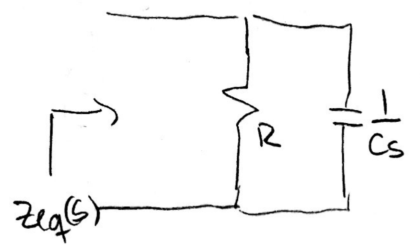
Homework 7

10.2)



$$Z_{eq} = \frac{(R)\left(\frac{1}{Cs}\right)}{R + \frac{1}{Cs}} = \frac{R}{RCs+1}$$

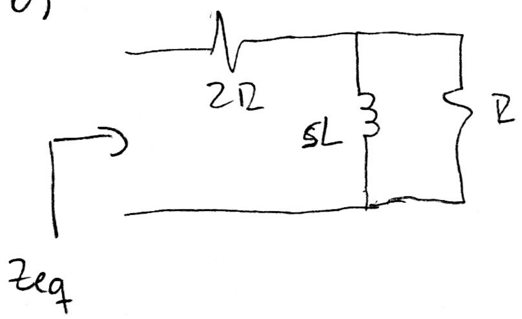
For the pole $s = -250K$ rad/s we have:

$RCs + 1 = 0 \Rightarrow s = \frac{-1}{RC}$, so we pick $C = 100pF$.

It follows that $R = \frac{-1}{sC} = \frac{-1}{(-250K)(100p)} = \frac{1}{(250 \times 10^3)(100 \times 10^{-12})}$

$= 40K\Omega$

10.8)



a) $Z_{eq} = 2R + (sL \| R)$

$$= 2R + \frac{sLR}{sL+R}$$

$$= \frac{2R(sL+R) + sLR}{sL+R}$$

$$= \frac{3sLR + 2R^2}{sL+R}$$

To find the poles, we have

$$sL + R = 0 \Rightarrow s = \frac{-R}{L}$$

For zeros, $3sLR + 2R^2 = 0 \Rightarrow s = \frac{-2R^{\cancel{2}}}{3L\cancel{R}} = \frac{-2R}{3L}$

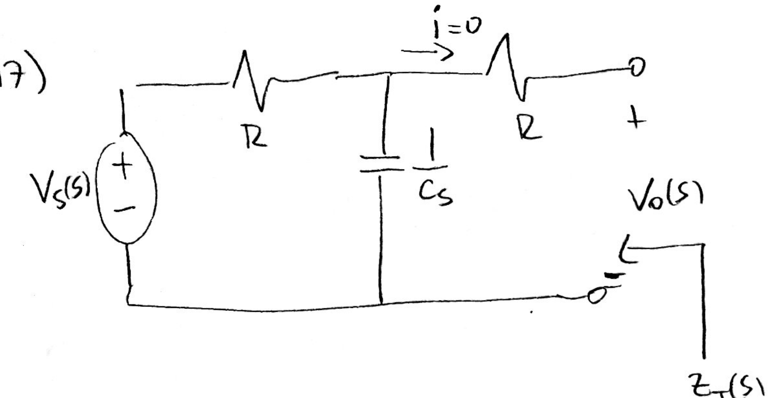
b) It is given a pole at -330 rad/s, it implies that $-330 = \frac{-R}{L}$.

Since the zero is located at $s = -\frac{2}{3}\frac{R}{L}$, we have that

$s = -\frac{2}{3}(330) = -220$. So the zero is located at $s = -220$ rad/s.

We pick $L = 10H$, so we have $R = (330)10 = 3.3K\Omega$.

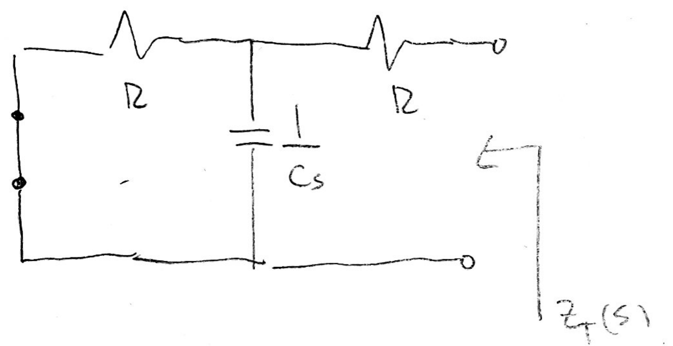
10.17)



a) Since $i=0$, $V_o(s)$ is the voltage in the capacitor, it follows

$$V_o(s) = \frac{\left(\frac{1}{Cs}\right)}{\frac{1}{Cs}+R}\left(V_s(s)\right) = \frac{V_s(s)}{1+RCs}$$

