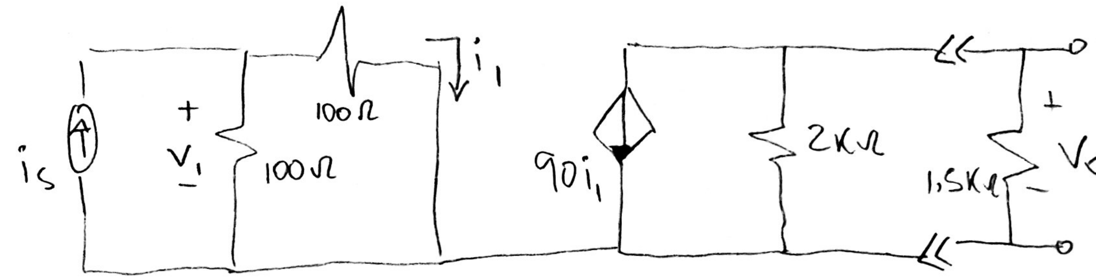
Homework 5

4.2)



Using current division, one has

$$i_1 = \frac{\frac{1}{100}}{\frac{1}{100}+\frac{1}{100}} i_s = \frac{1}{2} i_s$$

By noticing that the voltage in both 100Ω resistors is the same, we have

$$V_1 = 100 i_1 = 100\left(\tfrac{1}{2} i_s\right) = 50 i_s \quad (*)$$

For the second part of the circuit, one has (using current div.)

$$i_o = \frac{\frac{1}{1.5}}{\frac{1}{1.5}+\frac{1}{2}}(-90 i_1) = -51.42 i_1 = -51.42\left(\tfrac{1}{2} i_s\right) = -25.71 i_s$$

$$\Rightarrow \frac{i_o}{i_s} = -25.71$$

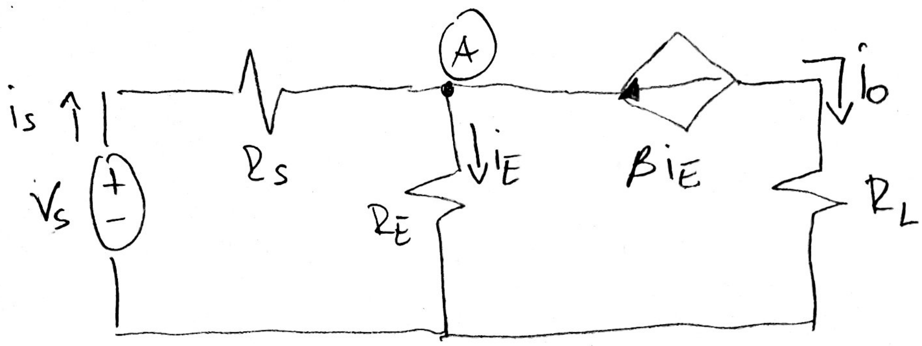
$$V_o = 1.5K\Omega\, i_o = 1.5K\Omega(-25.71 i_s) = 1.5K\Omega(-25.71)\left(\tfrac{1}{50} V_1\right) = -771.42 V_1$$

where we have used $(*)$ in the last expression. It follows:

$$\frac{V_o}{V_i} = -771.42$$

- When $i_s = 10mA$:
  - $V_1 = 50(10\times10^{-3}) = 500mV$. Then the power suppied by the current source is $P_s = V_1 i_s = (500\times10^{-3})(10\times10^{-3}) = 5mW$
  - The power delivered to the $1.5K\Omega$ resistor is $P_L = i_o^2\, 1.5K\Omega = (25.71\times10^{-3})^2(1.5\times10^{3}) = 99.15W$

4.7)



KCL:

