MAE140 - Linear Circuits - Winter 17

Final, March 21, 2017

Instructions

(i) The exam is open book. You may use your class notes and textbook. You may use a hand calculator with no communication capabilities.

(ii) You have 180 minutes

(iii) Do not forget to write your **name** and **student number**

(iv) On the questions for which the answers are given, please provide detailed derivations

(v) The exam has 5 questions for a total of 50 points and 2 bonus points

Good luck!

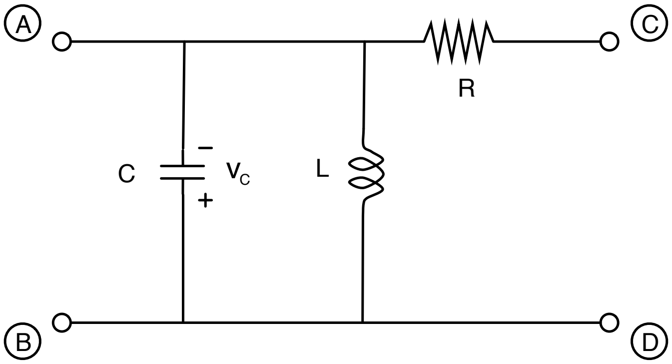

Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

All impedances should be given as a ratio of two polynomials.

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 1 into the s-domain.

Part II: [2 points] For the circuit you obtained in Part I, find the equivalent impedance as seen from terminals Ⓐ and Ⓑ.

Part III: [2 points] For the circuit you obtained in Part I, find the equivalent impedance as seen from terminals Ⓐ and Ⓒ.

Part IV: [2 points] For the circuit you obtained in Part I, find the equivalent impedance as seen from terminals Ⓒ and Ⓓ.

Part V: [3 points] If the initial condition of the capacitor is instead $v_C(0) = 10V$, what is the s-domain Thévenin equivalent of the circuit as seen from terminals Ⓒ and Ⓓ?

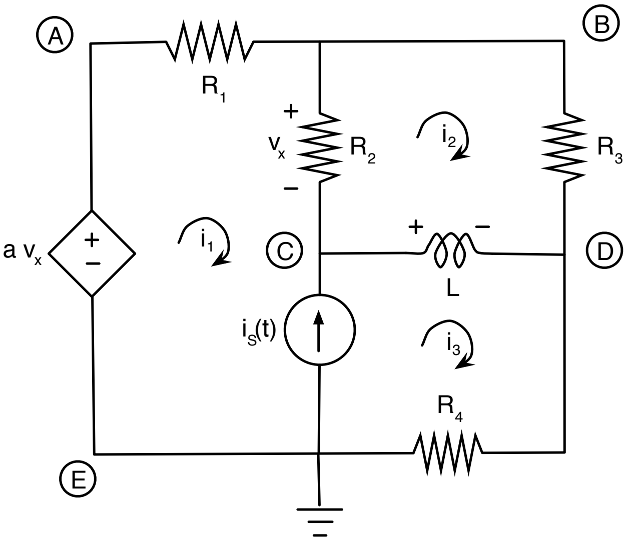

Figure 2: Nodal and Mesh Analysis Circuit for Question 2. a is a positive constant.

2. **Nodal and Mesh Analysis**

Part I: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its node-voltage equations. Use the reference node and other labels as shown in the figure. Do not assume zero initial conditions. Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

Part II: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its mesh-current equations. Use the mesh currents shown in the figure. Do not assume zero initial conditions. Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

Part III: [1 bonus point] Express the transform of the inductor voltage using your unknown variables of Part I. Also, express the transform of the inductor voltage using your unknown variables of Part II.

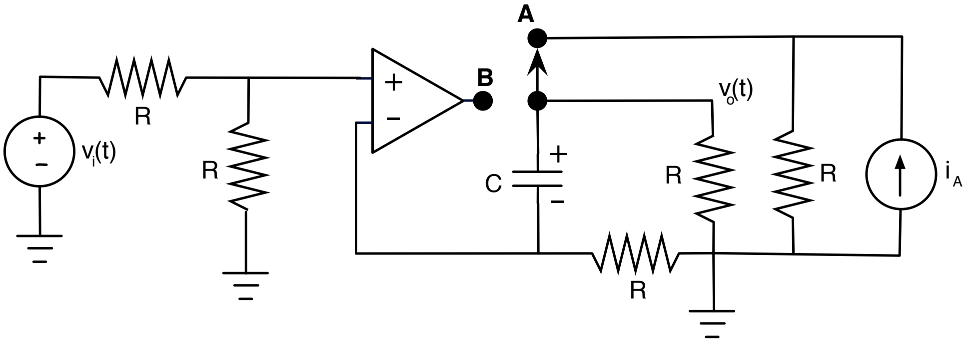

Figure 3: RCL circuit for Laplace Analysis for Question 3.

3. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 3. The value i_A of the current source at the right is constant. The switch is kept in position **A** for a very long time. At $t = 0$, it is moved to position **B**. Show that the initial condition for the capacitor is given by

$$v_C(0^-) = \frac{R}{2} i_A.$$

[Show your work]

Part II: [4 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the capacitor in which the initial condition appears as a current source. Use nodal analysis to express the output response transform $V_o(s)$ as a function of $V_i(s)$ and i_A.