b) We turn off all independent sources, i.e.,



$$Z_T(s) = R + \left(R \,\|\, \frac{1}{Cs}\right)$$

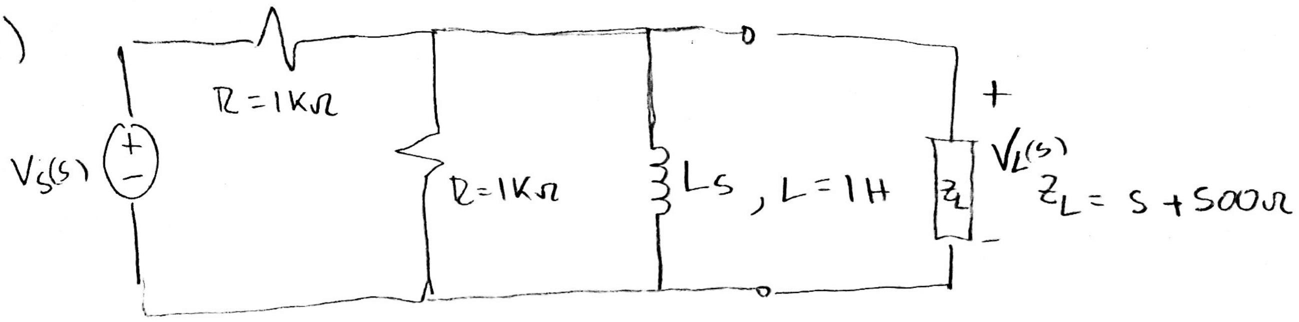
$$= R + \frac{(R)\left(\frac{1}{Cs}\right)}{R+\frac{1}{Cs}}$$

$$= R + \frac{R}{RCs+1}$$

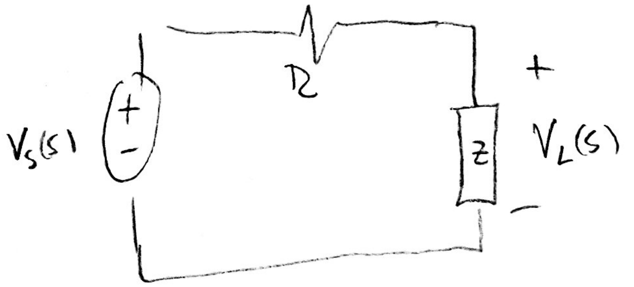
$$= \frac{R^2Cs+R+R}{RCs+1}$$

$$= \frac{R^2Cs+2R}{RCs+1}$$

10.22)



the circuit is equivalent to:



where $Z = R \| Ls \| Z_L = \left(R \| Ls \| (s+500)\right) = \left(\frac{1}{R} + \frac{1}{Ls} + \frac{1}{s+500}\right)^{-1}$

$$= \left(\frac{Ls(s+500) + R(s+500) + RLs}{RLs(s+500)}\right)^{-1}$$

$$= \frac{RLs(s+500)}{Ls^2 + s(500L + R + RL) + 500R}$$

Now, we use voltage division:

$$V_L(s) = \frac{Z}{Z+R} V_s(s) = \frac{(RLs^2 + 500RLs)\, V_s(s)}{(Ls^2 + (500L + R + RL)s + 500R)\left(\frac{RLs^2 + 500RLs}{Ls^2 + s(500L + R + RL) + 500R} + R\right)}$$

(Replacing values and reorganizing)
$$= \frac{(1k\,s^2 + 500k\,s)\, V_s(s)}{1k\,s^2 + 500k\,s + 1k(s^2 + (500 + 1k + 1k)s + 500k)}$$

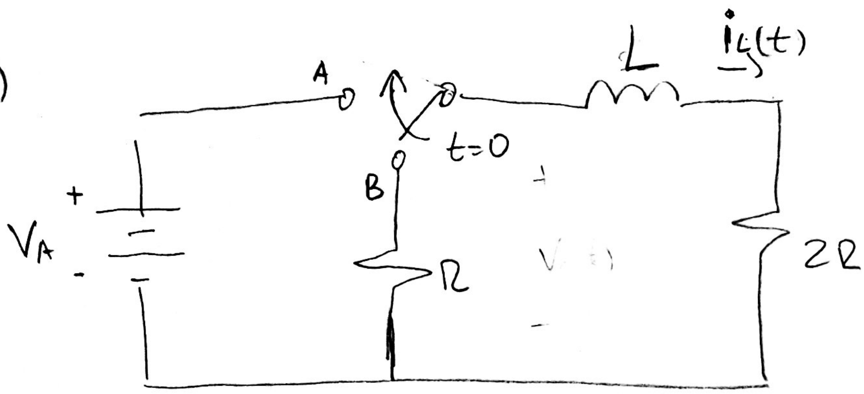
$$= \frac{(1\cancel{k}s^2 + 500\cancel{k}s)\, V_s(s)}{2\cancel{k}s^2 + (500\cancel{k} + 2500\cancel{k})s + 500.000\cancel{k}}$$

$$= \frac{\frac{1}{2} s (s + 500)\, V_s(s)}{s^2 + 1500 s + 250 \cdot 10^3}$$

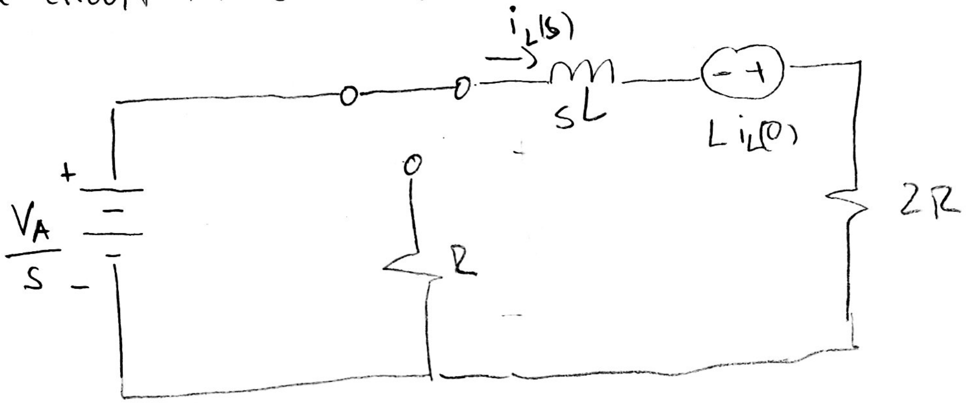
$$= \frac{\frac{1}{2} s (s + 500)\, V_s(s)}{(s + 191)(s + 1309)}$$

