}{10^{-1}s + 1}} = \frac{10^3}{10^3 + \dfrac{10^6}{s+10}}$$

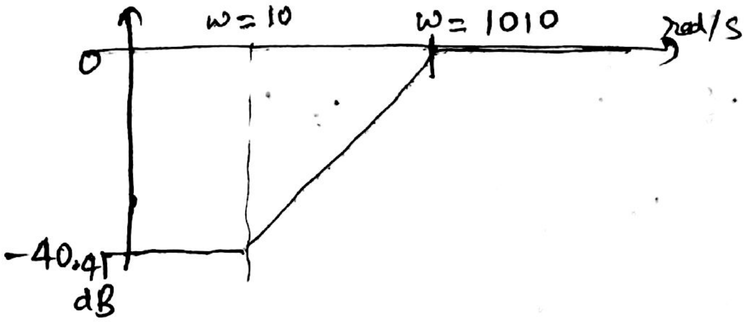
$$= \frac{1}{1 + \dfrac{10^3}{s+10}} = \frac{s+10}{s+1010}$$

a) DC gain : $\lim_{s\to 0} T_v(s) = \frac{10}{1010} = 0.0099$ in dB $= -40.41$ dB

∞ gain : $\lim_{s\to\infty} T_v(s) = 1$, in dB $= 0$

corner frequency $= 10$ (zero), 1010 (pole)

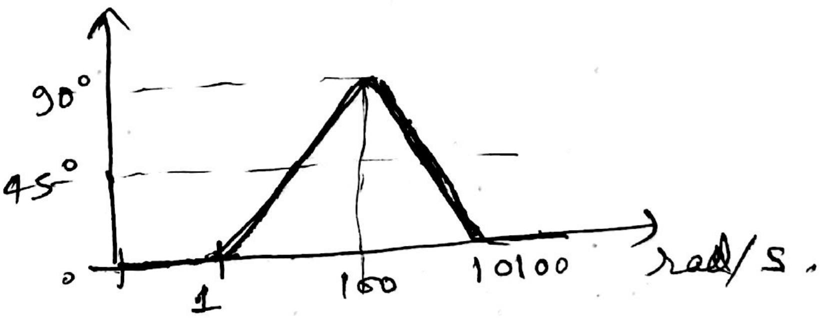
no critical point at $\omega = 0$, the slope of gain asymptote entering plot from left is 0 till it reaches, corner frequency 10, from where slope becomes +1. This happens till another corner frequency 1010 & slope change to 0 (+1 −1 = 0) & becomes straight (horizontal)



Similar observation is done for phase where phase starts increasing from 0.1×10 to 10×10 (rad/s) due to zero 10

& starts decreasing from 0.1×1010 to 10×1010 (rad/s)

combining these we get



For $V_1(t) = 10 \sin 100t \ V = 10 \cos\left(100t - \frac{\pi}{2}\right)$

$\omega = 100$ rad/s gain $= -20$ dB $= 0.1$

phase $= 90° = \pi/2$

$\therefore V_2(t) = 10 \times 0.1 \sin\left(100t + \frac{\pi}{2}\right)$

$= \sin\left(100t + \frac{\pi}{2}\right)$

or

$$\boxed{\begin{aligned} V_2(t) &= 10 \times 0.1 \cos\left(100t - \frac{\pi}{2} + \frac{\pi}{2}\right) \\ &= \cos(100t). \end{aligned}}$$

b) For actual output amplitude & phase for input, $\omega = 100$ rad/s.

$$|T_v(j100)| = \frac{\sqrt{10^2 + (100)^2}}{\sqrt{1010^2 + 100^2}} \approx 0.1.$$

$$\measuredangle T_v(j100) = \measuredangle \frac{10 + j100}{j100 + 1010} \approx 1.3724 \text{ rad} = 78.635^\circ$$

$\therefore\ v_2(t) = 10 \times 0.1 \sin(100t + 1.3724)$

$= \sin(100t + 1.3724)$

or $v_2(t) = 10 \times 0.1 \cos(100t - \frac{\pi}{2} + 1.3724) = \cos(100t - 0.1984)$