Part III: [2 points] Use partial fractions and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $i_A = 2\,m\text{A}$, $v_i(t) = 12e^{-t}u(t)\,V$, $C = 6\,m\text{F}$, and $R = 1$ KΩ is

$$v_o(t) = (5e^{-t} + 2)u(t).$$

Part IV: [2 points] Decompose the output voltage of Part III as (i) the sum of the natural and forced response, and (ii) the sum of the zero-state and zero-input response.

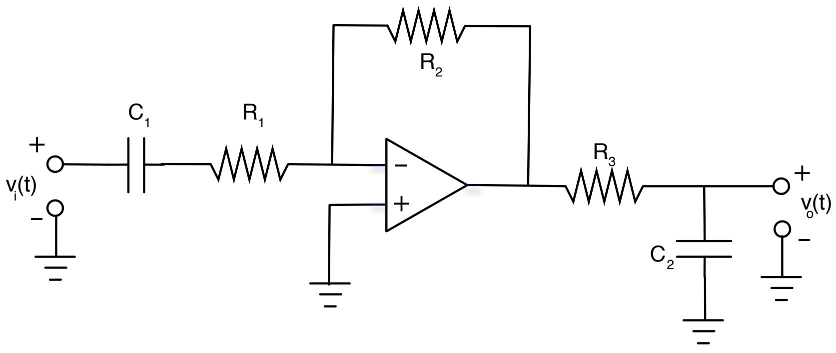

Figure 4: Frequency Response Analysis for Question 4.

4. **Frequency Response Analysis**

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 4 into the s-domain.

Part II: [2 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ is given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = -\frac{R_2 C_1 s}{(R_1 C_1 s + 1)(R_3 C_2 s + 1)}$$

[Show your work]

Part III [5.5 points] Let $R_1 = R_2 = R_3 = 1$ KΩ, $C_1 = 100\,\mu$F and $C_2 = 10\,\mu$F. Compute the gain and phase functions of $T(s)$. What are the DC gain and the ∞-freq gain? What are the corresponding values of the phase function? What are the cut-off frequencies? Sketch plots for the gain and phase functions. What type of filter is this one? [Explain your answer]

Part IV [1.5 points] Using what you know about frequency response, compute the steady state response $v_o^{SS}(t)$ of this circuit when $v_i(t) = \cos(50t + \frac{\pi}{4})$ using the same values of R_1, R_2, R_3, C_1, and C_2 as in Part III.

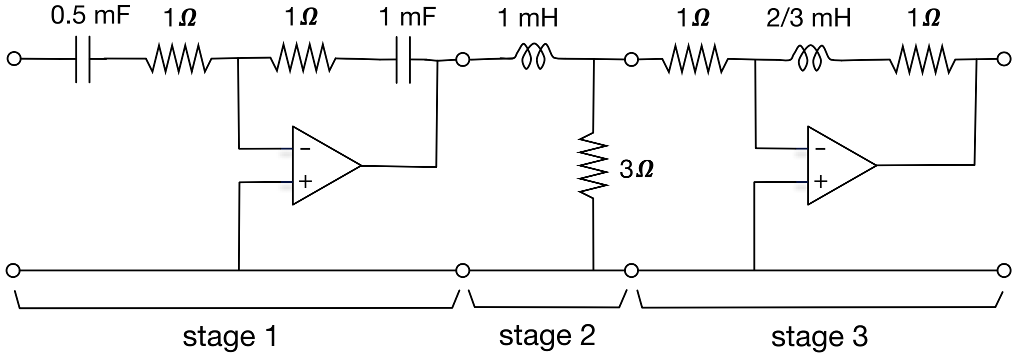

Figure 5: Circuit for Question 5.

5. **Loading and the Chain Rule**

A former instructor of MAE140 was given the task of designing a circuit with the following transfer function

$$T(s) = \frac{2s^2 + 5000s + 3 \cdot 10^6}{s^2 + 5000s + 6 \cdot 10^6}$$

He decomposed the transfer function as follows

$$T(s) = \Big(-\frac{s + 1000}{s + 2000}\Big)\Big(\frac{3000}{3000 + s}\Big)\Big(-\frac{s + 1500}{1500}\Big)$$

and came up with the design in Figure 5.

Part I: [2 points] Consider each stage in Figure 5 separately, and compute the transfer function of each one. Show that the product of the 3 transfer functions is equal to $T(s)$.

Part II: [1 point] In spite of Part I, explain why the circuit designed by the instructor is not a valid solution. Properly justify your answer.

Part III: [1 point] Could you fix the design provided by the instructor by re-ordering the stages without modifying anything else? Again, justify your answer.

Part IV: [1 point] Could you fix the design provided by the instructor by using one more op-amp without re-ordering the stages?

Part V: [5 points] Provide an alternative design solution that only uses 1 op-amp and components with the same values as those used in the instructor's design (i.e., 1Ω-resistors, $1\,m$H- and $2/3\,m$H-inductors, and $0.5\,m$F- and $1\,m$F-capacitors). Your design should be based on connecting two voltage dividers and one non-inverting op-amp. Make sure you properly justify that the chain rule applies.

Part VI: [1 bonus point] Name (i) one criteria for which your design in Part IV is better than your design in Part V, and (ii) one criteria for which your design in Part V is better than your design in Part VI.