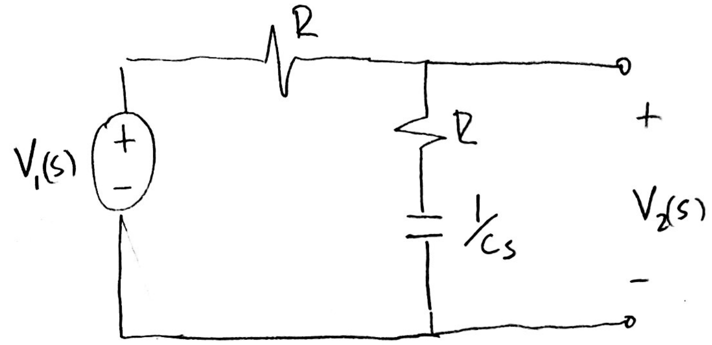
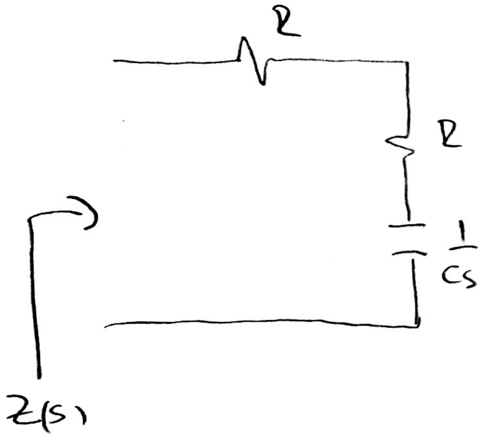
Homework 8:

11.1)



The driving point impedance is the impedance seen by the voltage source. This is shown in the Figure below:



Notice that all elements are in series, then

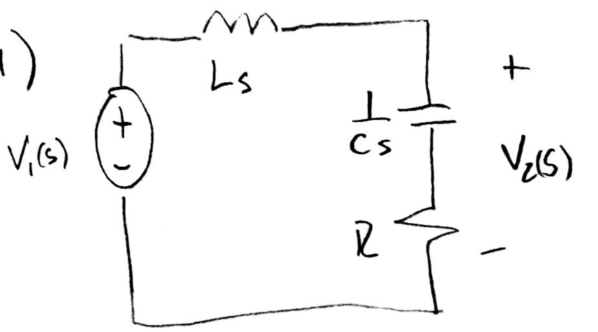
$$Z(s) = R + R + \frac{1}{Cs} = \frac{2RCs+1}{Cs}$$

Next, we use voltage division to find the voltage transfer function $T_V(s) = \frac{V_2(s)}{V_1(s)}$,

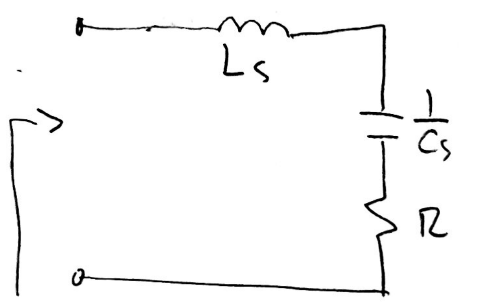
$$V_2(s) = \frac{R+\frac{1}{Cs}}{2R+\frac{1}{Cs}} V_1(s) = \frac{RCs+1}{2RCs+1} V_1(s)$$

Then, $T_V(s) = \dfrac{V_2(s)}{V_1(s)} = \dfrac{RCs+1}{2RCs+1}$

11.4)



a) The driving point impedance seen by the voltage source is given by the sum of impedances of the three elements, i.e.



Z(S)

$$Z(s) = Ls + \frac{1}{Cs} + R$$

$$= \frac{LCs^2 + 1 + RCs}{Cs}$$

we use voltage division to find $T_V(s) = \frac{V_2(s)}{V_1(s)}$

$$V_2(s) = \frac{\frac{1}{Cs} + R}{Ls + \frac{1}{Cs} + R} V_1(s) = \frac{1 + RCs}{LCs^2 + 1 + RCs} V_1(s) = \frac{\frac{R}{L}s + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} V_1(s)$$

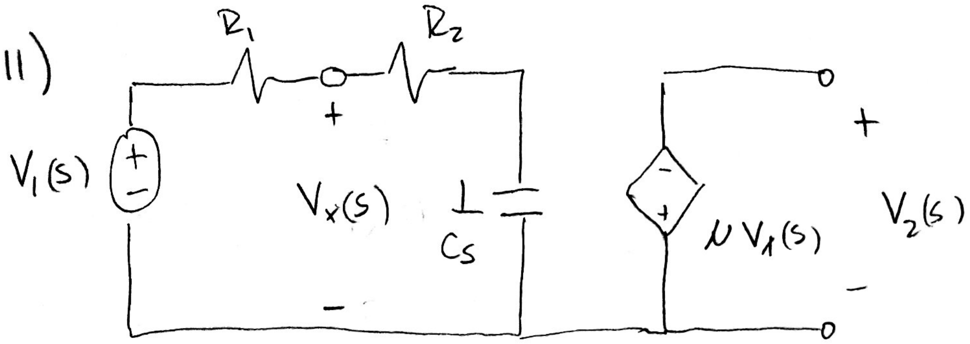
b) We are required to have poles $s = -9898$ rad/s and $s = -101$ rad/s, then the denominator of the transfer function should be $(s + 9898)(s + 101) = s^2 + 1 \times 10^4 s + 1 \times 10^6$.