Natural poles: $s = -191$ rad/s and $s = -1309$ rad/s.
Natural zeros: $s = 0$ and $s = -500$ rad/s

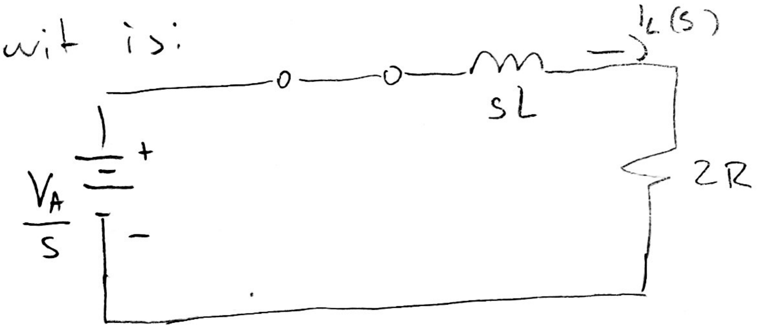
10.26)



The circuit in s domain:



Since the position of the switch was in B for a long time before moved to position A, then $i_L(0) = 0$. The equivalent circuit is:



$$i_L(s) = \frac{\frac{V_A}{s}}{sL + 2R} = \frac{V_A}{s(sL+2R)} = \frac{\frac{V_A}{L}}{s\left(s+\frac{2R}{L}\right)} = \frac{K_1}{s} + \frac{K_2}{s+\frac{2R}{L}}$$

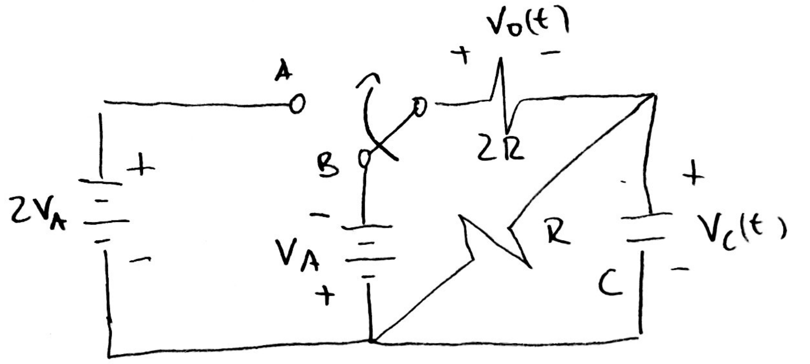
$$K_1 = s\, i_L(s)\Big|_{s=0} = \frac{\frac{V_A}{L}}{s+\frac{2R}{L}}\Bigg|_{s=0} = \frac{V_A}{2R}$$

$$K_2 = \left(s+\frac{2R}{L}\right) i_L(s)\Bigg|_{s=-\frac{2R}{L}} = \frac{\frac{V_A}{L}}{s}\Bigg|_{s=-\frac{2R}{L}} = -\frac{V_A}{2R}$$

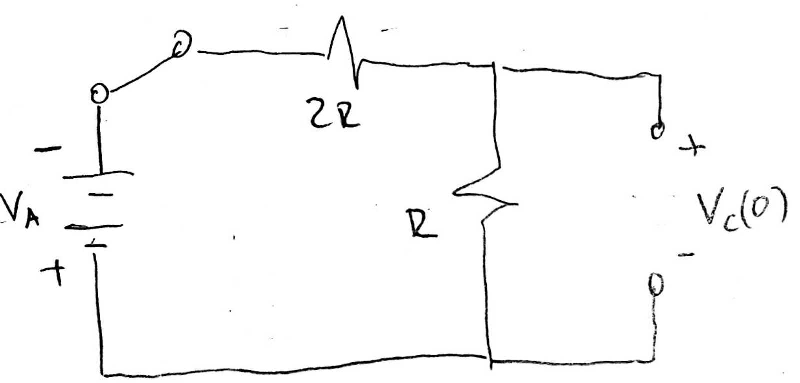
$$i_L(s) = \frac{\frac{V_A}{2R}}{s} - \frac{\frac{L\, V_A}{2R}}{s+\frac{2R}{L}}$$

$$i_L(t) = \frac{V_A}{2R} u(t) - \frac{V_A}{2R} e^{-2R/L\, t} u(t)$$

10.28)