Node A: $i_s - i_o - i_E = 0$ (*)

we know that $i_o = -\beta i_E \Rightarrow i_E = \frac{-1}{\beta} i_o$

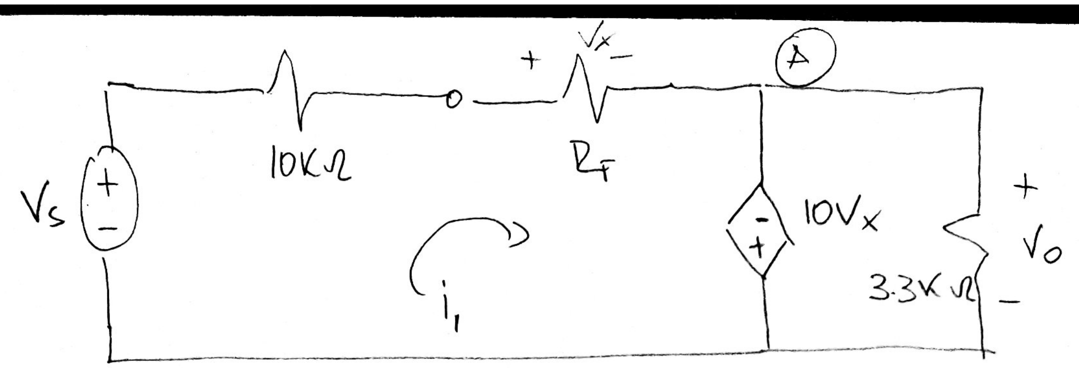
Replacing i_E in (*), one has

$$i_s - i_o + \frac{1}{\beta} i_o = 0$$

$$i_s = \left(1 - \frac{1}{\beta}\right) i_o = \left(\frac{\beta - 1}{\beta}\right) i_o$$

$$\Rightarrow \frac{i_o}{i_s} = \frac{\beta}{\beta - 1}$$

4.13)



$V_o = -10V_x$

KVL for i_1 : $-V_s + 10K\, i_1 + R_F\, i_1 + V_o = 0$

$$-V_s + (10K + R_F)\, i_1 + V_o = 0 \quad (*)$$

$V_x = R_F\, i_1 \Rightarrow -\frac{1}{10} V_o = R_F\, i_1 \Rightarrow i_1 = -\frac{1}{10R_F} V_o$

Replacing i_1 in (*) :

$$-V_s + (10K + R_F)\left(-\frac{1}{10R_F} V_o\right) + V_o = 0$$

$$\left(1 - \frac{10K + R_F}{10R_F}\right) V_o = V_s$$

$$(10R_F - 10K - R_F) V_o = 10R_F\, V_s \Rightarrow \boxed{\frac{V_o}{V_s} = \frac{10R_F}{9R_F - 10K}}$$

• For $K = \frac{V_o}{V_s}$ to go to infinity, the denominator should go to zero

$$9R_F - 10K = 0 \Rightarrow R_F = \frac{10 \times 10^3}{9} = 1.11 K\Omega$$

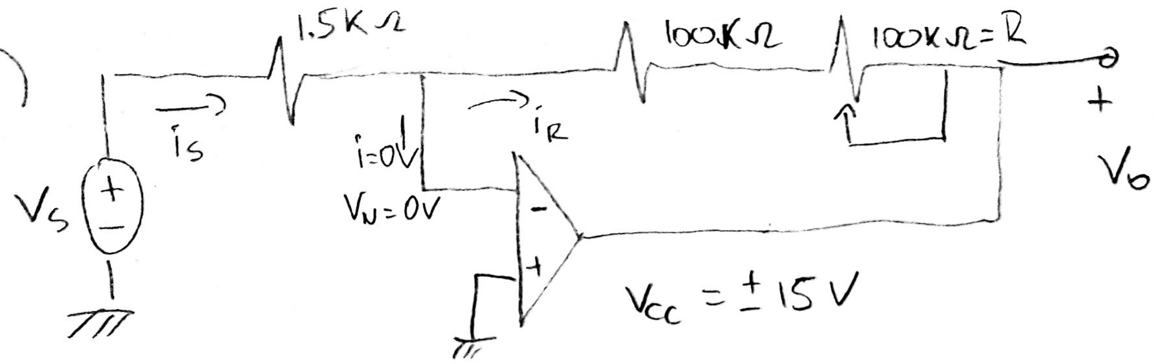
• To have $K = 2$, We need:

$$K = \frac{V_o}{V_s} = \frac{10R_F}{9R_F - 10 \times 10^3} = 2$$

$$\Rightarrow 18R_F - 20 \cdot 10^3 = 10R_F$$

$$8R_F = 20 \cdot 10^3 \Rightarrow \boxed{R_F = 2.5 K\Omega}$$

4.31)