Thus, we need $\frac{R}{L} = 1 \times 10^4$, pick $R = 1 \times 10^4 \,\Omega$ and we get $L = \frac{1 \times 10^4}{1 \times 10^4} = 1\,H$. Also, we have $\frac{1}{LC} = 1 \times 10^6$. Since $L = 1\,H$, we have $C = 1 \times 10^{-6}$ F.

The zeros under these condition satisfy $\frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow s = -\frac{1}{RC}$.

It follows that $s = -\frac{1}{1 \times 10^4 \cdot 1 \times 10^{-6}} = -1 \times 10^2$ rad/s. The other zero is $+\infty$.

11.11)



We use voltage division to find $V_x(s)$,

$$V_x(s) = \frac{R_2 + \frac{1}{Cs}}{R_1 + R_2 + \frac{1}{Cs}} V_1(s) = \frac{R_2 Cs + 1}{(R_1+R_2)Cs + 1} V_1(s)$$

Notice also that

$$V_2(s) = -\nu V_x(s) = -\nu \frac{(R_2 Cs + 1)\, V_1(s)}{(R_1+R_2)Cs + 1}$$

It follows,

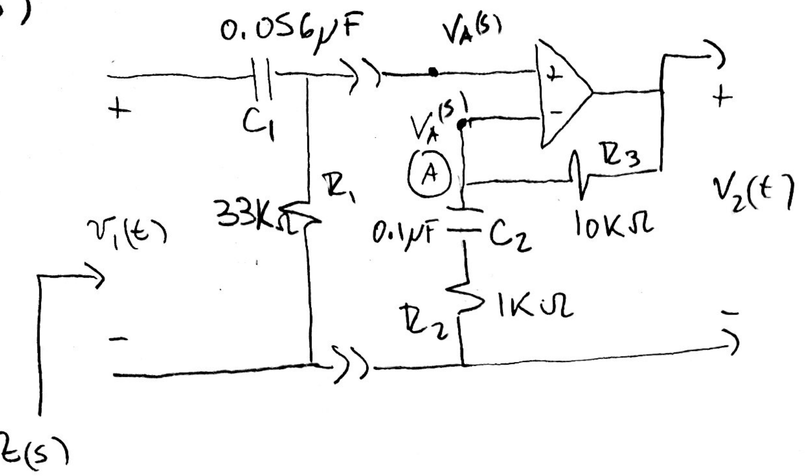
$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{-\nu (R_2 Cs + 1)}{(R_1+R_2)\, Cs + 1}$$

To have one pole $s = -100\text{K rad/s}$, we have that the denominator of $T_V(s)$ should satisfy $s = -\frac{1}{(R_1+R_2)C} = -100\text{K rad/s}$.

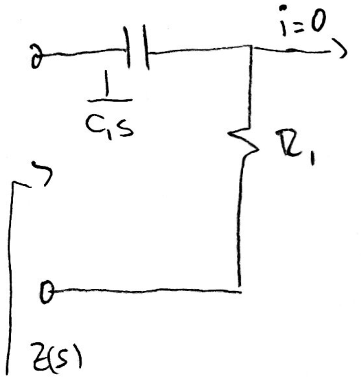
Let $R_1 = 500\Omega = R_2$, then $C = (100\times10^3 \times 1000)^{-1} = 1\times 10^8$ F.

Notice that the gain factor ν doesn't affect in the location of the pole.

11.15)



The driving point impedance is given by $Z(s) = \frac{1}{C_1 s} + R_1$



since there is no current in the input terminals of the OPAMP

To find $T_V(s) = \frac{V_2(s)}{V_1(s)}$, we first find $V_A(s)$ using voltage div.

$$V_A(s) = \frac{R_1}{\frac{1}{C_1 s} + R_1} V_1(s) = \frac{R_1 C_1 s}{R_1 C_1 s + 1} V_1(s) = \frac{s}{s + \frac{1}{R_1 C_1}} V_1(s)$$

It follow that $T_V(s) = \frac{s}{s + \frac{1}{R_1 C_1}} = \frac{s}{s + \frac{1}{33K \times 0.056\mu}} = \frac{s}{s + 541.12}$

Using KCL at (A), we have

$$-\left(\frac{1}{R_2 + \frac{1}{C_2 s}}\right) V_A(s) = \frac{1}{R_3}\left(V_A(s) - V_2(s)\right)$$

$$R_3\left(\frac{C_2 s}{R_2 C_2 s + 1} + \frac{1}{R_3}\right) V_A(s) = V_2(s)$$

$$V_2(s) = \frac{\cancel{R_3}(R_3 C_2 s + R_2 C_2 s + 1)}{R_2 \cancel{R_3} C_2 s + \cancel{R_3}} V_A(s) = \frac{\left(\frac{R_3}{R_2} + 1\right) s + \frac{1}{R_2 C_2}}{s + \frac{1}{R_2 C_2}} V_A(s)$$

11.15) cont.

$$V_2(s) = \frac{\left(\frac{10K}{1K}+1\right)s + \frac{1}{1K\times 0.1\mu}}{s + \frac{1}{1K\times 0.1\mu}} V_A(s)$$

$$V_2(s) = \left(\frac{11s + 10^4}{s + 10^4}\right)\left(\frac{s}{s+541.12}\right) V_1(s)$$

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{(11s+10^4)s}{(s+10^4)(s+541.12)} \qquad (2)$$

The poles are located at $s=-10^4$ and $s=-541.12$.