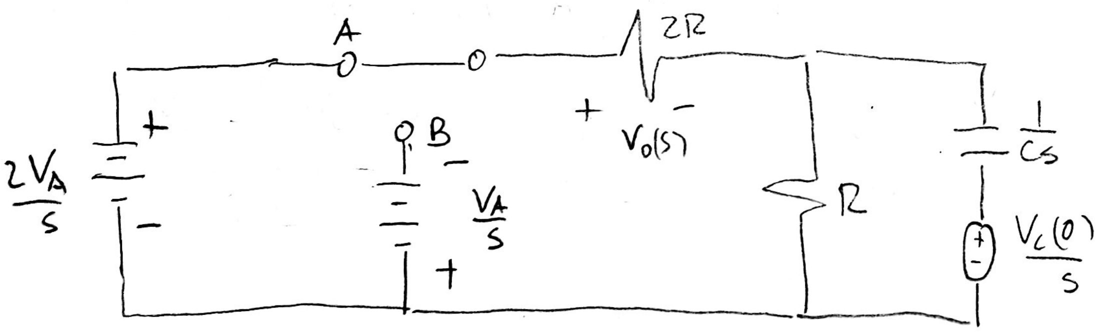
First, we find the initial contition when the switch is in position B for $t < 0$. The corresponding circuit is shown below:



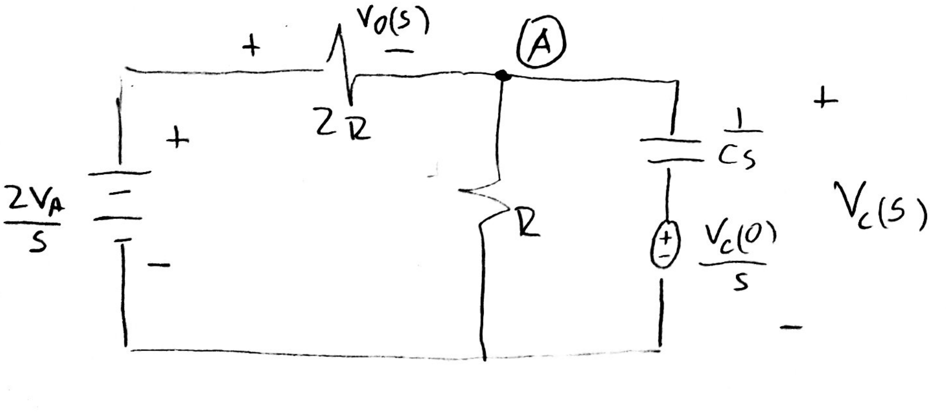
Since the circuit for $t < 0$ is in dc for long time, then The capacitor acts as an open circuit. It follows

$$V_c(0) = \frac{R}{3R}(-V_A) = \frac{-1}{3} V_A$$

Next, we show the circuit in s domain when $t \geq 0$



which is equivalent to:



Using KCL at node A, we have

$$\frac{\frac{2V_A}{s} - V_c(s)}{2R} - \frac{V_c(s)}{R} + \frac{V_c(s) + \frac{V_A}{3s}}{1/Cs} = 0$$

$$\left(\frac{1}{2R} + \frac{1}{R} + Cs\right) V_c(s) = \frac{\cancel{2}V_A}{\cancel{2}Rs} - \frac{C}{3} V_A$$

$$\left(\frac{3 + 2RCs}{2\cancel{R}}\right) V_c(s) = \left(\frac{3 - RCs}{3\cancel{R}s}\right) V_A$$

$$V_c(s) = \frac{(6 - 2RCs)V_A}{9s + 6RCs^2} = \frac{-\frac{1}{3}\left(s - \frac{3}{RC}\right)V_A}{s\left(s + \frac{3}{2RC}\right)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + \frac{3}{2RC}}$$

$$K_1 = sV_c(s)\Big|_{s=0} = \frac{-\frac{1}{\cancel{3}}\left(-\frac{\cancel{3}}{\cancel{R}C}\right)V_A}{\frac{3}{2\cancel{R}C}} = \frac{2}{3}V_A$$

$$K_2 = \left(s + \frac{3}{2RC}\right)V_c(s)\Big|_{s=-\frac{3}{2RC}} = \frac{-\frac{1}{3}\left(-\frac{3}{2RC} - \frac{3}{RC}\right)V_A}{-3/2RC} = \frac{-\left(\frac{1}{2} + 1\right)V_A}{3/2} = -V_A$$

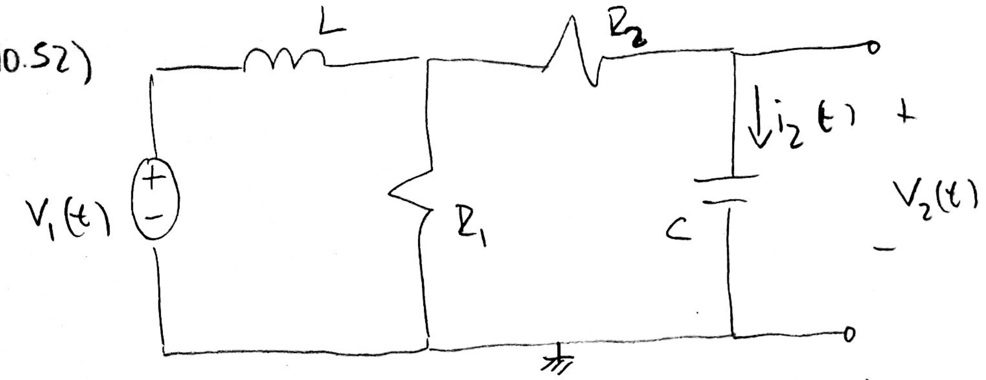
$$V_c(s) = \frac{\frac{2}{3}V_A}{s} - \frac{V_A}{s + \frac{3}{2RC}}$$

