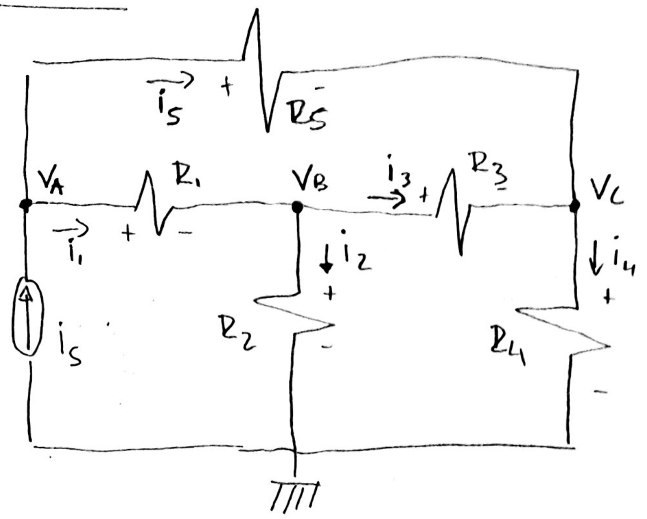
Homework 3

3.1)



1) KCL:

A: $i_s - i_1 - i_5 = 0$

B: $i_1 - i_2 - i_3 = 0$

C: $i_5 + i_3 - i_4 = 0$

2) Relate element currents to node voltages:

$i_1 = \frac{1}{R_1}(V_A - V_B)$, $i_2 = \frac{V_B}{R_2}$, $i_3 = \frac{1}{R_3}(V_B - V_C)$, $i_4 = \frac{V_C}{R_4}$

$i_5 = \frac{1}{R_5}(V_A - V_C)$

3) Node voltage equations:

A: $i_s - \frac{1}{R_1}(V_A - V_B) - \frac{1}{R_5}(V_A - V_C) = 0$

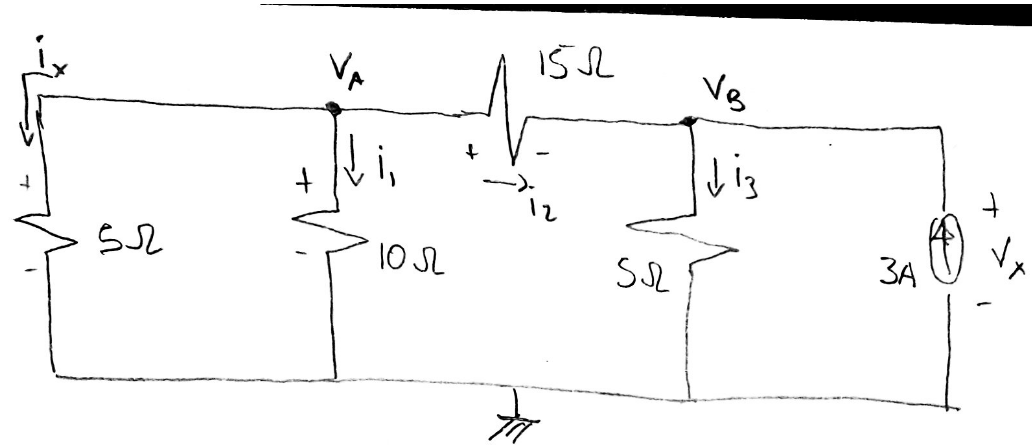
B: $\frac{1}{R_1}(V_A - V_B) - \frac{V_B}{R_2} - \frac{1}{R_3}(V_B - V_C) = 0$

C: $\frac{1}{R_5}(V_A - V_C) + \frac{1}{R_3}(V_B - V_C) - \frac{V_C}{R_4} = 0$

4) Node voltage equations in the form $Ax = b$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_5} & -\frac{1}{R_1} & -\frac{1}{R_5} \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_5} & -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix}$$

3.3)



1) <u>KCL</u>

A: $i_x + i_1 + i_2 = 0$

B: $i_3 - i_2 - 3A = 0$

2) Relate element currents to node voltages:

$i_1 = \frac{V_A}{10\Omega}$, $i_2 = \frac{1}{15\Omega}(V_A - V_B)$, $i_3 = \frac{V_B}{5\Omega}$, $i_x = \frac{V_A}{5\Omega}$

3) Node-Voltage equations:

A: $\frac{1}{5}V_A + \frac{V_A}{10} + \frac{1}{15}(V_A - V_B) = 0$

B: $\frac{V_B}{5} - \frac{1}{15}(V_A - V_B) - 3A = 0$

a) Node-Voltage equations in the form $Ax = b$

$$\begin{bmatrix} \frac{1}{5} + \frac{1}{10} + \frac{1}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{1}{5} + \frac{1}{15} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 0 \\ 3A \end{bmatrix}$$

b)

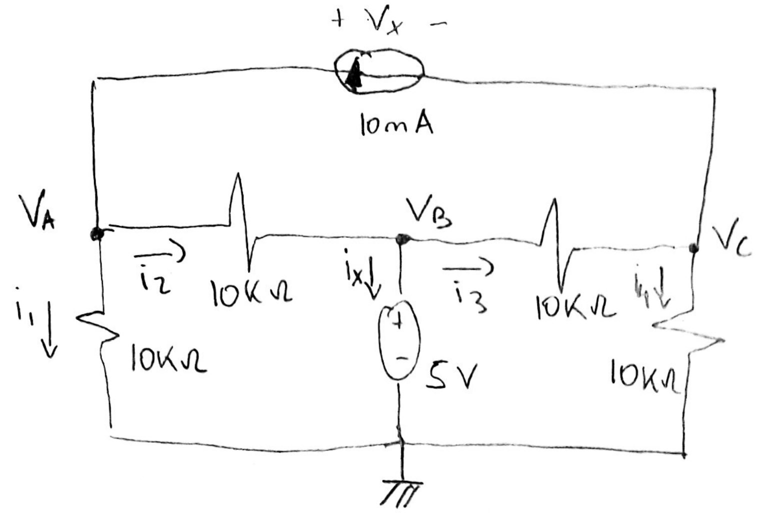
$$A^{-1} = \frac{1}{\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{15}\right)\left(\frac{1}{5}+\frac{1}{15}\right) - \left(\frac{1}{15}\right)} \begin{bmatrix} \frac{1}{5} + \frac{1}{5} & \frac{1}{15} \\ \frac{1}{15} & \frac{1}{5} + \frac{1}{10} + \frac{1}{15} \end{bmatrix} = \begin{bmatrix} 2.8571 & 0.7143 \\ 0.7143 & 3.928 \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = A^{-1}\begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 2.14\,V \\ 11.78\,V \end{bmatrix}$$

c) $V_x = V_B = 11.78V$

$i_x = \frac{V_A}{5\Omega} = \frac{2.14}{5\Omega} = 0.428\ A$

3.5)



KCL:

A: $i_1 + i_2 - 10mA = 0$

B: $i_x + i_3 - i_2 = 0$

C: $i_4 - i_3 + 10mA = 0$

Relate element currents to node Voltages:

$i_1 = \frac{V_A}{10K\Omega}$, $i_2 = \frac{1}{10K\Omega}(V_A - V_B)$, $i_3 = \frac{1}{10K\Omega}(V_B - V_C)$

$i_4 = \frac{V_C}{10K\Omega}$, $V_B = 5V$ (By inspection)

Node-Voltage equations:

A: $\frac{V_A}{10K\Omega} + \frac{1}{10K\Omega}(V_A - 5V) - 10mA = 0$

C: $\frac{V_C}{10K\Omega} - \frac{1}{10K\Omega}(5V - V_C) + 10mA = 0$

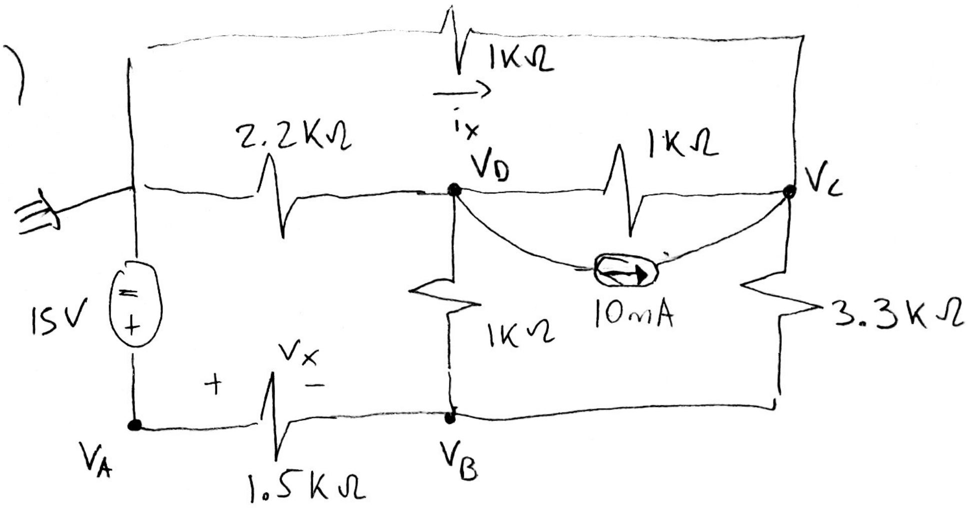
a) In matrix form $Ax = b$:

$$\begin{bmatrix} \frac{1}{10K} + \frac{1}{10K} & 0 \\ 0 & \frac{1}{10K} + \frac{1}{10K} \end{bmatrix} \begin{bmatrix} V_A \\ V_C \end{bmatrix} = \begin{bmatrix} 10\times10^3 + \frac{5}{10K} \\ -10\times10^3 + \frac{5}{10K} \end{bmatrix}$$

b) $\begin{bmatrix} V_A \\ V_C \end{bmatrix} = \begin{bmatrix} 52.5V \\ -47.5V \end{bmatrix}$

c) $V_x = V_A - V_C = 52.5V - (-47.5) = 100V$, $i_x = i_2 - i_3 = -0.5mA$

3.8)



By inspection $V_A = 15\,V$

Nodal-voltage equations in matrix form $Ax = b$

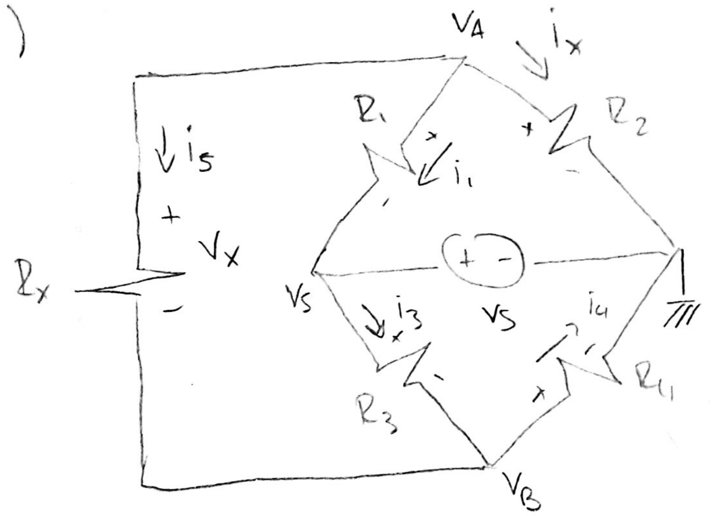
)
$$\begin{bmatrix} \frac{1}{1.5K} + \frac{1}{1K} + \frac{1}{3.3K} & \frac{-1}{3.3K} & \frac{-1}{1K} \\ \frac{-1}{3.3K} & \frac{1}{3.3K} + \frac{1}{1K} + \frac{1}{1K} & \frac{-1}{1K} \\ \frac{-1}{1K} & \frac{-1}{1K} & \frac{1}{2.2K} + \frac{1}{1K} + \frac{1}{1K} \end{bmatrix} \begin{bmatrix} V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} \frac{15}{1.5K} \\ 10 \times 10^{-3} \\ -10 \times 10^{-3} \end{bmatrix}$$

$$\begin{bmatrix} V_B \\ V_C \\ V_D \end{bmatrix} = A^{-1} x = \begin{bmatrix} 6.286\,V \\ 5.482\,V \\ 0.7203\,V \end{bmatrix}$$

) $V_x = V_A - V_B = 15\,V - 6.286\,V = \mathbf{8.72\,V}$

$$i_x = -\frac{V_C}{1K} = \frac{-5.482}{1K} = -5.482\,mA$$

3.11)



Node A: $i_1 + i_x + i_s = 0$

$$\frac{1}{R_1}(V_A - V_S) + \frac{1}{R_2}(V_A) + \frac{1}{R_x}(V_A - V_B) = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_x}\right) V_A - \frac{1}{R_x} V_B = \frac{1}{R_1} V_S$$

