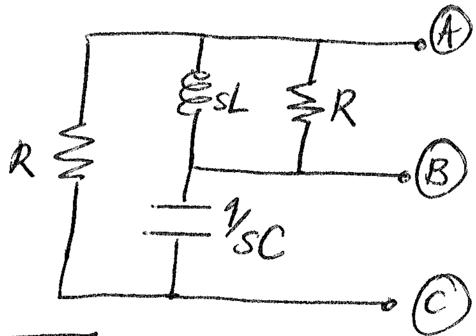
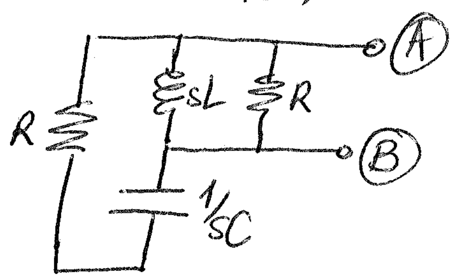
1.- $\boxed{\text{Part I.}}$ As stated, we assume <u>zero</u> initial conditions to draw



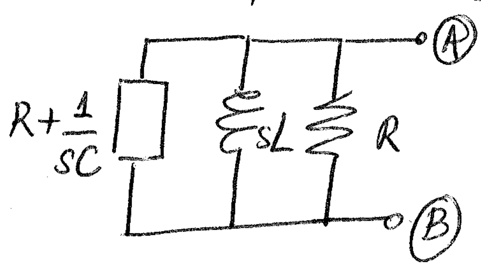
[+1 point]

$\boxed{\text{Part II}}$ We redraw slightly the circuit above as seen from terminals Ⓐ,Ⓑ;



[+1 point]

One clearly sees that the resistor R on the left is in series w/ the impedance $1/sC$. Hence, we draw



[+1 point]

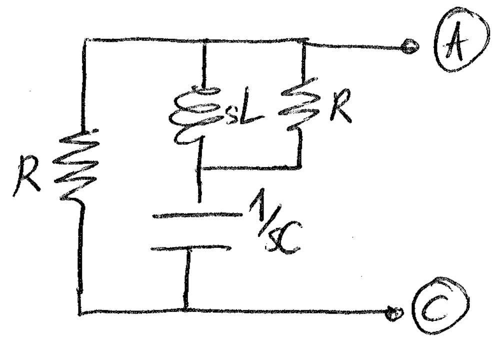
All three impedances are in parallel. Therefore

$$Z_{AB}(s) = R \,||\, sL \,||\left(R + \frac{1}{sC}\right) = \frac{1}{\frac{1}{R} + \frac{1}{sL} + \frac{sC}{RCs+1}} =$$

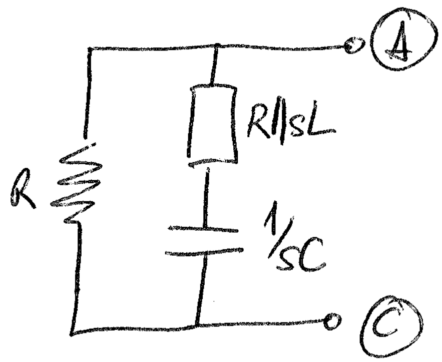
$$= \frac{1}{\frac{sL(RCs+1) + R^2Cs + R + RCLs^2}{RLs(RCs+1)}} = \frac{R^2CLs^2 + RLs}{2RCLs^2 + (L+R^2C)s + R} \quad \blacksquare$$

[+1 point]

Part III As before, we slightly redraw the circuit as



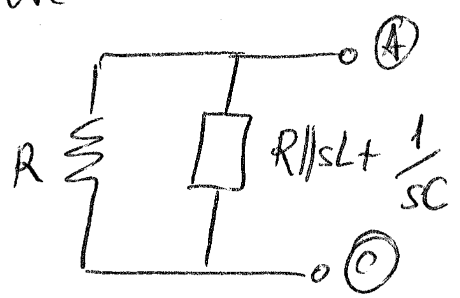
The resistor R on the right is in parallel w/ the impedance sL.