$i_s = i_R$

$$\frac{V_s}{1.5K} = \frac{0 - V_o}{100K + R}$$

$$-\left(\frac{100K + R}{1.5K}\right) = \frac{V_o}{V_s}$$

when $R = 0 \Rightarrow \frac{V_o}{V_s} = \frac{-100K}{1.5K} = -66.66$

when $R = 100K\Omega \Rightarrow \frac{V_o}{V_s} = \frac{-200K}{1.5K} = -133.33$

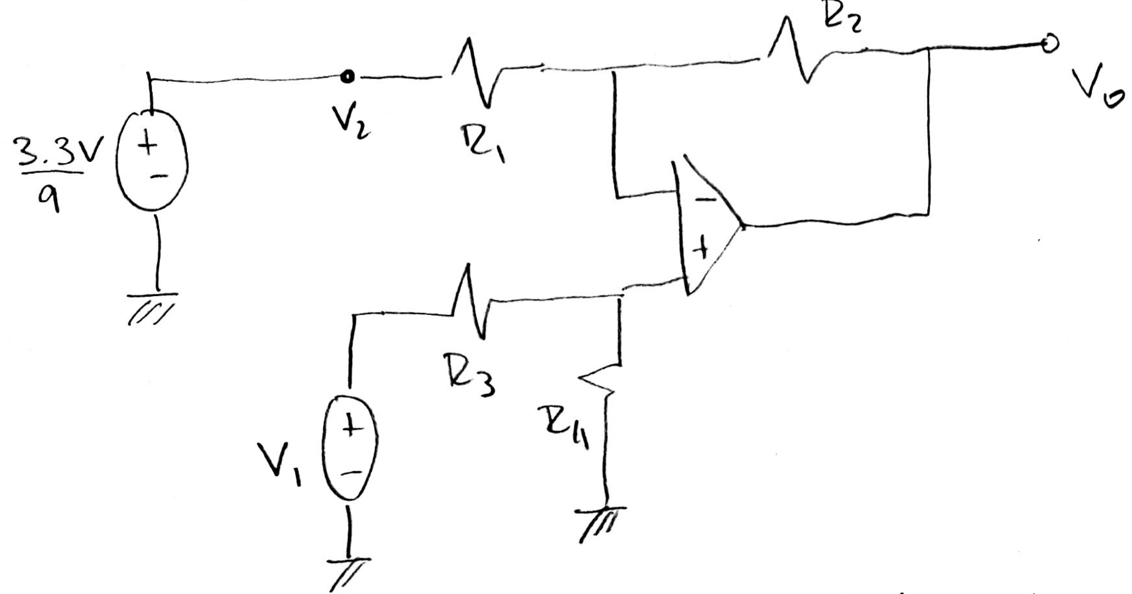
So, the range of $\frac{V_o}{V_s}$ is from -133.33 to -66.66.

Another way to do it:

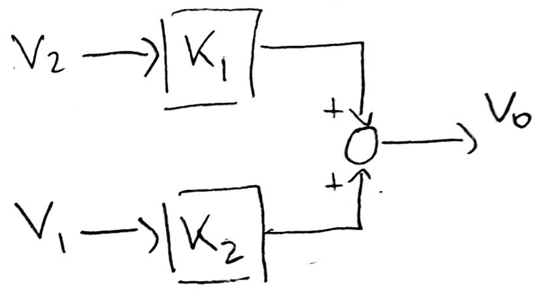
Notice that the circuit is an inverting amplifier with $R_1 = 1.5K\Omega$ and R_2 takes values from $100K\Omega$ to $200K\Omega$. Recall that the gain is $K = \frac{V_o}{V_s} = \frac{-R_2}{R_1}$

It follows that $\frac{V_o}{V_s}$ ranges from -133.33 to -66.66.