Node B:

$$\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_x}\right) V_B - \frac{1}{R_x} V_A = \frac{1}{R_3} V_S$$

b) We solve using the matrix form $Ax = b$

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_x} & -\frac{1}{R_x} \\ -\frac{1}{R_x} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_x} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} V_S \\ \frac{1}{R_3} V_S \end{bmatrix}$$

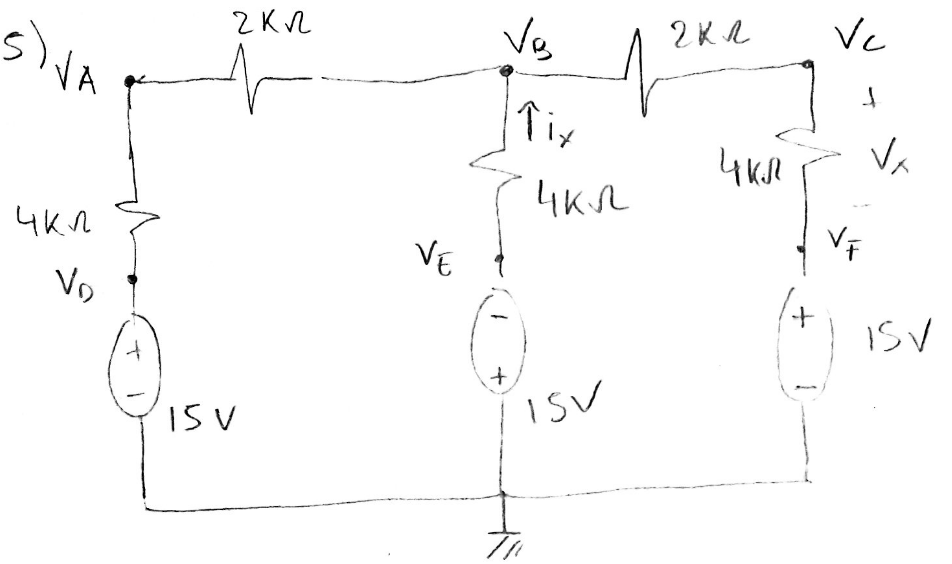
when $R_1 = 1K\Omega$, $R_2 = 1.5K\Omega$, $R_3 = 500\,\Omega$, $R_4 = 2K\Omega$, $R_x = 100\Omega$, $V_S = 15V$

$$\begin{bmatrix} V_A \\ V_B \end{bmatrix} = A^{-1} \begin{bmatrix} \frac{1}{1.5k} 15 \\ \frac{1}{500} 15 \end{bmatrix} = \begin{bmatrix} 10.63\ V \\ 10.90\ V \end{bmatrix}$$

$$V_x = V_A - V_B = 10.63 - 10.90 = -0.27V$$

$$i_x = \frac{V_A}{R_2} = \frac{10.63V}{1.5K\Omega} = 7.1\ mA$$

3.15)



By inspection:

$V_D = 15V$, $V_E = -15$, $V_F = 15V$

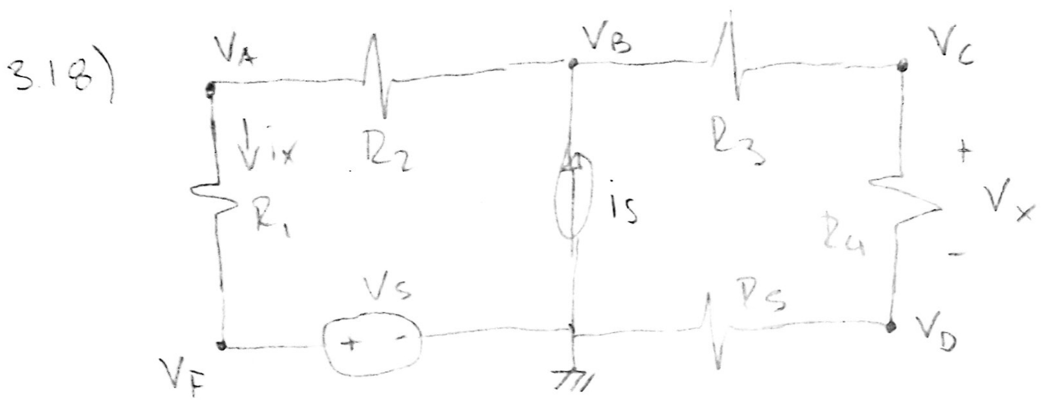
a)
$$\begin{bmatrix} \frac{1}{2K}+\frac{1}{4K} & -\frac{1}{2K} & 0 \\ -\frac{1}{2K} & \frac{1}{2K}+\frac{1}{4K}+\frac{1}{2K} & -\frac{1}{2K} \\ 0 & -\frac{1}{2K} & \frac{1}{2K}+\frac{1}{4K} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \frac{1}{4K}(15) \\ \frac{1}{4K}(-15) \\ \frac{1}{4K}(15) \end{bmatrix}$$