The zeros are located at $s=0$ and $s=-\frac{10^4}{11}$.

Another way to find the transfer function: Notice that we have two stages. One stage is a voltage divider and the other stage is a non-inverting OPAMP. The transfer function is the product for each stage because the chain rule applies here since the two stages do not load each other. It follows for the first stage we have (Using the result in (1)):

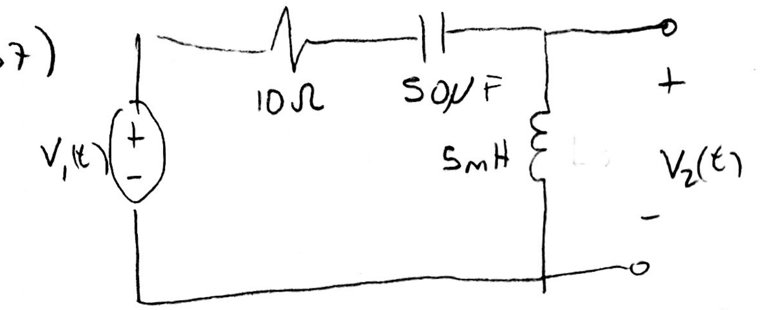
$$T_{V_1}(s) = \frac{s}{s+541.12}$$

For the second stage (non-inverting OPAM):

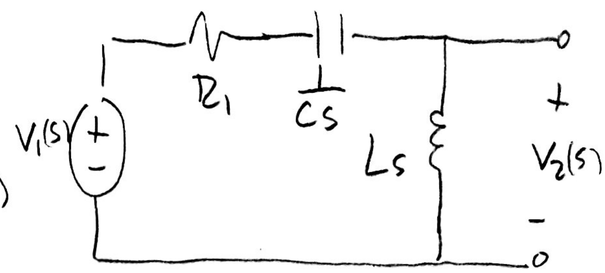
$$T_{V_2}(s) = \frac{R_2 + \frac{1}{C_2 s} + R_3}{R_2 + \frac{1}{C_2 s}} = \frac{\frac{R_2+R_3}{R_2}s + \frac{1}{R_2C_2}}{s+\frac{1}{R_2C_2}} = \frac{11s+10000}{s+10000}$$

$T_V(s) = T_{V_1}(s)\,T_{V_2}(s) = \dfrac{s(11s+10000)}{(s+10000)(s+541.12)}$, which gives the same result as (2).

11.37)



=)



First, we find the transfer function by using voltage div.

$$\frac{V_2(s)}{V_1(s)} = T(s) = \frac{Ls}{Ls + \frac{1}{Cs} + R} = \frac{CLs^2}{CLs^2 + RCs + 1} = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

Replacing $R_1 = 10\Omega$, $C = 50\mu F$, and $L = 5mH$, we get

$$T(s) = \frac{s^2}{s^2 + 2000s + 4\times10^6}$$

Replacing $s = jw$, we have

$$T(jw) = \frac{-w^2}{-w^2 + j2000w + 4\times10^6}$$

The magnitude and phase of T at $w = 2000$ is given by

$$|T(j2000)| = \frac{|-(2000)^2|}{|-(2000)^2 + j(2000)(2000) + 4\times10^6|} = \frac{4\times10^6}{|\cancel{-4\times10^6} + j4\times10^6 + \cancel{4\times10^6}|}$$

$$= 1$$

$$\angle T(j2000) = \angle(-2000)^2 - \angle(j4\times10^6) = 180^\circ - 90^\circ = 90^\circ$$

The steady-state output is

$$V_{2ss}(t) = 25\,|T(j2000)|\cos(2000t + \angle T(j2000))$$

$$= 25\cos(2000t + 90^\circ)$$

11.37) cont.

The poles are located at $s = \frac{-2000 \pm \sqrt{2000^2 - (4)(4\times10^6)}}{2}$

$= -1000 \pm 1732.1j$ rad/s

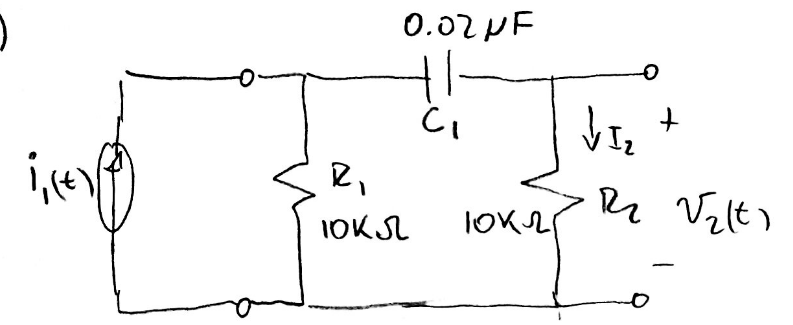
with $v_1 = 5V$, we have that $\omega = 0$ and then

