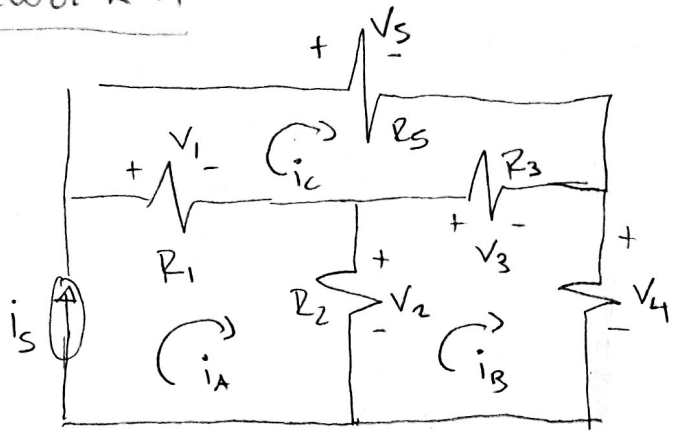
Homework 4

3.1)



KVL: By inspection $i_A = i_S$

Mesh B: $-V_2 + V_3 + V_4 = 0$

Mesh C: $-V_1 + V_S - V_3 = 0$

Element voltages

$V_1 = R_1 (i_S - i_C)$, $V_2 = R_2 (i_S - i_B)$, $V_3 = R_3 (i_B - i_C)$

$V_4 = R_4 i_B$, $V_S = R_S i_C$

Mesh-analysis:

Mesh B: $-R_2 (i_S - i_B) + R_3 (i_B - i_C) + R_4 i_B = 0$

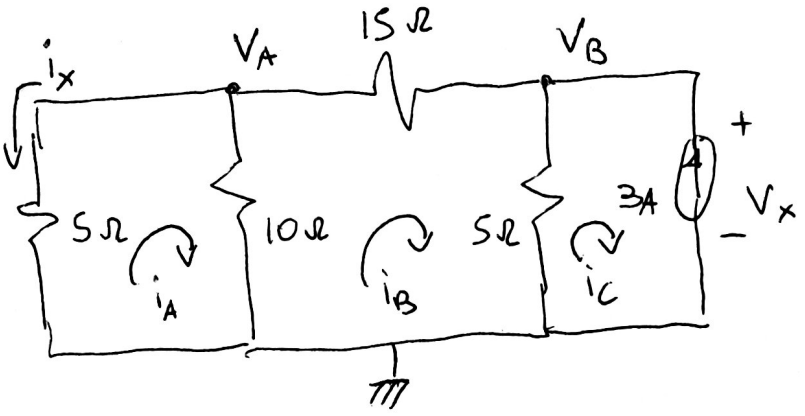
$$(R_2 + R_3 + R_4) i_B - R_3 i_C = R_2 i_S$$

Mesh C: $+(R_1 + R_2 + R_S) i_C - R_3 i_B = R_1 i_S$

In matrix form

$$\begin{bmatrix} R_2 + R_3 + R_4 & -R_3 \\ -R_3 & R_1 + R_3 + R_S \end{bmatrix} \begin{bmatrix} i_B \\ i_C \end{bmatrix} = \begin{bmatrix} R_2 i_S \\ R_1 i_S \end{bmatrix}$$

3.3)



By inspection $i_C = -3A$

a)

$$\begin{bmatrix} 5+10 & -10 \\ -10 & 10+15+5 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 0 \\ -5(3) \end{bmatrix}$$

$$\begin{bmatrix} 15 & -10 \\ -10 & 30 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 0 \\ -15 \end{bmatrix}$$

b)

$$\begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} -0.4286\ A \\ -0.6429\ A \end{bmatrix}$$

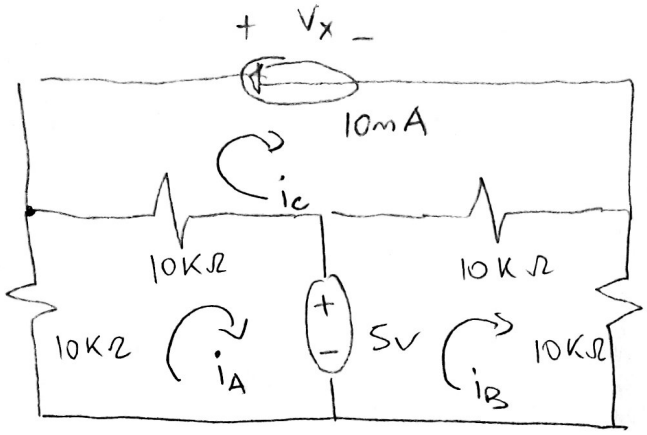
$V_A = -5\Omega\, i_A = -5(-0.4286) = 2.14\ V$