$$V_c(t) = \frac{2}{3}V_A u(t) - V_A e^{-3t/2RC} u(t)$$

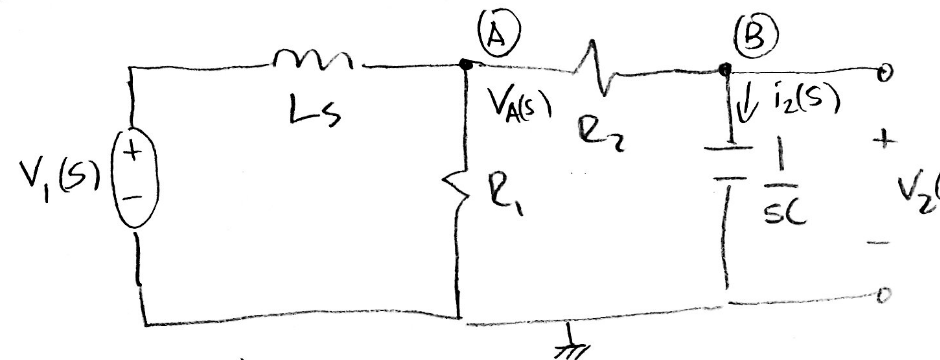
$$V_o(s) = \frac{2V_A}{s} - V_c(s) = \frac{2V_A}{s} - \frac{\frac{2}{3}V_A}{s} + \frac{V_A}{s + \frac{3}{2RC}} = \frac{\frac{4}{3}V_A}{s} + \frac{V_A}{s + \frac{3}{2RC}}$$

$$V_o(t) = \frac{4}{3}V_A u(t) + V_A e^{-\frac{3t}{2RC}} u(t)$$

10.52)



a) The circuit in s-domain with initial condition equal to zero:



Node-Voltage equations:

$$\left(\frac{1}{Ls}+\frac{1}{R_1}+\frac{1}{R_2}\right)V_A(s)-\frac{1}{R_2}V_2(s)=\frac{V_1(s)}{Ls}$$ (Node A) (*)

$$-\frac{1}{R_2}V_A(s)+\left(\frac{1}{R_2}+sC\right)V_2(s)=0$$ (Node B)

b)

$$V_A(s)=\left(\frac{1+R_2Cs}{\cancel{R_2}}\right)\cancel{R_2}\,V_2(s)$$ (From node-Voltage at node B)

we replace V_A in (*):

$$\left(\frac{R_1R_2+R_2Ls+R_1Ls}{R_1R_2Ls}\right)(1+R_2Cs)V_2(s)-\frac{1}{R_2}V_2(s)=\frac{V_1(s)}{Ls}$$

$$\left((R_1R_2+R_2Ls+R_1Ls)(1+R_2Cs)-R_1Ls\right)V_2(s)=R_1R_2V_1(s)$$

$$V_2(s)=\frac{R_1\cancel{R_2}\,V_1(s)}{\cancel{R_2}(R_1+R_2)LCs^2+(\cancel{R_1}+\cancel{R_2}-\cancel{R_1})Ls+R_1R_2^{\cancel{2}}Cs+R_1\cancel{R_2}}$$

$$=\frac{R_1V_1(s)}{(R_1+R_2)LCs^2+(R_1R_2C+L)s+R_1}$$

10.52) cont:

c) The natural poles are located at

$$s = \frac{-(R_1R_2C + L) \pm \sqrt{(R_1R_2C+L)^2 - 4(R_1+R_2)LCR_1}}{2(R_1+R_2)LC}$$

d) $V_1(t) = 5u(t) \Rightarrow V_1(s) = \frac{5}{s}$, Replacing, we have

$$V_2(s) = \frac{5R_1}{\left((R_1+R_2)LCs^2 + (R_1R_2C+L)s + R_1\right)s}$$

$$= \frac{5(500)}{s\left((1K)(0.5)(1\mu)s^2 + (500^2(1\mu) + 0.5)s + 500\right)}$$

$$= \frac{2500}{s(500\times10^{-6}s^2 + 0.75s + 500)}$$

$$= \frac{5\times10^6}{s(s^2 + 1500s + 10\times10^5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 1500s + 10\times10^5}$$