$$\left(R \| sL = \frac{RLs}{R+sL} \right)$$

[+1 point]

We combine the two impedances in series to get



$$\left(R \| sL + \frac{1}{sC} = \frac{RCLs^2+sL+R}{LCs^2+RCs} \right)$$

[+1 point]

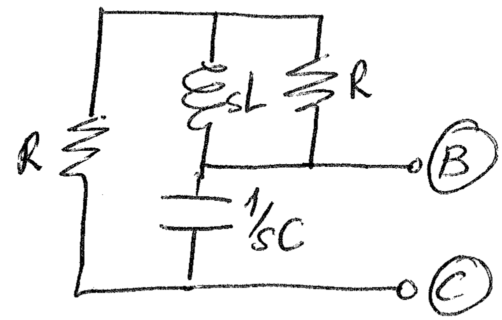
Finally, combining the two impedances in parallel

$$Z_{AC}(s) = R \,\|\, \left(R \| sL + \frac{1}{sC} \right) = \frac{1}{\frac{1}{R} + \frac{LCs^2+RCs}{RCLs^2+sL+R}} =$$

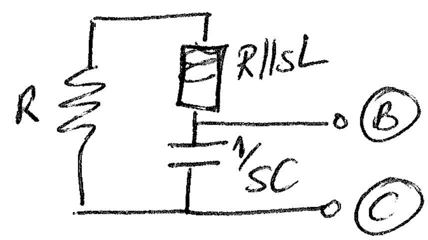
$$= \frac{R^2CLs^2+RLs+R^2}{RCLs^2+sL+R+RLCs^2+R^2Cs} = \frac{R^2CLs^2+RLs+R^2}{2RCLs^2+(L+R^2C)s+R}$$

[+1 point]

Part IV As before, we slightly redraw the circuit as

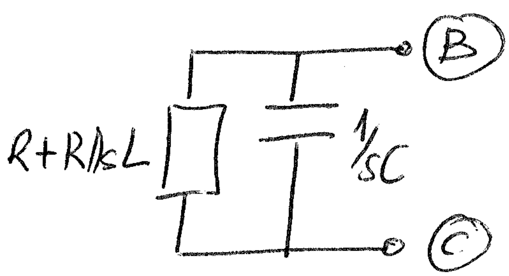


Combining the resistor on the right w/ the impedance sL (in parallel),



$$\left(R \| sL = \frac{RLs}{R+sL} \right)$$ [+1 point]

The resistor R is in series w/ R||sL, hence



$$\left(R + R \| sL = \frac{2RLs+R^2}{R+sL} \right)$$ [+1 point]

Finally, combining the two impedances in parallel

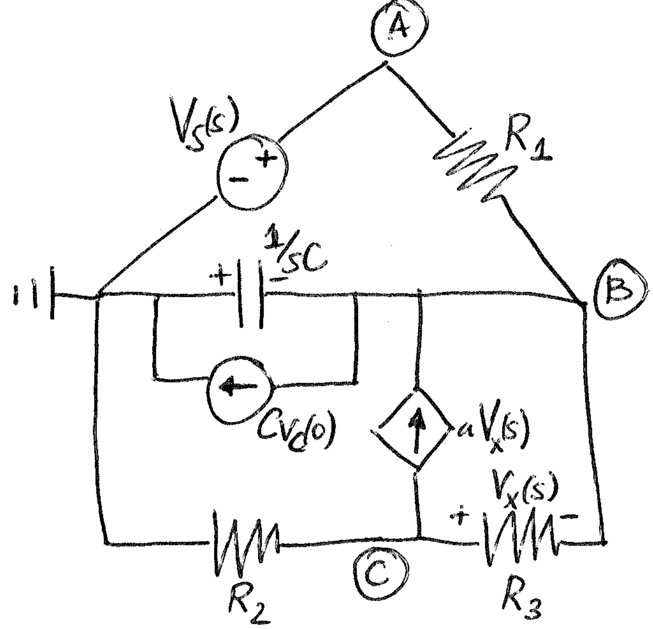
$$Z_{BC}(s) = \frac{1}{sC} \,\|\, (R + R\|sL) = \frac{1}{sC + \dfrac{R+sL}{2RLs+R^2}} =$$

$$= \frac{2RLs+R^2}{2RLCs^2 + R^2Cs + R + sL} = \frac{2RLs+R^2}{2RLCs^2 + (L+R^2C)s + R} \qquad \blacksquare$$