$V_B = 5(i_B - i_C) = 5(-0.6429 + 3) = 11.78V$

c) $i_x = -i_A = 0.4286\ A$

$V_x = V_B = 11.78V$

3.5)



By inspection $i_C = -10mA$

a)

$$\begin{bmatrix} 10K+10K & 0 \\ 0 & 10K+10K \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 10K(-10mA)-5 \\ 10K(-10mA)+5 \end{bmatrix}$$

$$\begin{bmatrix} 20K & 0 \\ 0 & 20K \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} -105 \\ -95 \end{bmatrix}$$

b)

$$\begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} -5.25mA \\ -4.75mA \end{bmatrix}$$

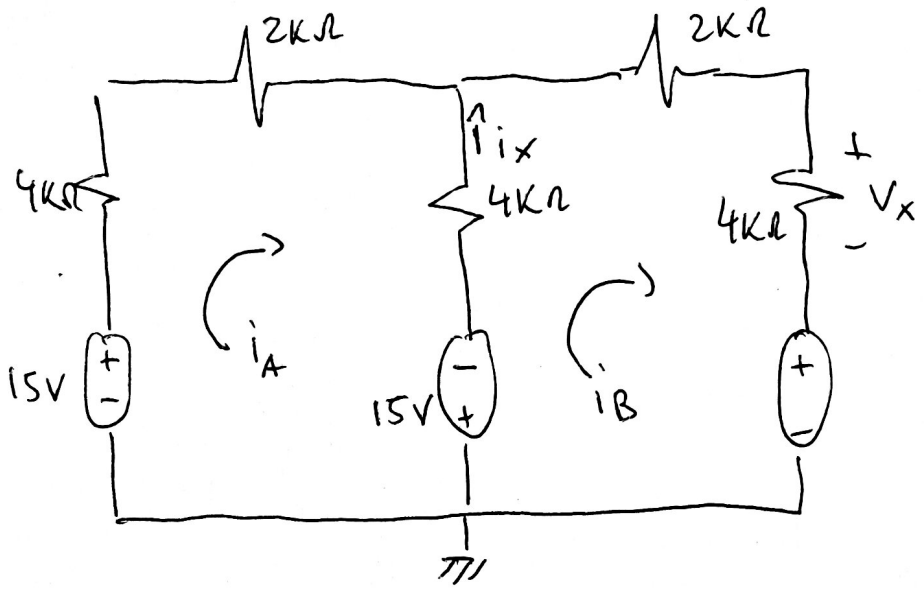
$$V_A = -10K(-5.25mA) = 52.5V$$

$$V_C = 10K\, i_B = 10K(-4.75mA) = -47.5V$$

c) $V_x = V_A - V_C = 52.5V + 47.5V = 100V$

$i_x = i_A - i_B = -5.25mA + 4.75mA = -0.5mA$

3.15)



a)

$$\begin{bmatrix} 4K+2K+4K & -4K \\ -4K & 4K+2K+4K \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 15+15 \\ -15-15 \end{bmatrix}$$

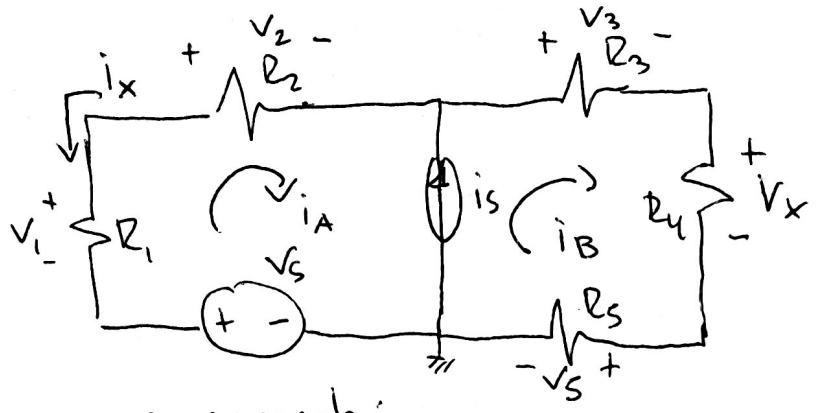
$$\begin{bmatrix} 10K & -4K \\ -4K & 10K \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 30 \\ -30 \end{bmatrix}$$