$$5\times10^6 = A(s^2 + 1500s + 10\times10^5) + Bs^2 + Cs$$

$A + B = 0 \Rightarrow B = -5$

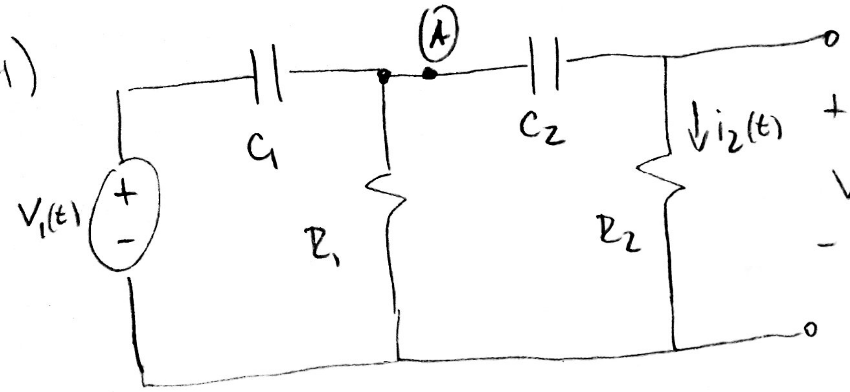
$1500A + C = 0 \Rightarrow C = -7500$

$5\times10^6 = 10\times10^5 A \Rightarrow A = 5$

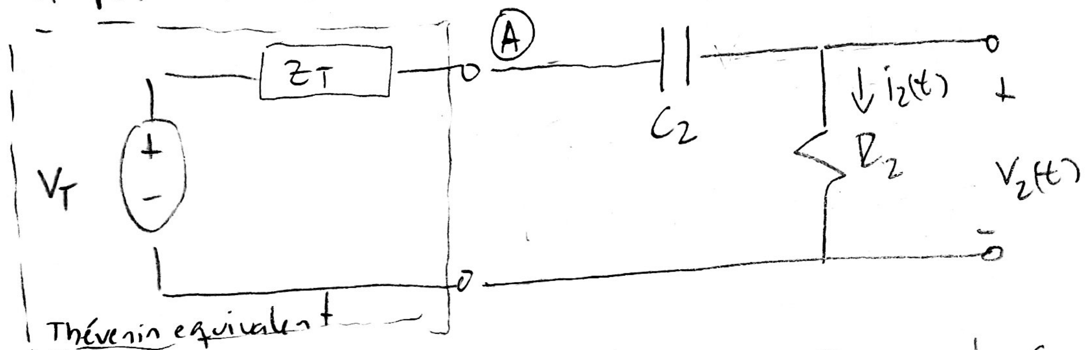
$$V_2(s) = \frac{5}{s} - \frac{5s + 7500}{s^2 + 1500s + 1\times10^6} = \frac{5}{s} - \frac{5(s+1500)}{(s+750)^2 + (661.44)^2}$$

$$V_2(t) = 5u(t) - 5e^{-750t}\cos(661.44t)u(t) - 5.67e^{-750t}\sin(661.44t)u(t)$$

10.54)



when $V_1(t) = \frac{1}{s}$, the Thévenin equivalent circuit to the left of point A is shown below, with $Z_T(s) = \frac{10^6}{s+10^3}$ Ω and $V_T(s) = \frac{1}{s+10^3}$ V-s.



We have to select the values of R_2 and C_2 s.t:

$$V_2(s) = \frac{s}{s^2+3000s+10^6}$$

We apply voltage division:

$$V_2(s) = \frac{R_2 V_T(s)}{Z_T + \frac{1}{C_2 s} + R_2} = \frac{R_2 \frac{1}{s+10^3}}{\frac{10^6}{s+10^3} + \frac{1}{C_2 s} + R_2}$$

$$= \frac{\frac{\cancel{(s+10^3)} \cancel{R_2} C_2 s}{\cancel{C_2}\cancel{(s+10^3)}}}{10^6 C_2 s + s + 10^3 + R_2 C_2 (s+10^3) s} = \frac{s}{s^2 + \left(10^3 + \frac{10^6}{R_2} + \frac{1}{R_2 C_2}\right) s + \frac{10^3}{R_2 C_2}}$$

we need: $\left(10^3 + \frac{10^6}{R_2} + \frac{1}{R_2 C_2}\right) = 3000$ and $\frac{10^3}{R_2 C_2} = 10^6$

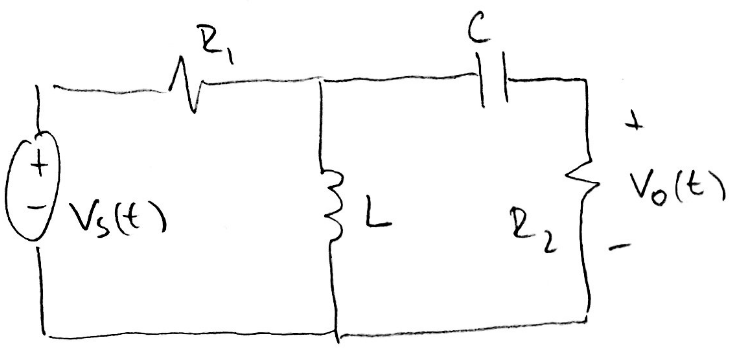
$\Rightarrow R_2 C_2 = 10^{-3}$

$$\left(10^3 + \frac{10^6}{R_2} + \frac{1}{10^{-3}}\right) = 3000$$

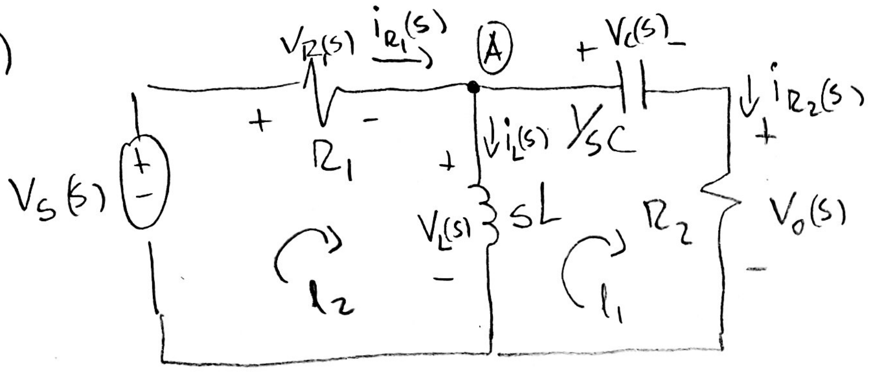
$R_2 = (3000 - 2 \times 10^3)^{-1} 10^6 = 1 \text{K}\Omega$