4.33) Design $V_o = 5V_1 - 3.3V$. We design a substractor



Block diagram:



$V_o = K_1 V_2 + K_2 V_1$

where

$K_1 = -\frac{R_2}{R_1}$

$K_2 = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)$

We pick $R_3 = R_4 = 10K\Omega$ and $R_1 = 10K\Omega$, $R_2 = 90K\Omega$

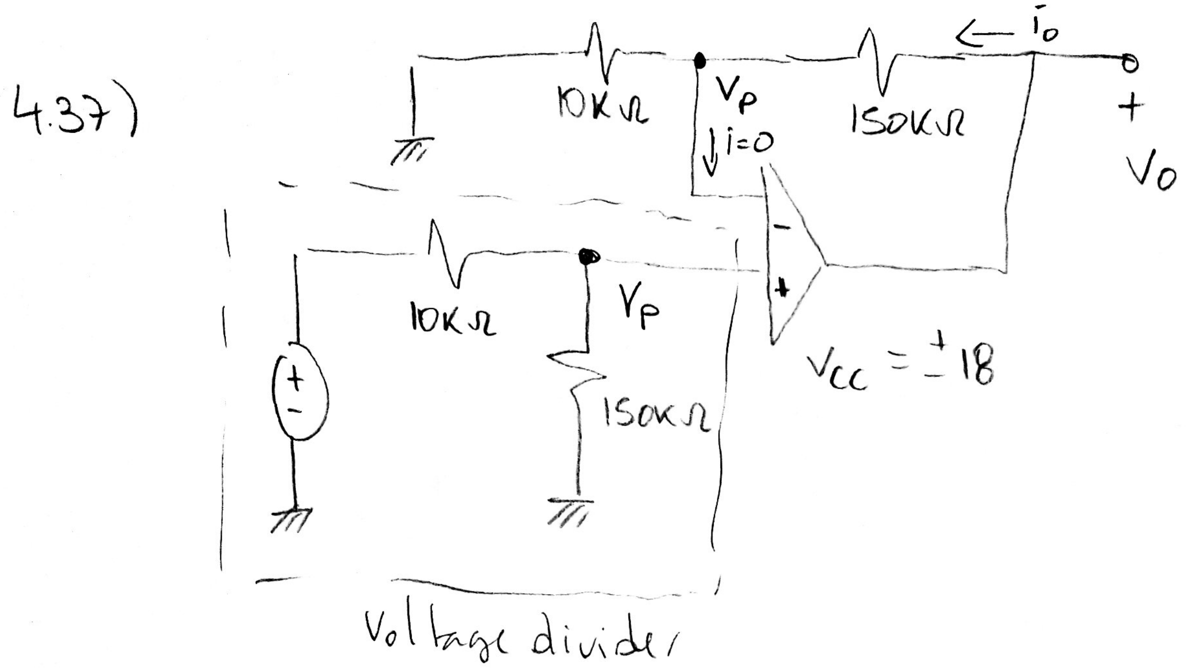
Then, $K_2 = \left(\frac{10K + 90K}{10K}\right)\left(\frac{10K}{10K + 10K}\right) = \frac{100}{20} = 5$

and $K_1 = -\frac{90K}{10K} = -9$

Therefore, $V_o = K_1 V_2 + K_2 V_1 = -9\left(\frac{3.3V}{9}\right) + 5V_1$

$= -3.3V + 5V_1$

4.37)



Notice that the circuit is a voltage divider connected to a noninverting amplifier with gain $K = \left(\frac{150K + 10K}{10K}\right) = 16$

$$V_p = \frac{150}{150+10} V_s = \frac{15}{16} V_s \quad \text{(voltage div.)}$$

a) The output voltage is $V_o = K V_p = 16\left(\frac{15}{16} V_s\right) = 15 V_s$

b) Find i_o when $V_s = 1V$:

$$V_o = 15(1V) = 15V$$

$$i_o = \frac{V_o - V_p}{150K} = \frac{\cancel{15}^{1} V_s - \frac{\cancel{15}^{1}}{16} V_s}{\cancel{150}_{10} K} = \frac{(16-1)V_s}{160K} = \frac{3}{32K} V_s$$