$T(0) = 0$

and

$V_{2ss}(t) = 0\ V$

11.41)



a) if $i_1(t) = 10\cos(500t)$ mA, find i. We use current division

$$I_2(s) = \frac{\left(\frac{1}{Cs} + R_1\right)^{-1} I_1(s)}{\frac{1}{R_1} + \left(\frac{1}{Cs} + R_2\right)^{-1}} = \frac{\frac{Cs}{1+R_1Cs}}{\frac{1}{R_1} + \frac{Cs}{1+R_2Cs}} I_1(s) = \frac{R_1Cs}{1+R_2Cs+R_1Cs}$$

$$= \frac{\frac{R_1}{(R_1+R_2)}s}{s + \frac{1}{(R_1+R_2)C}} I_1(s) = \frac{\frac{10K}{20K}}{s + \frac{1}{(20K)0.02\mu}} I_1(s) = \frac{\frac{1}{2}s}{s+2500} I_1(s)$$

It follows that

$$T(s) = \frac{\frac{1}{2}s}{s+2500}$$

when $s = jw$, then

$$T(jw) = \frac{j\frac{1}{2}w}{jw + 2500}$$

For $w = 500$, we have

$$|T(j500)| = \frac{|j250|}{|j500+2500|} = \frac{250}{\sqrt{500^2 + 2500^2}} = 0.0981$$

$$\angle T(j500) = 90° - \tan^{-1}\left(\frac{500}{2500}\right) = 90° - 11.31° = 78.69°$$

Then $i_{2ss}(t) = (10\,mA)(0.0981)\cos(500t + 78.69°)$

$$= 0.980\cos(500t + 78.69°)\ mA$$

11.41) cont.

b) When $i_1(t) = 10\cos(2500t)$ mA, we have that $\omega = 2500$,

$$T(j2500) = \frac{j1250}{2500 + j2500}$$

$$|T(j2500)| = \frac{1250}{\sqrt{2500^2 + 2500^2}} = 0.3536$$

$$\angle T(j2500) = 90^\circ - \tan^{-1}\left(\frac{2500}{2500}\right) = 90 - 45^\circ = 45^\circ$$

$$i_{2ss}(t) = 3.536\cos(2500t + 45^\circ) \text{ mA}$$

c) When $i_1(t) = 10\cos(12500t)$ mA

$$T(j12500) = \frac{j6250}{j12500 + 2500}$$

$$|T(j12500)| = \frac{6250}{\sqrt{12500^2 + 2500^2}} = 0.4903$$

$$\angle T(j12500) = 90^\circ - \tan^{-1}\left(\frac{12500}{2500}\right) = 90^\circ - 78.69^\circ = 11.31^\circ$$

$$i_{2ss}(t) = 4.903\cos(12500t + 11.31^\circ) \text{ mA}$$

12.3) a) At $\omega = 100$ rad/s , $|T(j\omega)| = 3.5$. and $T_{max} = 10$

$$\# \text{ of dB down} = 20\log_{10}(3.5) - 20\log_{10}(10)$$

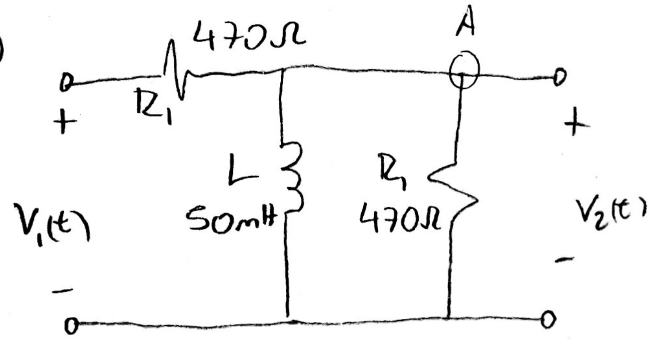
$$= 20\log_{10}\left(\frac{3.5}{10}\right) = -9.1186 \text{ dB}$$

b) The cutoff frequency ω_c occurs when $|T(j\omega_c)| = \frac{1}{\sqrt{2}} T_{max}$, then ω_c occurs at $\frac{1}{\sqrt{2}}(10) = 7.071$, which corresponds approximately when $\omega \approx 30$ rad/s.

Recall that an octave is any frequency range whose end points have 2:1 ratio. Then, one octave after the cutoff frequency we have $\omega = 60$ rad/s with $|T(j\omega)| \approx 5$. It follows

$$\# \text{ of dB down} = 20\log_{10}\left(\frac{5}{10}\right) = -6.0206 \text{ dB}$$

12.5)



Use KCL at (A):

$$\frac{V_1(s) - V_2(s)}{R_1} - \frac{V_2(s)}{sL} - \frac{V_2(s)}{R_1} = 0$$

$$V_2(s)\left(\frac{1}{R_1} + \frac{1}{sL} + \frac{1}{R_1}\right) = \frac{1}{R_1} V_1(s)$$

Note that the transfer function $T_V(s)$ can be obtained using voltage division.

$$V_2(s)\left(\frac{2sL + R_1}{R_1 L s}\right) = \frac{1}{R_1} V_1(s)$$

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{Ls}{2Ls + R_1} = \frac{50m\,s}{2(50m)s + 470} = \frac{\frac{1}{2}s}{s + 4700}$$

a) $$T(jw) = \frac{j\frac{1}{2}w}{jw + 4700}$$

Dc gain is given when $w = 0$, it follows

$$|T(j0)| = 0$$

The infinite freq. gain:

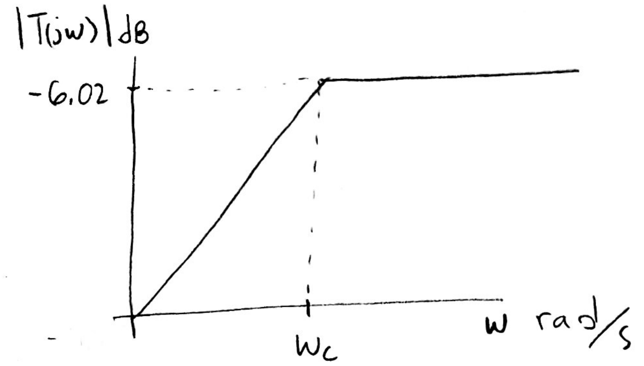
$$\lim_{w \to \infty} |T(jw)| = \lim_{w \to \infty} \frac{|j\frac{1}{2}w|}{|jw + 4700|} = \frac{1}{2}$$

Cutoff freq $w_c = 4700$

Since it has the form of a high-pass filter (see lecture Notes page 214)

The filter is high-pass

b) straight line approximation:



c) Gain at $w = 0.25 w_c = (0.25)4700 = 1175$

$$|T(j1175)| = \frac{|j587.5|}{|j1175 + 4700|} = \frac{587.5}{\sqrt{1175^2 + 4700^2}} = 0.1213$$

or -18.33 dB

Gain at $w = 0.5 w_c = 2350$

$$|T(j2350)| = \frac{j1175}{j2350 + 4700} = \frac{1175}{\sqrt{2350^2 + 4700^2}} = 0.2236$$

or -13.01 dB

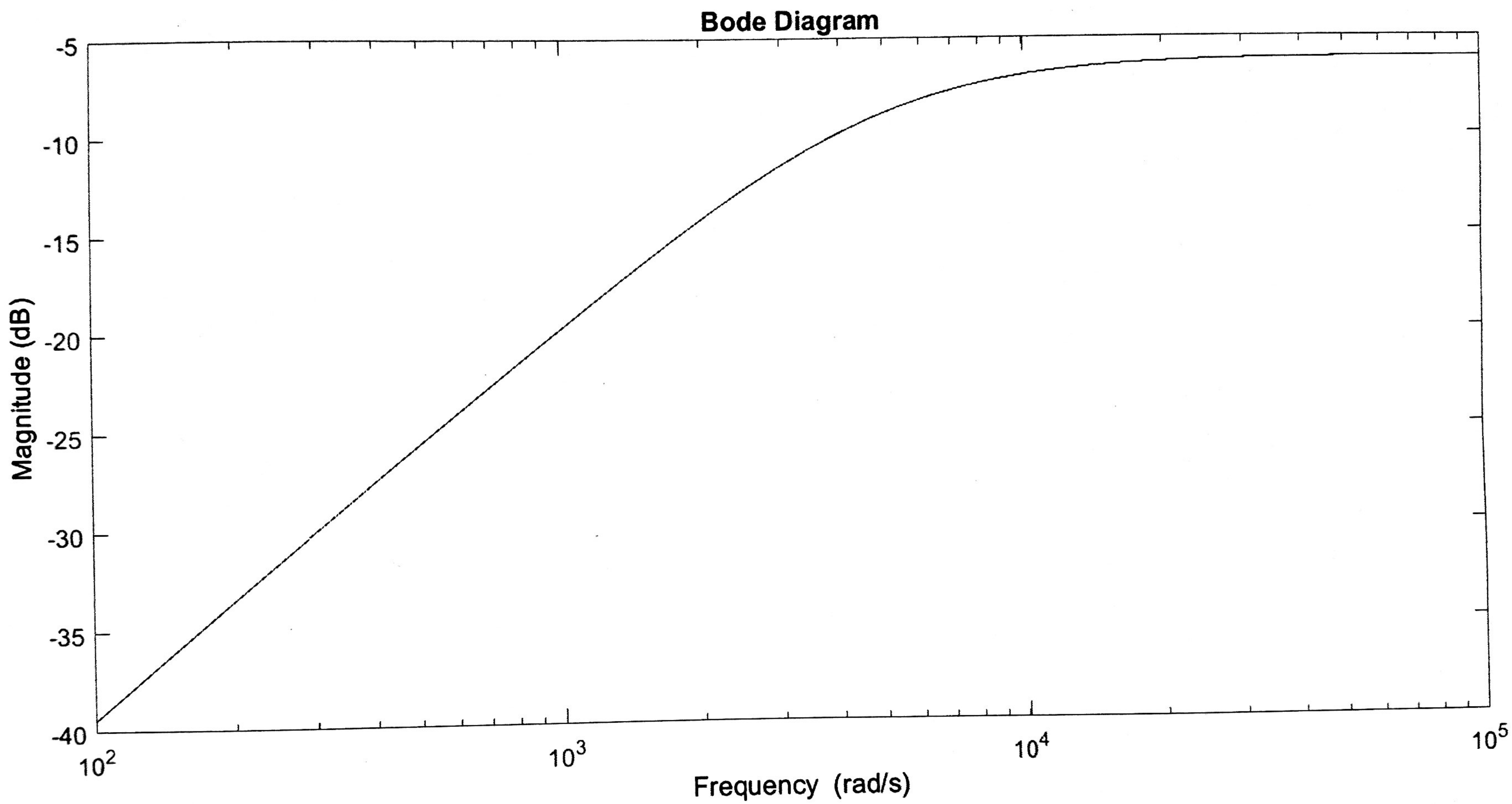
Gain when $w = w_c = 4700$

$$|T(j4700)| = \frac{|j2350|}{\sqrt{4700^2 + 4700^2}} = 0.3536 \quad \text{or } -9.03 \text{ dB}$$