$C_2 = 1 \mu F$

10.71)



a)



b) We use the unit ouput method to find $\frac{V_o(s)}{V_s(s)}$:

We assume $V_o(s) = 1\ V\text{-}s$, then

$$i_{R_2}(s) = \frac{V_o(s)}{R_2} = \frac{1}{R_2}$$

$$V_C(s) = \frac{i_{R_2}(s)}{sC} = \frac{1}{R_2 C s}$$

KVL at l_1 : $-V_L(s) + V_c(s) + V_o(s) = 0$

$$V_L(s) = \frac{1}{R_2 C s} + 1 = \frac{1 + R_2 C s}{R_2 C s}$$

$$i_L(s) = \frac{V_L(s)}{sL} = \frac{1 + R_2 C s}{sL(R_2 C s)} = \frac{1 + R_2 C s}{L R_2 C s^2}$$

KCL at node A : $i_{R}(s) = i_L(s) + i_{R_2}(s) = \frac{1 + R_2 C s}{R_2 L C s^2} + \frac{1}{R_2}$

$$= \frac{\cancel{R_2} + R_2^{\cancel{2}} C s + \cancel{R_2} L C s^2}{R_2^{\cancel{2}} L C s^2} = \frac{1 + R_2 C s + L C s^2}{R_2 L C s^2}$$

10.71) cont:

$$V_{R_1}(s) = R_1\, i_{R_1}(s) = \frac{R_1(LCs^2 + R_2Cs + 1)}{R_2LCs^2}$$

KVL at ℓ_2: $\quad -V_S(s) + V_{R_1}(s) + V_L(s) = 0$

$$V_S(s) = \frac{R_1LCs^2 + R_1R_2Cs + R_1}{R_2LCs^2} + \frac{1 + R_2Cs}{R_2Cs}$$

$$= \frac{R_1LCs^2 + R_1R_2Cs + R_1 + Ls + LR_2Cs^2}{R_2LCs^2}$$

Therefore,

$$\frac{V_o(s)}{V_S(s)} = \frac{1}{V_S(s)} = \frac{R_2LCs^2}{(R_1+R_2)LCs^2 + (R_1R_2C + L)s + R_1}$$

c) We have

$$\frac{V_o(s)}{V_S(s)} = \frac{\frac{R_2}{R_1+R_2}s^2}{\underbrace{s^2 + \frac{R_1R_2C+L}{(R_1+R_2)LC}s + \frac{R_1}{(R_1+R_2)LC}}_{s^2 + 2\zeta\omega_0 s + \omega_0^2}}$$

with $\zeta = 0.707$ and $\omega_0 = 707$ rad/s, we have

$$2\zeta\omega_0 = 2(0.707)(707) = 999.698 = \frac{R_1R_2C + L}{(R_1+R_2)LC} = \frac{500^2C + L}{1000LC} \quad (*)$$

$$\omega_0^2 = 707^2 = 499849 = \frac{R_1}{(R_1+R_2)LC} = \frac{5\cancel{0}\cancel{0}}{10\cancel{0}\cancel{0}LC} = \frac{1}{2LC} \quad (**)$$

From $(*)$, we have $\quad 999.698 \times 10^3 LC = 250\times10^3 C + L \quad (***)$

and from $(**)$ $\quad 999698LC = 1 \Rightarrow C = \frac{1}{999698L}$

From $(***)$ $\quad 1 = \frac{250\times10^3}{999698L} + L \Rightarrow L^2 - L + \frac{250\times10^3}{999698} = 0$

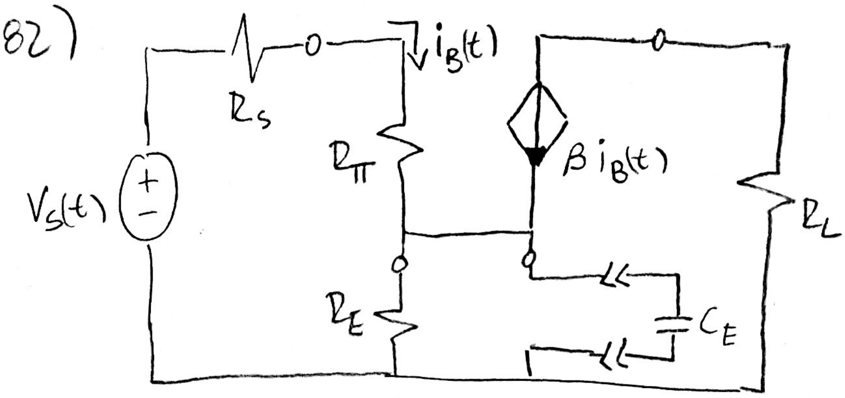
10.71) cont

$$L^2 - L + 0.25 = 0$$

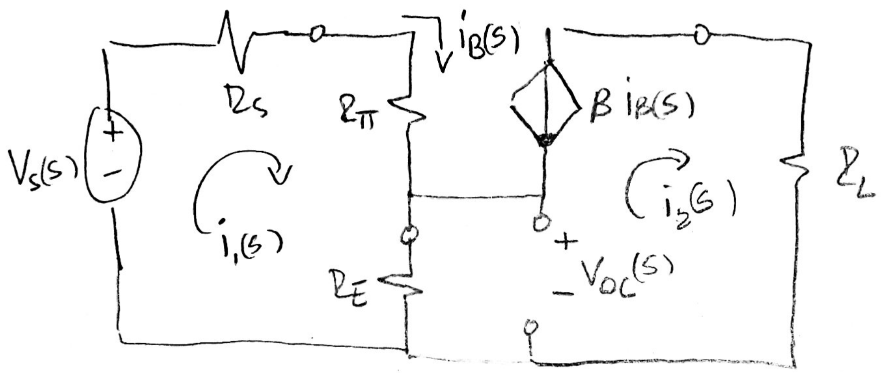
$$L = 0.5 \text{ H}$$

$$C = \frac{1}{999698(\frac{1}{2})} = 2 \mu F$$

10.82)