b) $$\begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 2.1429\,mA \\ -2.1429\,mA \end{bmatrix}$$

c) $i_x = i_B - i_A = -2.1429\,mA - 2.1429\,mA = -4.2858\,mA$

$V_x = 4K(i_B) = 4K(-2.1429\,mA) = -8.5716\,V$

3.18)



We use super mesh:

$$i_S = i_B - i_A$$

$$-V_1 + V_2 + V_3 + V_x + V_S - V_S = 0$$

$$+R_1 i_A + R_2 i_A + R_3 i_B + R_4 i_B + R_5 i_B - V_S = 0$$

in matrix form

a)

$$\begin{bmatrix} R_1+R_2 & R_3+R_4+R_5 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} V_S \\ i_S \end{bmatrix}$$

b)

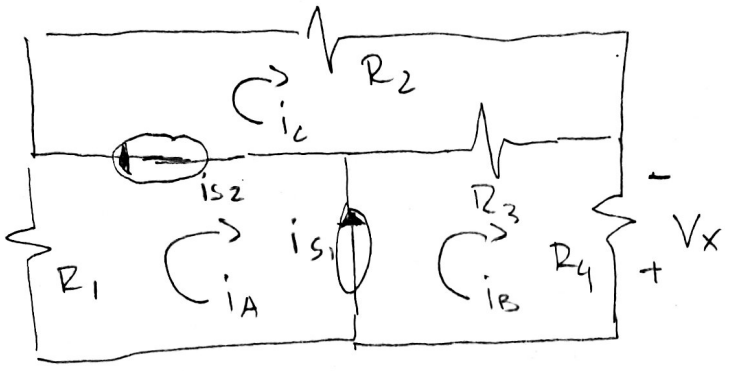
$$\begin{bmatrix} 2.7K+1.5K & 680K+2.2K+3.3K \\ -1 & 1 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 12V \\ 10mA \end{bmatrix}$$

$$\begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} -4.7977\,mA \\ 5.2023\,mA \end{bmatrix}$$

$$V_x = R_4(i_B) = 2.2K\,(5.2023\,mA) = 11.44\,V$$

$$i_x = -i_A = 4.7977\,mA$$

.19)



a) $i_{s_1} = i_B - i_A \quad (1)$

$i_{s_2} = i_C - i_A \quad (2)$

Supermesh:

$R_1 i_A + R_2 i_C + R_4 i_B = 0 \quad (3)$

$(1)-(2) \Rightarrow i_{s_1} - i_{s_2} - i_B + i_C = 0$

$\Rightarrow i_B = i_{s_1} - i_{s_2} + i_C \quad (4)$

Replace $i_A = i_B - i_{s_1}$ in (3):

$R_1 (i_B - i_{s_1}) + R_2 i_C + R_4 i_B = 0 \quad (5)$

Use (4) in (5):

$R_1 (\cancel{i_{s_1}} - i_{s_2} + i_C - \cancel{i_{s_1}}) + R_2 i_C + R_4 (i_{s_1} - i_{s_2} + i_C) = 0$

$-R_1 i_{s_2} + (R_1 + R_2 + R_4) i_C + R_4 (i_{s_1} - i_{s_2}) = 0$

$\Rightarrow i_C = \dfrac{R_1 i_{s_2} - R_4 (i_{s_1} - i_{s_2})}{R_1 + R_2 + R_4} = \dfrac{(R_1 + R_4) i_{s_2} - R_4 i_{s_1}}{R_1 + R_2 + R_4}$