d) The code in matlab to generate the magnitud plot:

```
>> T=tf([1/2 0],[1 4700])
>> bodemag(T)
```

The bode plot is shown in the next page.


Bode Diagram
-5
-10
-15
-20
-25
-30
-35
-40
Magnitude (dB)
10^2
10^3
10^4
10^5
Frequency (rad/s)

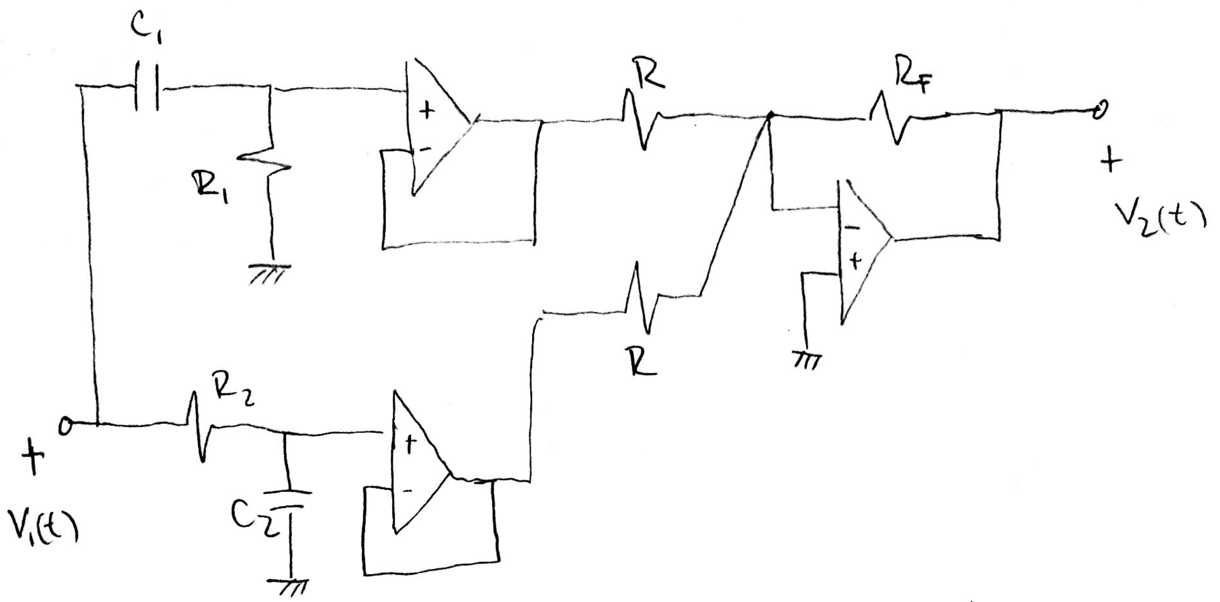
e) At two octaves before the cutoff we have