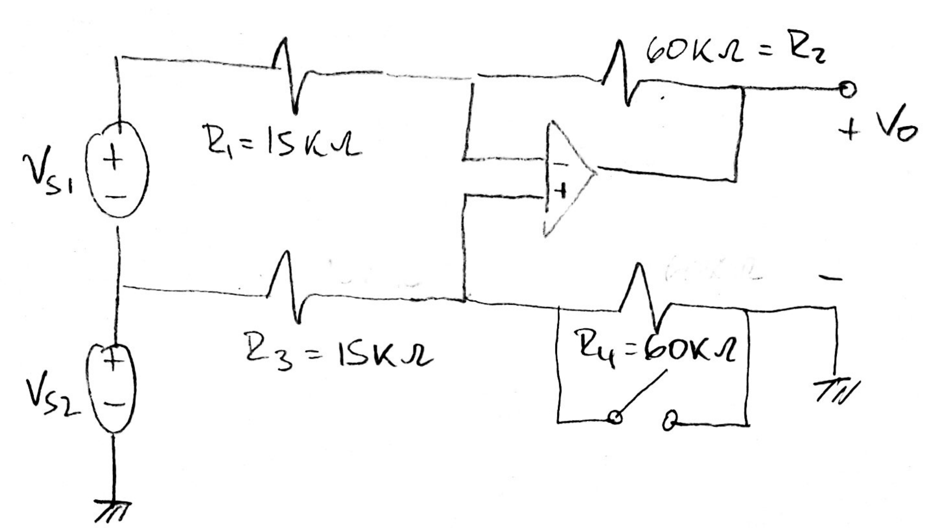
$$i_o = \frac{3}{32K}(1V) = 93.75 \mu A$$

when $V_s = 2V$:

$V_o = 15(2V) = 30$. Therefore, V_o is saturated to 18V.

Then, $i_o = \dfrac{V_o}{150K + 10K} = \dfrac{18V}{160K} = 112 \mu A$

4.44)



open switch:

The circuit is connected to a substractor with inputs $V_{s1} + V_{s2}$ and V_{s2}. It follows that

$$V_o = -\frac{R_2}{R_1}(V_{s1} + V_{s2}) + \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) V_{s2}$$

$$= -\frac{60K}{15K}(V_{s1} + V_{s2}) + \left(\frac{75}{15}\right)\left(\frac{60}{75}\right) V_{s2}$$

$$= -4V_{s1} - 4V_{s2} + 4V_{s2}$$

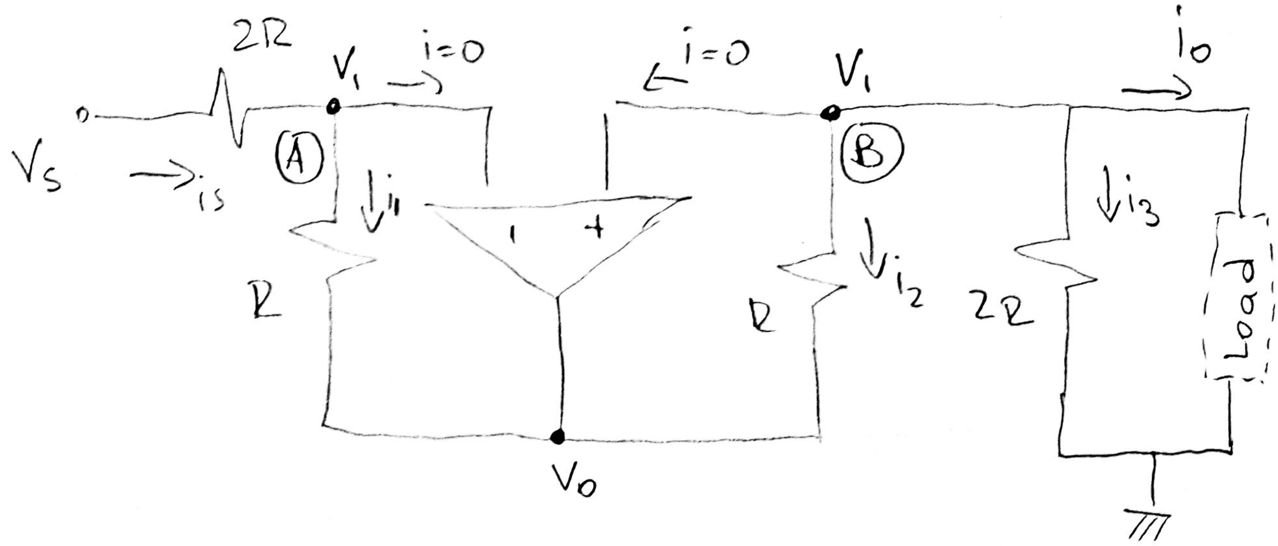
$$\boxed{V_o = -4V_{s1}}$$

Closed switch:

R_4 is shorted out. The positive input of the OPAMP is connected to ground directly. Then, the circuit is an inverting amplifier with gain $K = -\left(\frac{60K}{15K}\right) = -4$ and input voltage of $V_{s1} + V_{s2}$. Therefore,

$$\boxed{V_o = -4(V_{s1} + V_{s2})}$$

4.51)



<u>Node A:</u>

$$i_s = i_1$$

$$\frac{V_s - V_1}{2\cancel{R}} = \frac{V_1 - V_o}{\cancel{R}} \Rightarrow V_s - V_1 = 2(V_1 - V_o)$$

$$V_1 = \frac{1}{3}(V_s + 2V_o)$$

<u>Node B:</u>

$$i_2 + i_3 + i_o = 0$$

$$i_o = -i_2 - i_3$$

$$= -\left(\frac{V_1 - V_o}{R}\right) - \left(\frac{V_1}{2R}\right)$$

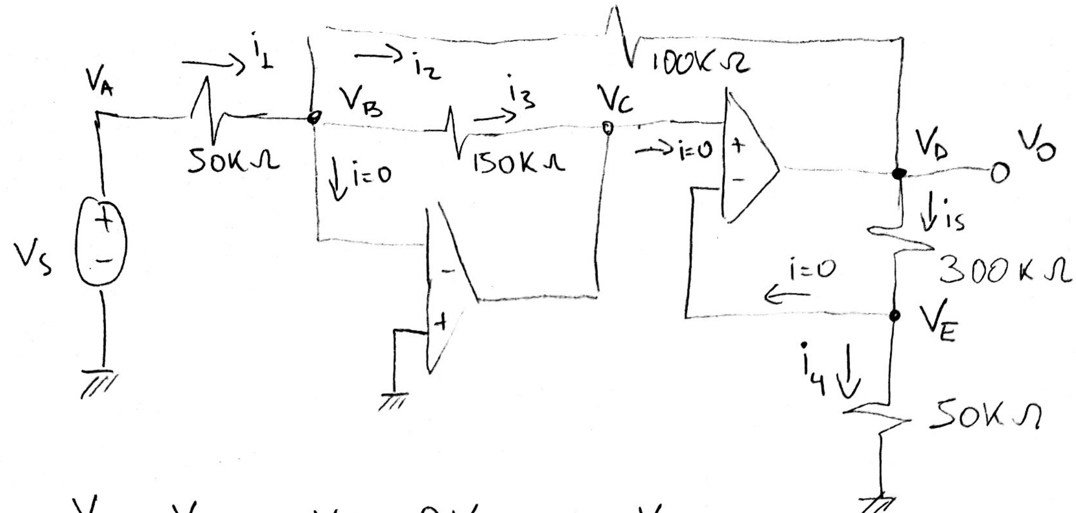
$$= -\left(\frac{\frac{1}{3}(V_s + 2V_o) - V_o}{R}\right) - \left(\frac{\frac{1}{3}(V_s + 2V_o)}{2R}\right)$$

$$= -\frac{1}{3R}\left(V_s + \frac{1}{2}V_s\right) - \left(\frac{2}{3R} - \frac{1}{R} + \frac{1}{3R}\right)V_o$$

$$= -\frac{1}{\cancel{3}R}\left(\frac{\cancel{3}}{2}V_s\right) - \overset{0}{\cancel{\left(\frac{1}{R} - \frac{1}{R}\right)}}V_o$$

$$\boxed{i_o = -\frac{1}{2R}V_s}$$

4.60)



$V_A = V_S$, $V_D = V_o$, $V_B = 0V$, $V_C = V_E$

KCL:

Node B: $i_1 - i_2 - i_3 = 0$

$$\frac{V_S - 0}{50K\Omega} - \frac{0 - V_o}{100K\Omega} - \frac{0 - V_E}{150K\Omega} = 0$$

$$V_E = -150\left(\frac{V_S}{50} + \frac{V_o}{100}\right)$$

$$= -3V_S - \frac{3}{2}V_o \qquad (1)$$

Node E: $i_4 - i_5 = 0$

$$\frac{V_E}{\cancel{50K\Omega}_{1}} - \frac{V_o - V_E}{\cancel{300K\Omega}_{6}} = 0 \Rightarrow 6V_E - V_o + V_E = 0$$

$$V_o = 7V_E \qquad (2)$$

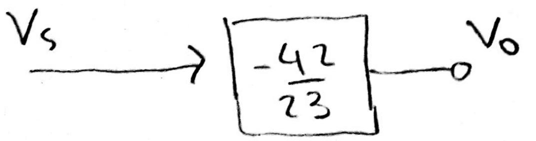
Replace (1) in (2):

$$V_o = 7\left(-3V_S - \frac{3}{2}V_o\right)$$

$$V_o\left(1 + \frac{21}{2}\right) = -21V_S$$

$$\boxed{V_o = -\frac{42}{23}V_S}$$

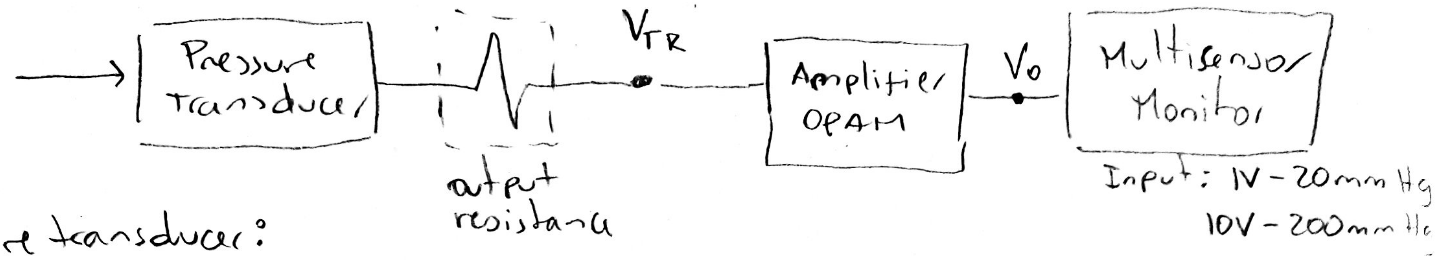
b)



4.86) Given: Pressure (P) in mm Hg

$V_{TR} = (0.06P - 0.75)\,mV$

diagram:



Pressure transducer:

i) When $P = 20\,mmHg$, then $V_{TR} = (0.06(20) - 0.75)\,mV = 0.45\,mV$.
The designed circuit should produce $V_o = 1V$

ii) When $P = 200\,mm\,Hg$, $V_{TR} = 11.25\,mV$. The designed circuit shold produce $V_o = 10V$.

We have to fit a line of the form $V_o = K V_{TR} + b$;
where

$$K = \frac{10V - 1V}{11.25mV - 0.45mV} = \frac{9V}{10.8mV} = 833$$

Now, we proceed to find b. For that, we pick a point $(\underbrace{0.45mV}_{V_{TR}}, \underbrace{1V}_{V_o})$