$$\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = A^{-1} \begin{bmatrix} \frac{15}{4K} \\ -\frac{15}{4K} \\ \frac{15}{4K} \end{bmatrix} = \begin{bmatrix} 6.4286\,V \\ 2.1429\,V \\ 6.4286\,V \end{bmatrix}$$

c) $V_x = V_C - V_F = 6.4286V - 15V = -8.5715\,V$

$$i_x = \frac{V_E - V_B}{4K} = \frac{-15V - 2.1429V}{4K} = -4.3\,mA$$

3.18)



By inspection $V_F = V_S$

a)

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} & 0 & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} & 0 \\ 0 & -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R_4} \\ 0 & 0 & -\frac{1}{R_4} & \frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} \frac{V_S}{R_1} \\ i_S \\ 0 \\ 0 \end{bmatrix}$$

b) $R_1 = 2.7K\Omega$, $R_2 = 1.5K\Omega$, $R_3 = 680\Omega$, $R_4 = 2.2K\Omega$, $R_5 = 3.3K\Omega$
$i_S = 10mA$, $V_S = 12V$

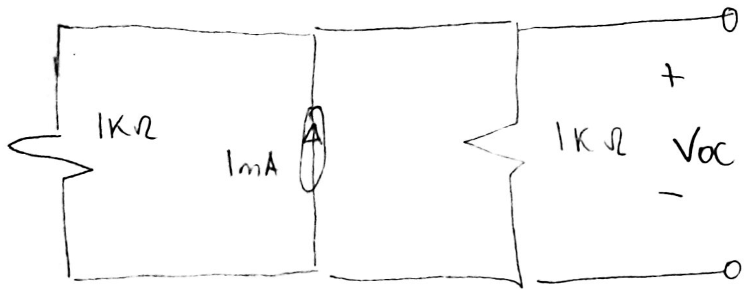
we solve using

$$\begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = A^{-1} \begin{bmatrix} \frac{12}{2.7K} \\ 10mA \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 24.9538V \\ 32.1503V \\ 28.6127V \\ 17.1676V \end{bmatrix}$$

$V_x = V_C - V_D = 28.6127V - 17.1676V = 11.4451V$

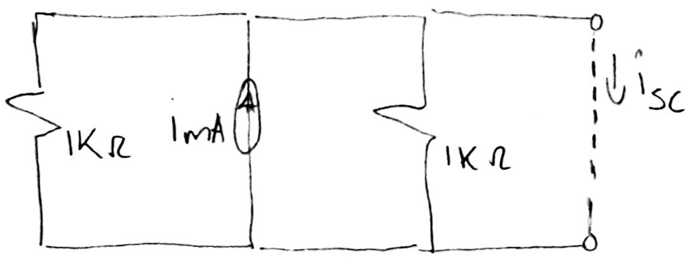
$i_x = \frac{V_A - V_F}{R_1} = \frac{24.9538V - 12V}{2.7K\Omega} = \frac{12.9538V}{2.7K\Omega} = 4.8mA$

3.52)



V_T, R_T, i_N

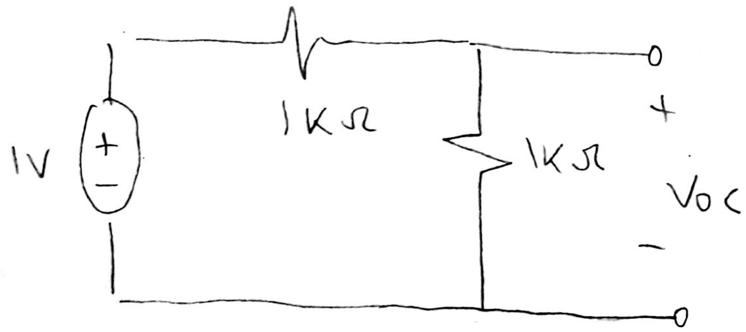
First we find the current in short circuit i_{sc}:



$i_{sc} = 1mA$

Note that i_{sc} is the Norton current i_N.