$\omega = \frac{1}{4}\omega_c = 1175 \, rad/s$, It follows

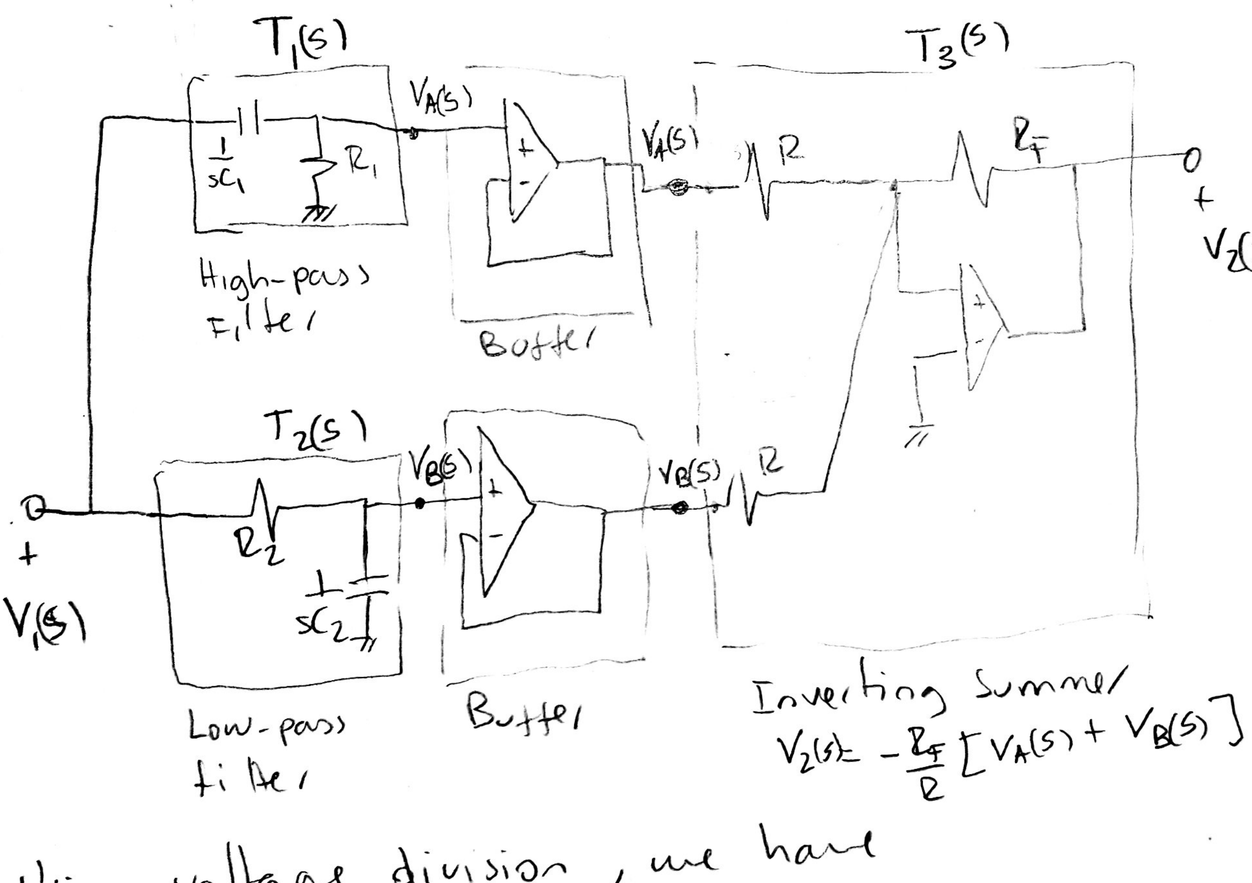
$$|T(j1175)| = \frac{|j587.5|}{|4700 + j1175|} = 0.1213 \quad or \quad -18.33\,dB$$

$$\#\,dB\ down = 20\log_{10}\left(\frac{0.1213}{0.5}\right) = -12.30\ dB$$

12.34)



a) Below is the block diagram of the circuit:



Using voltage division, we have

$$T_1(s) = \frac{R_1}{R_1 + \frac{1}{C_1 s}} = \frac{s}{s + \frac{1}{R_1 C_1}}$$

$$T_2(s) = \frac{\frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}} = \frac{\frac{1}{R_2 C_2}}{s + \frac{1}{R_2 C_2}}$$

12.34) cont.

$$T_3(s) = -\frac{R_F}{R}\left[T_1(s) + T_2(s)\right] = \frac{-R_F}{R}\left[\frac{s}{s+\frac{1}{R_1C_1}} + \frac{\frac{1}{R_2C_2}}{s+\frac{1}{R_2C_2}}\right]$$

$$= \frac{s\left(s+\frac{1}{R_2C_2}\right) + \frac{1}{R_2C_2}\left(s+\frac{1}{R_1C_1}\right)}{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}$$

$$= \frac{s^2 + \frac{2}{R_2C_2}s + \frac{1}{R_1R_2C_1C_2}}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}}$$

b) Since the circuit is a bandstop filter, there are two cutoff frequencies, i.e., the lower cutoff frequency W_{c1} and the upper cutoff frequency W_{c2}.

The lower cutoff freq. W_{c1} is controlled by the low pass filter with the elements R_2 and C_2, i.e., $W_{c1} = \frac{1}{R_2C_2}$

The upper cutoff freq. W_{c2} is controlled by the high pass-filter with the elements R_1 and C_1, i.e., $W_{c2} = \frac{1}{R_1C_1}$

It is required $W_{c1} = 400\,\text{K rad/s}$ and $W_{c2} = 4000\,\text{K rad/s}$.

Recall that for a high pass filter $W_{c1} = \frac{1}{R_2C_2} = 400\,\text{K rad/s}$.

Pick $R_2 = 10\,\text{K}\Omega$ and then $C_2 = \frac{1}{400\text{K} \cdot 10\text{K}} = 250\,\text{pF}$

Recall that for a low pass filter $W_{c2} = \frac{1}{R_1C_1} = 4000\,\text{K rad/s}$

Pick $R_1 = 10\,\text{K}\Omega$ and then $C_1 = \frac{1}{4000\text{K} \cdot 10\text{K}} = 25\,\text{pF}$.