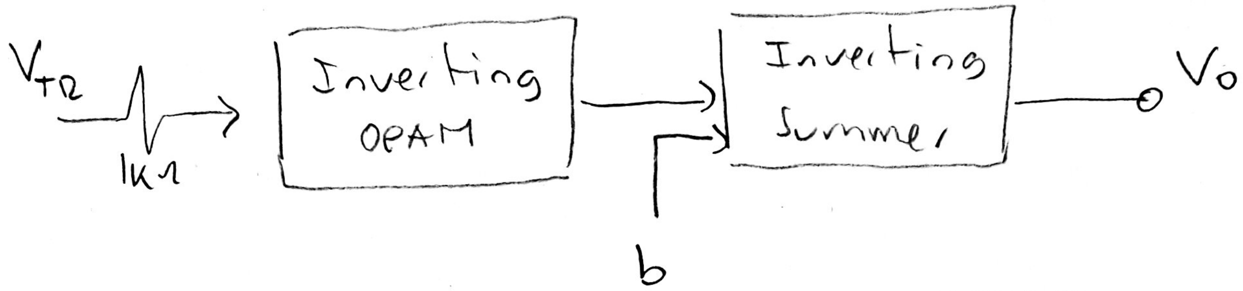
$$V_o = K(0.45mV) + b = 1V$$

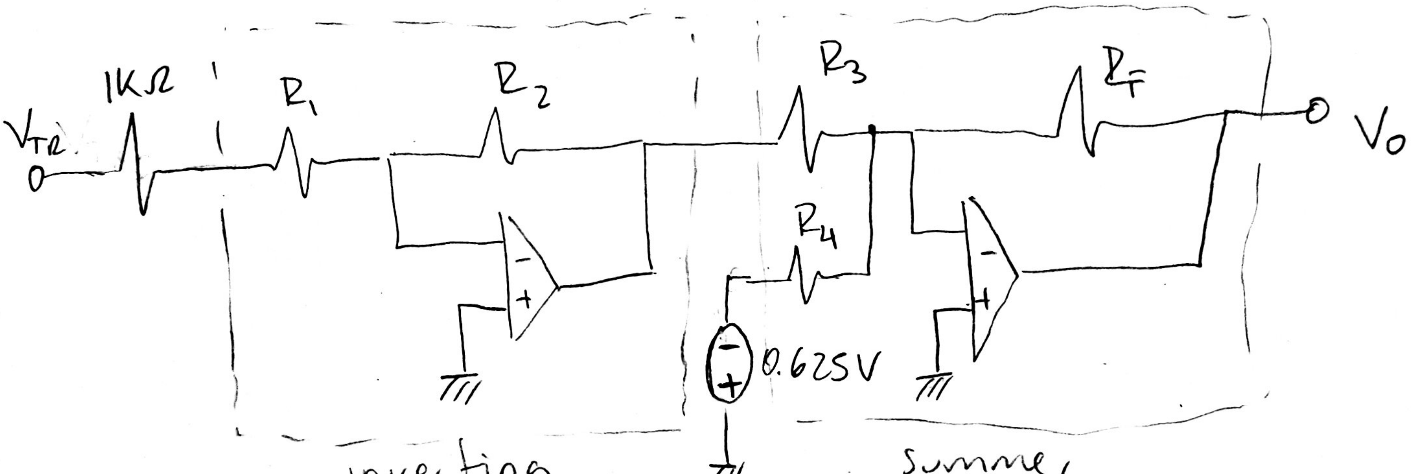
$$833(0.45mV) + b = 1V$$

$$\boxed{b = 0.625\,V}$$

The equation we have to satisfy is: $\boxed{V_o = 833\,V_{TR} + 0.625V}$

For the circuit, we use the following idea





inverting

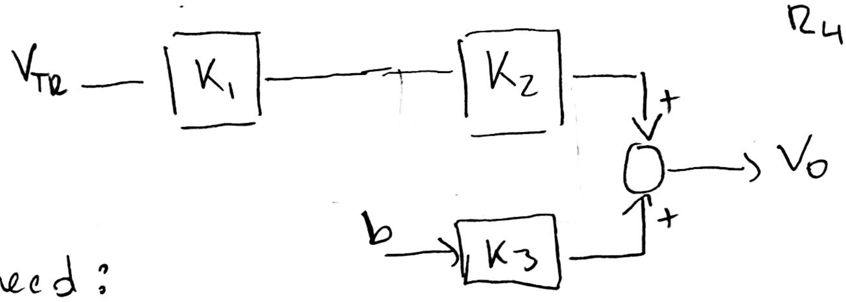
$K_1 = -\frac{R_2}{R_1}$

summer

$K_2 = -\frac{R_F}{R_3}$

$K_3 = -\frac{R_F}{R_4}$

=



What we need:

$K_1 K_2 = 833$ and $K_3 = -1$

$R_F = 100K\Omega$, $R_4 = 100K\Omega$ $\Rightarrow K_3 = -1$

$R_3 = 1K\Omega \Rightarrow K_2 = -100$

$R_2 = 500K\Omega$, $R_1 = 60K\Omega$ $\Rightarrow K_1 = -8.33$

$\Rightarrow V_{TR} = 833\, V_{TR} + 0.625V$

Notice that R_1 much bigger than 1KΩ. Other option is to include a buffer as the input of our designed circuit