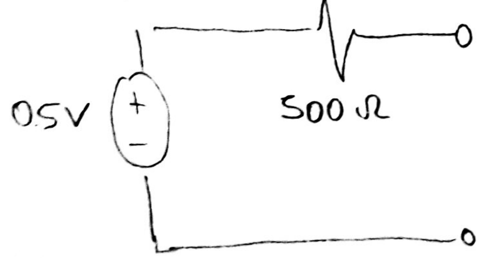
Now we find V_{oc} (Voltage when the circuit is open) using source transformation and voltage division:



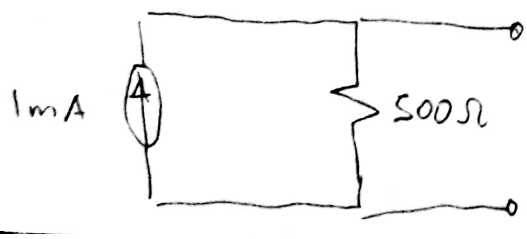
$V_{oc} = \frac{1}{2}(1V) = \frac{1}{2}V$

$$R_T = \frac{V_{oc}}{i_{sc}} = \frac{1/2}{1\times10^{-3}} = 500\ \Omega$$

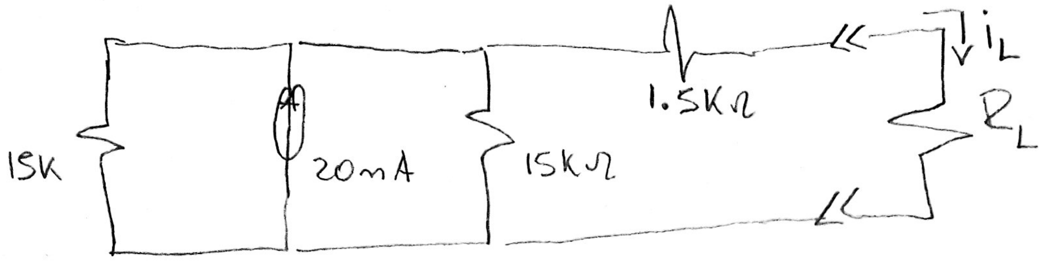
Then, the Thévenin equivalent is:



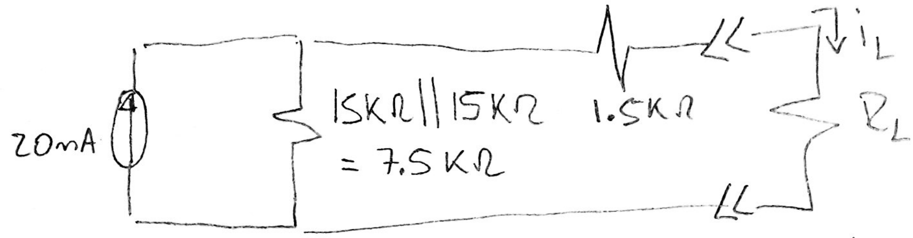
The Norton equivalent:



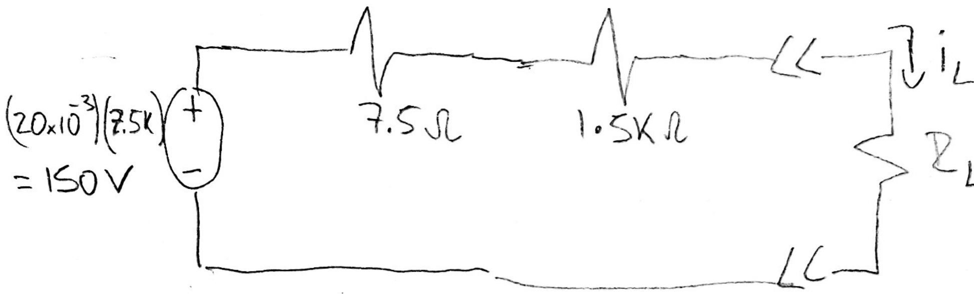
3.57)



Note that the circuit above is equivalent to:



which is equivalent to: (using source transformation)