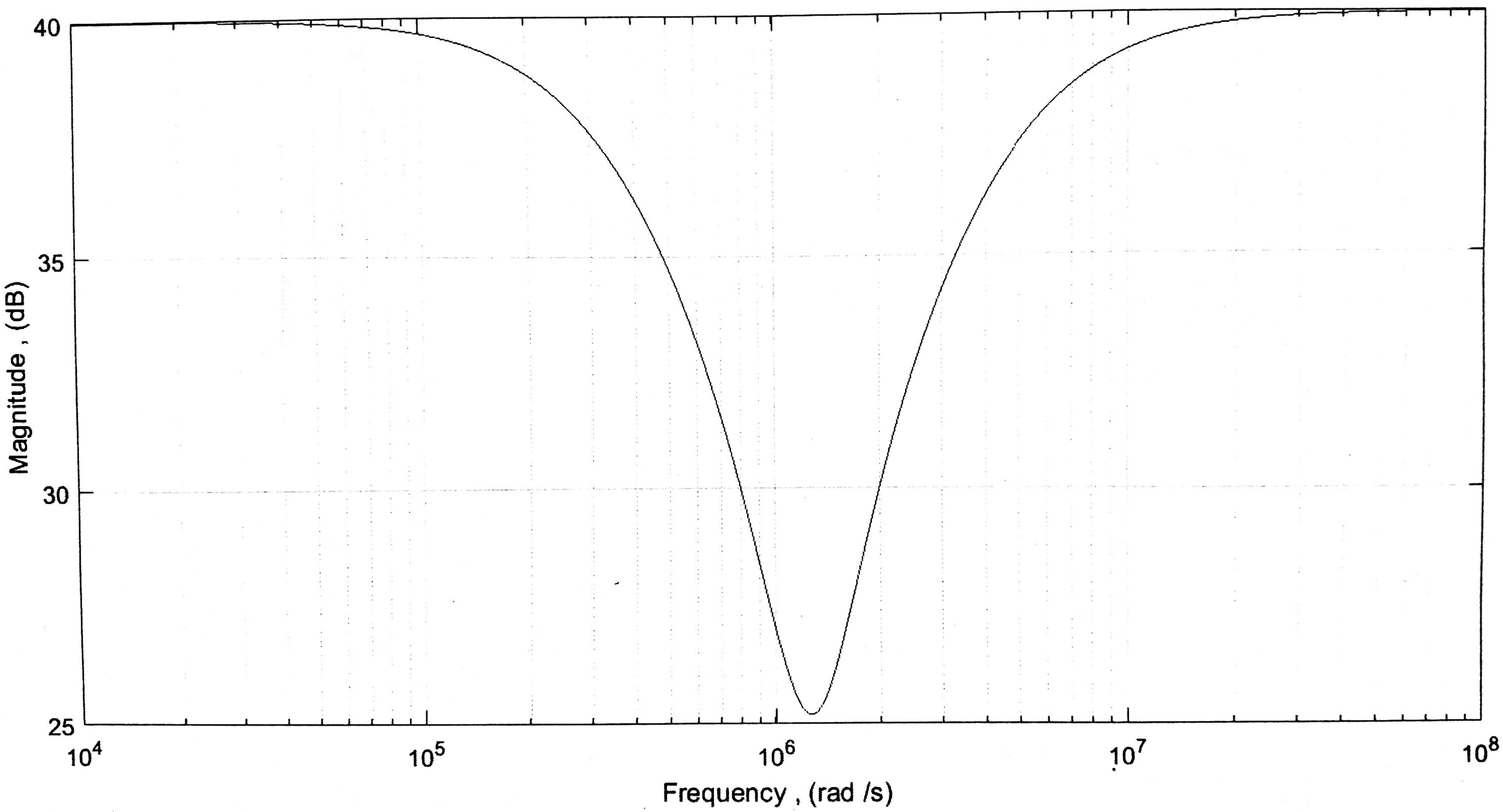
It is given the passband gain equal to 40dB or 100.
The band pass gain in this circuit is given by $\frac{R_F}{R}$, so Pick $R = 10\,\text{K}\Omega$ and we get $R_F = 10\,\text{K}\Omega(100) = 1\,\text{M}\Omega$.

c) Matlab code:

```
C1 = 25e-12;
C2 = 250e-12;
R1 = 10e3;
R2 = 10e3;
R = 10e3;
RF = 1e6;
syms s;
T1 = R1/(R1+ 1/(C1s));
T2 = (1/(C2s))/(R2+ 1/(C2s));
T = -RF/R * (T1 +T2);
w = logspace(4, 8, 1000);
Tjw = subs(T, s, i*w);
Mag = abs(Tjw);
MagdB = 20 log10(Mag);
semilogx(w, MagdB)
grid on
xlabel('Frequency (rad/s)')
ylabel('Magnitud (dB)')
```

After looking to the bode plot, the magnitude response does not satisfy the specifications because the lower and upper cutoff frequencies are not exactly as the required.

The specifications cannot be met for this circuit since w_{c1} and w_{c2} are too close together.



Magnitude , (dB)
40
35
30
25
10^4
10^5
10^6
10^7
10^8
Frequency , (rad /s)

12.38) Notice that the circuit for this problem is the same as the one in Problem 12.34.

The designed circuit looks to be righ since it contains a high pass filter and a low pass filter that together makes a stop band circuit. (See prob. 12.34)

The high pass fiter has cutoff Freq. $\omega_{c_2} = \frac{1}{R_1 C_1} = \frac{1}{10K(0.1\mu)} = 1000$ rad/s

The low pass fiter has cutoff freq $\omega_{c_1} = \frac{1}{R_2 C_2} = \frac{1}{(10K)(0.01\mu)} = 10000$ rad/s

The problem with the circuit is that the upper and lower cutoff frequencies are assigned with wrong values.

It should be that $\omega_{c_1} = 1000$ rad/s and $\omega_{c_2} = 10.000$ rad/s

To correct the design, we can swap the values of the capacitors of each fiter.