c) Using matlab plot

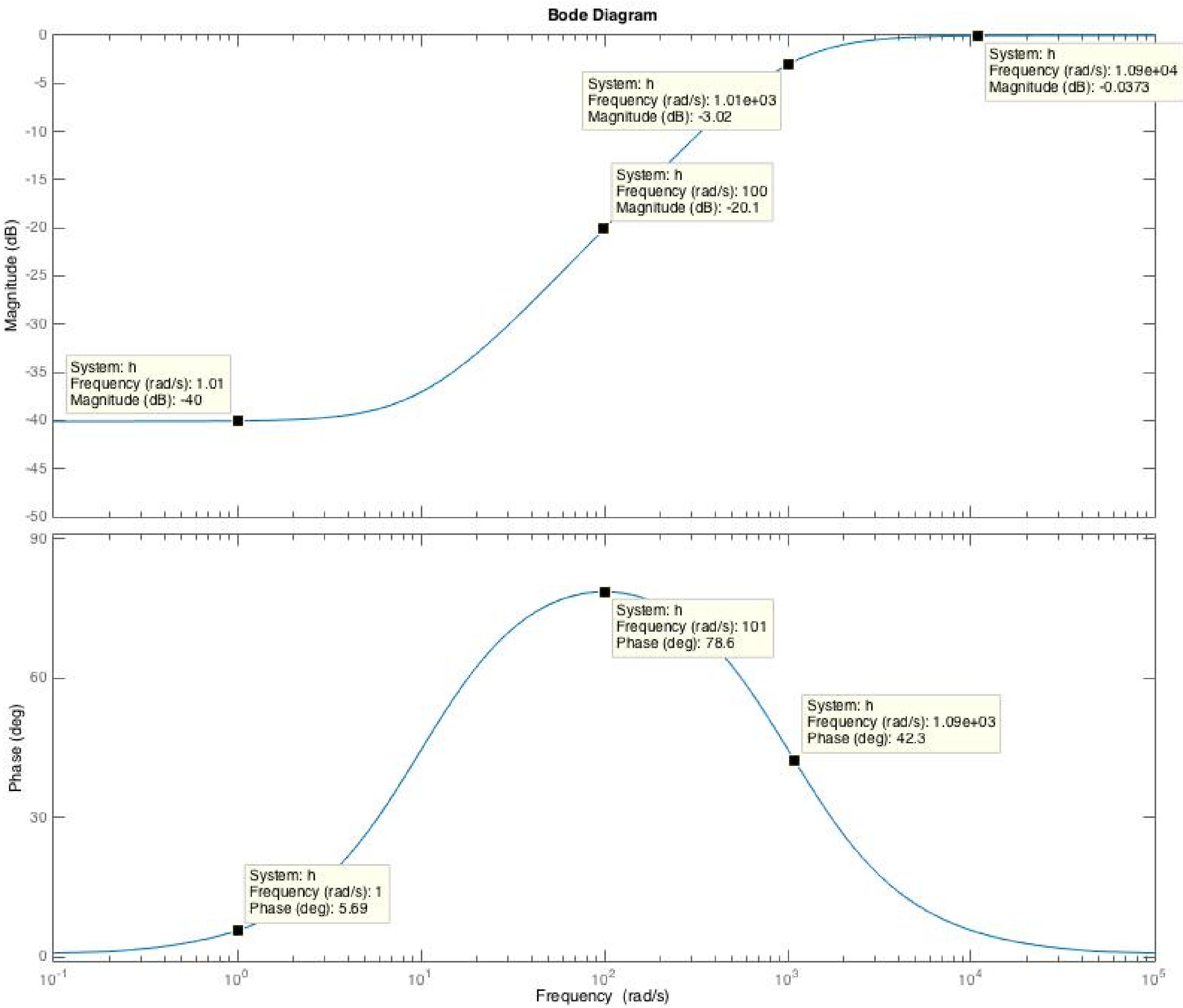
$|T_v(j0)|_{dB} = -40$ $\quad \measuredangle T_v(j0) = 5.69$

$|T_v(j100)|_{dB} = -20.1\,dB$, $\quad \measuredangle T_v(j100) = 78.6$.

$|T_v(j1010)|_{dB} = -3.02\,dB$, $\quad \measuredangle T_v(j1010) = 42.3$

$|T_v(j10100)|_{dB} = -0.0373\,dB$, $\quad \measuredangle T_v(j10100) \approx 0$

Hence, we see that the computations conform to computations done in b & c.


Bode Diagram
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
0
-5
-10
-15
-20
-25
-30
-35
-40
-45
-50
90
60
30
0
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
10^{5}
System: h
Frequency (rad/s): 1.09e+04
Magnitude (dB): -0.0373
System: h
Frequency (rad/s): 1.01e+03
Magnitude (dB): -3.02
System: h
Frequency (rad/s): 100
Magnitude (dB): -20.1
System: h
Frequency (rad/s): 1.01
Magnitude (dB): -40
System: h
Frequency (rad/s): 101
Phase (deg): 78.6
System: h
Frequency (rad/s): 1.09e+03
Phase (deg): 42.3
System: h
Frequency (rad/s): 1
Phase (deg): 5.69