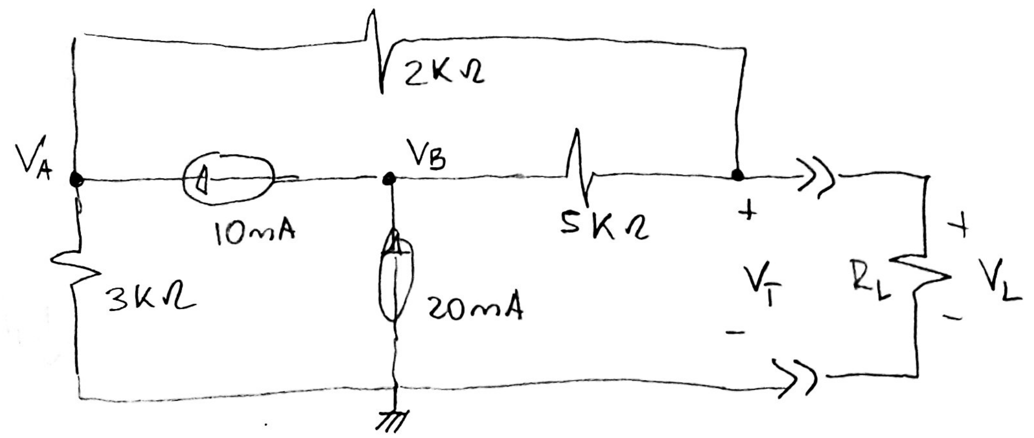
We use a source transformation again:

Norton equivalent

$i_N = 16.67 mA$ and $R_N = 9K\Omega$

- When $R_L = 4.7K\Omega$, $i_L = \dfrac{\frac{1}{4.7}}{\frac{1}{9}+\frac{1}{4.7}}(16.67\,mA) = 10.95\,mA$
- When $R_L = 15K\Omega$, $i_L = \dfrac{\frac{1}{15}}{\frac{1}{9}+\frac{1}{15}}(16.67\,mA) = 6.25\,mA$
- When $R_L = 68K\Omega$, $i_L = \dfrac{\frac{1}{68}}{\frac{1}{9}+\frac{1}{68}}(16.67\,mA) = 1.95\,mA$

3.62)



We find V_T when the circuit is open. For that we use Nodal equations in the matrix form $Ax = b$

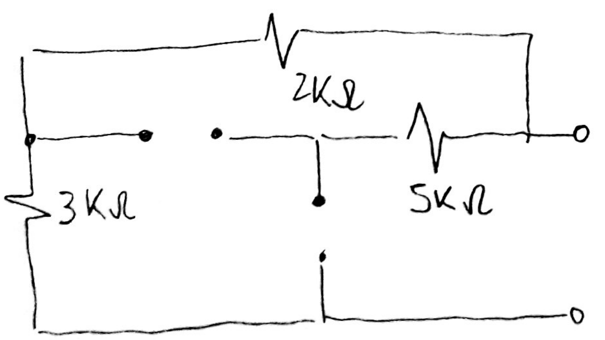
$$\begin{bmatrix} \frac{1}{3K} + \frac{1}{2K} & 0 & -\frac{1}{2K} \\ 0 & \frac{1}{5K} & -\frac{1}{5K} \\ -\frac{1}{2K} & -\frac{1}{5K} & \frac{1}{2K} + \frac{1}{5K} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_T \end{bmatrix} = \begin{bmatrix} 10 \times 10^{-3} \\ 20 \times 10^{-3} - 10 \times 10^{-3} \\ 0 \end{bmatrix}$$

Solving $x = A^{-1} b$ we have

$$\begin{bmatrix} V_A \\ V_B \\ V_T \end{bmatrix} = \begin{bmatrix} 60\text{ V} \\ 130\text{ V} \\ 80\text{ V} \end{bmatrix}$$

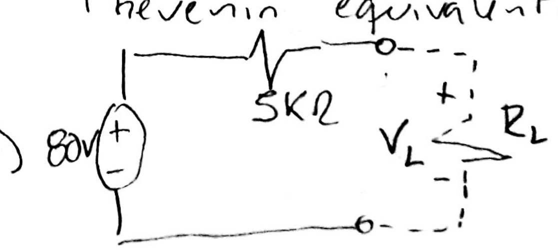
Therefore $V_T = 80V$.

We find next R_T: (By turning off all sources)



$R_T = 5K\Omega$

Thévenin equivalent:



When $R_L = 2.5K\Omega$, $V_L = \frac{R_L}{R_T + R_L} V_T = \frac{2.5}{5+2.5} \cdot (80V) = 26.66\text{ V}$