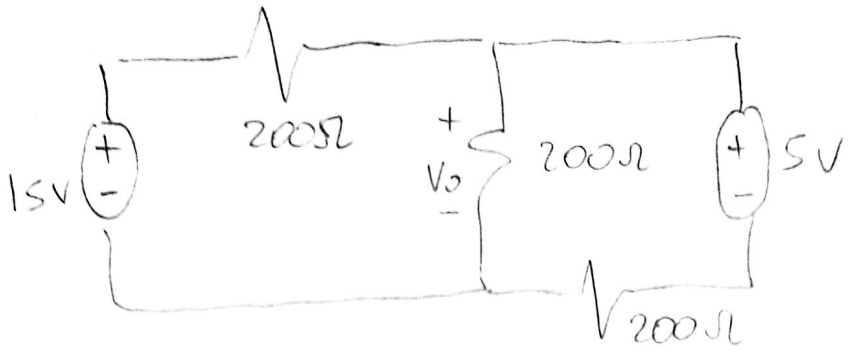
$i_B = i_{s_1} - i_{s_2} + \dfrac{(R_1 + R_4) i_{s_2} - R_4 i_{s_1}}{R_1 + R_2 + R_4} = \dfrac{(R_1 + R_2) i_{s_1} - R_2 i_{s_2}}{R_1 + R_2 + R_4}$

$i_A = i_B - i_{s_1} = \dfrac{(R_1 + R_2) i_{s_1} - R_2 i_{s_2}}{R_1 + R_2 + R_4} - i_{s_1}$

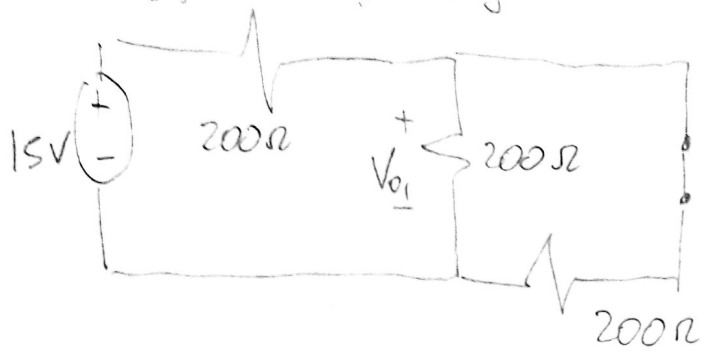
$= \dfrac{-R_4 i_{s_1} - R_2 i_{s_2}}{R_1 + R_2 + R_4}$

b) $V_x = -R_4 i_B = -R_4 \left(\dfrac{(R_1 + R_2) i_{s_1} - R_2 i_{s_2}}{R_1 + R_2 + R_4} \right)$

3.40)

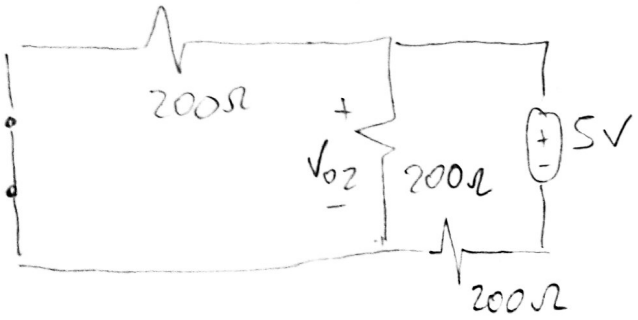


Turn off one voltage source:



$$V_{o_1} = \frac{100}{200+100}(15V) = 5V$$

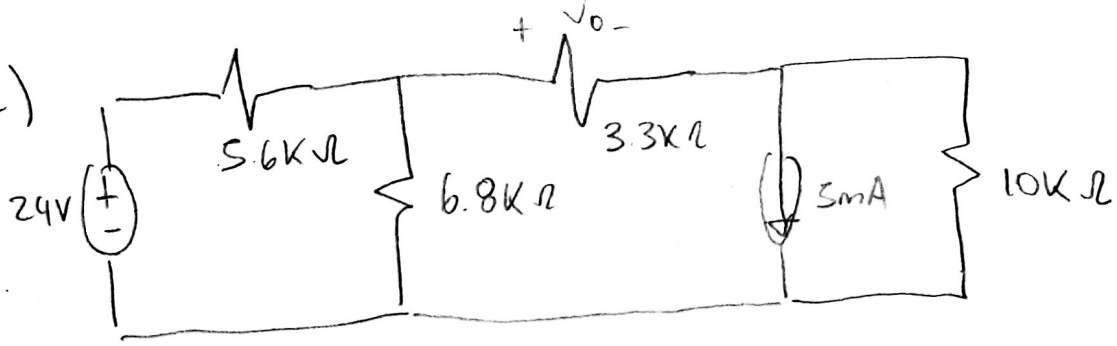
Next, we turn off the other voltage source:



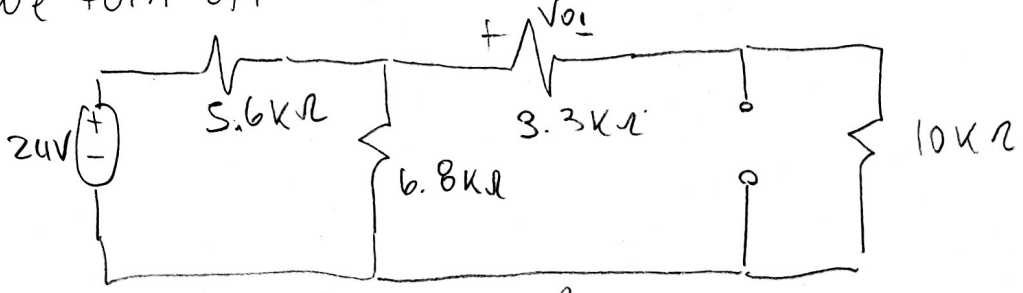
$$V_{o_2} = \frac{100}{300}(5V) = 1.66V$$

It follows that $V_o = V_{o_1} + V_{o_2} = 5 + 1.66 = 6.66V$

3.42)



We turn off the current source:

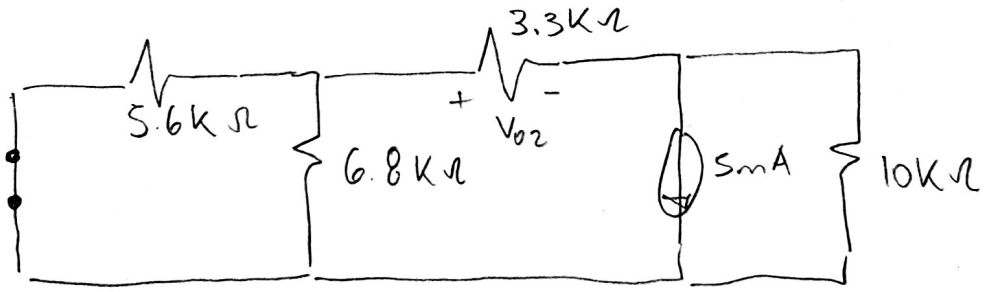


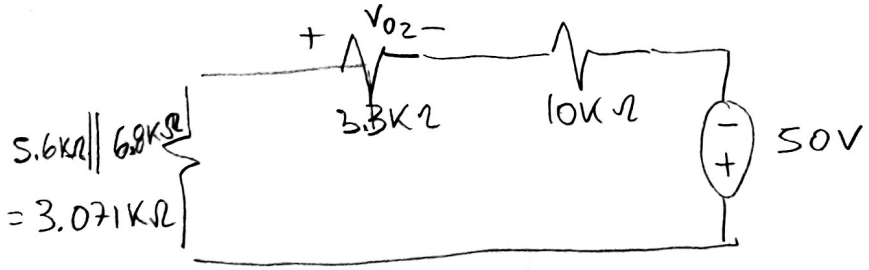
we use source transformation:

$$i_1 = \frac{\frac{1}{13.3}}{\frac{1}{13.3} + \frac{1}{5.6} + \frac{1}{6.8}} \cdot \frac{24}{5.6K} = 0.8039$$

$$V_{o_1} = i_1 \cdot 3.3K\Omega = 2.653\,V$$

Next, we turn off the voltage source:



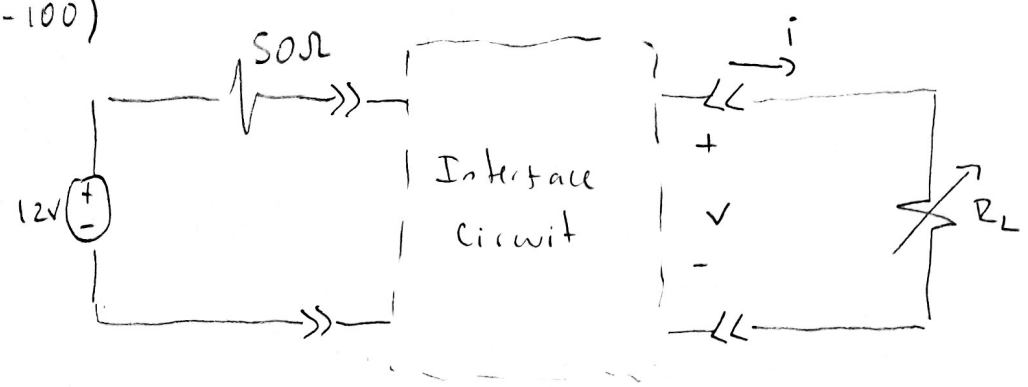


$$V_{o_2} = \frac{3.3}{3.3 + 10 + 3.071} \cdot (50V)$$

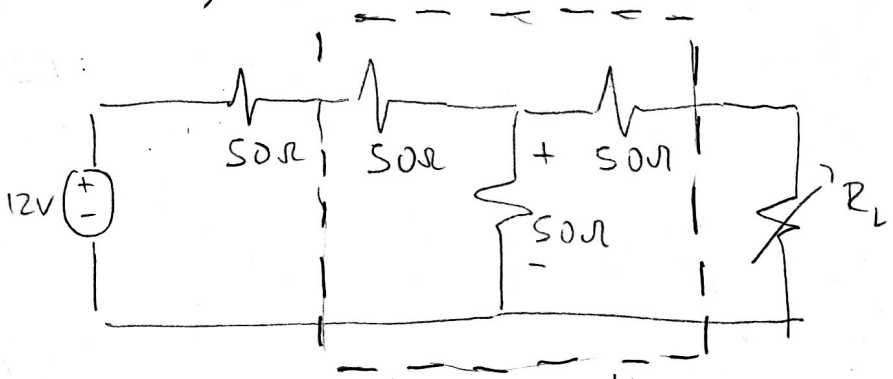
$$= 10.078\,V$$

$$V_o = V_{o_1} + V_{o_2} = 2.653V + 10.078V = 12.7318\,V$$

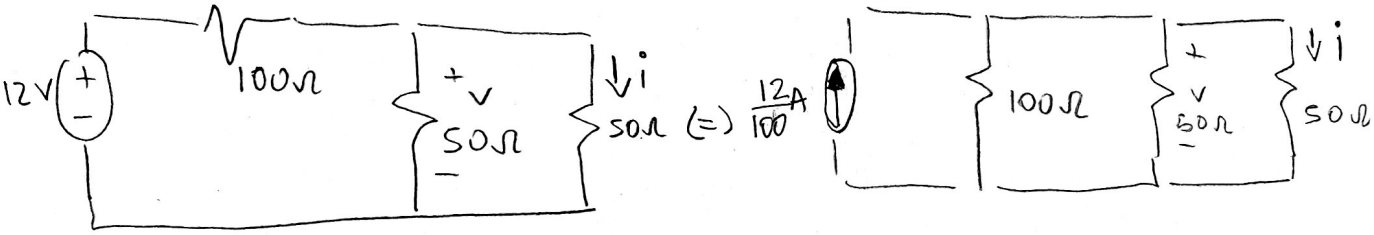
3-100)



$V \leq 4V$, $i \leq 50mA$

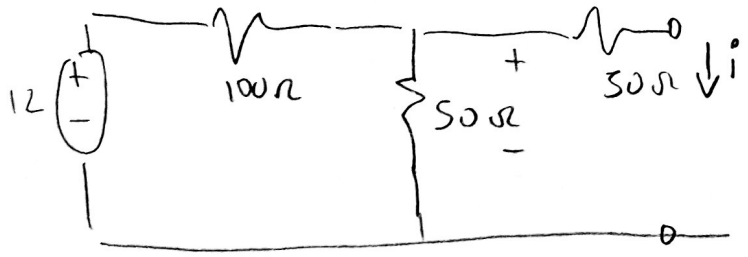


When $R_L = 0$ (short circuit)



$$i = \frac{\frac{1}{50}}{\frac{1}{100}+\frac{1}{50}+\frac{1}{50}}\left(\frac{12A}{100}\right) = 48mA \quad , \quad V = i \times 50\Omega = 48mA \times 50\Omega = 2.4V$$

When $R_L \to +\infty$ (open circuit)



We use voltage division:

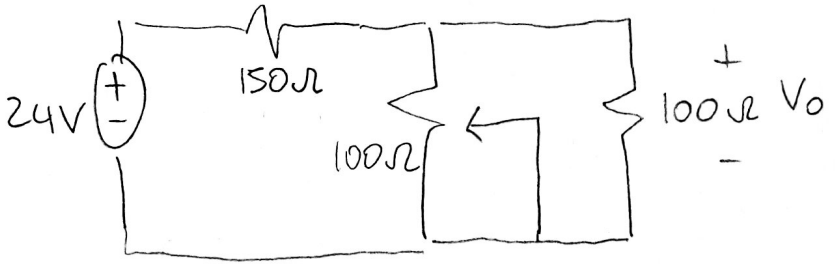
$$V = \frac{50}{100+50}\,12 = 4V$$

$$i = 0$$

Therefore, the interface circuit meets the required specification

3.102)

Circuit 1:

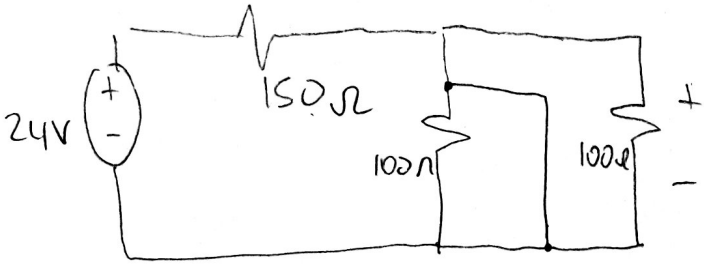


Required specifications:

1- Deliver 0 − 6 V

2- Power source 3W

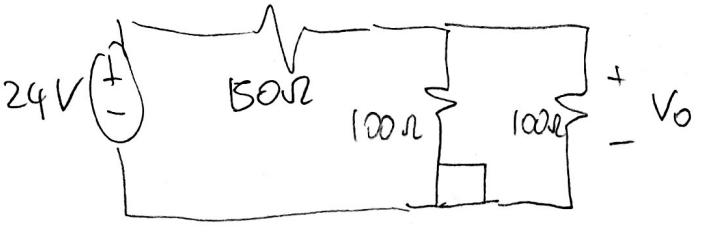
When the circuit is connected at the top:



$V_o = 0V$

$P_1 =$ Delivered Power By the voltage source $= \left(\frac{24V}{150\Omega}\right)24V = 3.84W$

When the circuit is connected at the bottom:

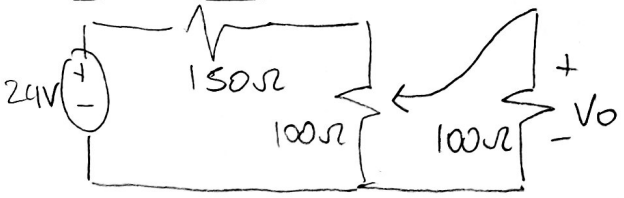


$V_o = \frac{50}{150+50}(24V) = 6V$

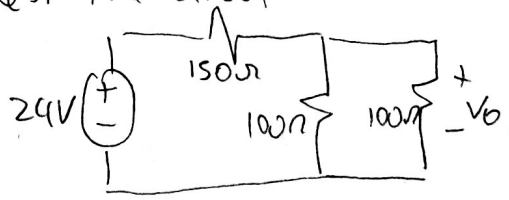
$P_2 =$ Delivered Power By the voltage source $= \left(\frac{24V}{200\Omega}\right)24V = 2.88W$

- Notice that $P_1 > 3W$, then circuit 1 doesn't satisfy the required specifications.

Circuit 2:



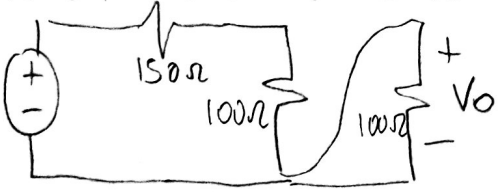
When the circuit is connected at the top:



$V_o = \frac{50}{200}(24) = 6V$

$P_1 = \left(\frac{24}{200}\right)24 = 2.88W$

When the circuit is connected at the bottom:



$V_o = 0$

$P_2 = \left(\frac{24}{250}\right)(24) = 2.304\,W$

Circuit 2 meets the required specifications!