To find the Thévenin equivalent, first we find $V_{OP}(s)$ as shown in the next plot:



We proceed by by mesh-current analysis:

$$-V_s(s) + (R_s + R_\pi + R_E)\, i_1(s) - R_E\, i_2(s) = 0 \qquad (1)$$

$$i_2(s) = -\beta\, i_B(s) = -\beta\, i_1(s) \qquad (2)$$

Where we used the fact: $i_B(s) = i_1(s)$. Next we replace (2) in (1):

$$-V_s(s) + (R_s + R_\pi + R_E + \beta R_E)\, i_1(s) = 0 \Rightarrow i_1(s) = \frac{V_s(s)}{R_s + R_\pi + R_E + \beta R_E}$$

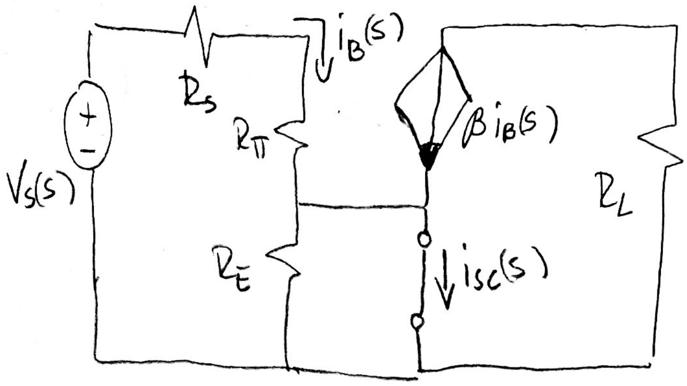
Notice that $V_{OC}(s)$ is the voltage in the resisto R_E, then

$$V_{OC}(s) = R_E\,(i_1(s) - i_2(s)) = R_E\,(i_1(s) + \beta\, i_1(s))$$

$$= \frac{(1+\beta) R_E\, V_s(s)}{R_s + R_\pi + R_E + \beta R_E}$$

10.82) cont:

Next, we find the short-circuit current:



$$i_{sc}(s) = i_B(s) + \beta i_B(s) = (1+\beta)\, i_B(s) \qquad (3)$$

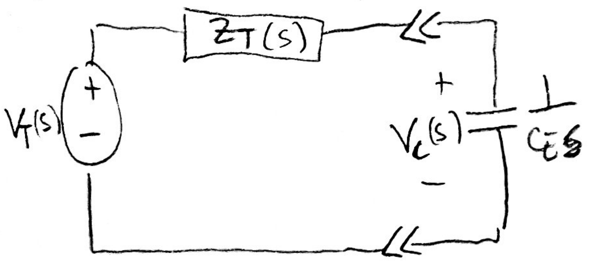
$$i_B(s) = \frac{V_S(s)}{R_S + R_\pi} \qquad (4)$$

Replacing (4) in (3): $\quad i_{sc}(s) = \dfrac{(1+\beta)\, V_S(s)}{R_S + R_\pi}$

It follows that: $\quad Z_T = \dfrac{V_{oc}(s)}{i_{sc}(s)} = \left(\dfrac{\cancel{(1+\beta)} R_E \cancel{V_S(s)}}{R_S + R_\pi + (1+\beta) R_E}\right)\left(\dfrac{R_S + R_\pi}{\cancel{(1+\beta) V_S(s)}}\right)$

$$= \frac{R_E (R_S + R_\pi)}{R_S + R_\pi + (1+\beta) R_E}$$

The Thévenin equivalent is



Using the values: $R_S = 10K\Omega$, $R_\pi = 2K\Omega$, $R_E = 3.3K\Omega$, $R_L = 1K\Omega$, and $\beta = 70$, we have:

$$V_T(s) = V_{oc}(s) = \frac{(1+70)(3.3K)\, V_S(s)}{10K + 2K + (1+70)3.3K} = 0.95\, V_S(s)$$

$$Z_T(s) = \frac{3.3K(10K + 2K)}{10K + 2K + (1+70)3.3K} = 160.78\,\Omega$$

Next, we find the pole associated with the capacitor. For that we use voltage division:

$$V_C(s) = \frac{\frac{1}{C_E s}}{Z_T(s) + \frac{1}{C_E s}} V_T(s) = \frac{V_T(s)}{Z_T(s) C_E s + 1} = \frac{V_T(s)}{160.78 C_E s + 1} \qquad (5)$$

10.82) cont

We are given $s = -300$, and from (5) we have that $160.78\, C_E\, s + 1 = 0 \Rightarrow s = \dfrac{-1}{160.78\, C_E} = -300$

which gives $C_E = \dfrac{1}{(160.78)(300)} = 2.07 \times 10^{-5}$ F