[+1 point]

2.- $\boxed{\text{Part I}}$

We do not assume zero initial conditions, as stated. We use a current source to capture the initial condition of the capacitor, taking good care of getting the direction correctly [because we are going to use nodal analysis].



[+ 1 point]
(1/2 correct drawing,
1/2 correct initial condition)

The presence of the voltage source represents a problem that we need to take care of. Fortunately, ground has been chosen wisely and then, by Method 2, we deduce

$$V_A(s) = V_S(s)$$ [+1 point]

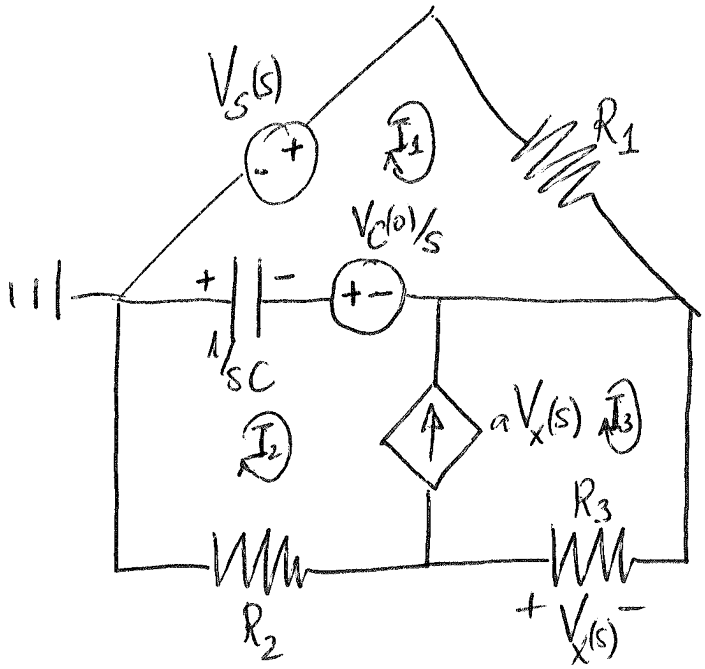
Therefore, we only write KCL equations for nodes B and C.

KCL @ B
$$\frac{1}{R_1}(V_B(s)-V_A(s)) + sC\,(V_B(s)) + \frac{1}{R_3}(V_B(s)-V_C(s)) =$$
$$= a\,V_X(s) - CV_C(0)$$ [+1 point]

KCL @ C
$$\frac{1}{R_2}V_C(s) + \frac{1}{R_3}(V_C(s)-V_B(s)) = -aV_X(s)$$ [+1 point]

Finally, we take care of the dependent source: $V_X(s) = V_C(s) - V_B(s)$
This gives a total of 4 eqs in 4 unknowns. [+1 point]

Part II Again, we do not assume zero initial conditions. Because we are gonna use mesh analysis, we use a voltage source to take care of the initial condition.



[+1 point]
($\frac{1}{2}$ correct drawing, $\frac{1}{2}$ correct initial condition)

To take care of the current source, we need to use a supermesh (method 3) by combining meshes 2 and 3. The equation for the supermesh is

$$I_3(s) - I_2(s) = aV_x(s)$$ [+1 point]

KVL at supermesh takes the form

$$\frac{1}{sC}(I_2(s) - I_1(s)) + \frac{V_C(0)}{s} + R_3(I_3(s)) + R_2 I_2(s) = 0$$ [+1 point]

KVL at mesh 1 is

$$R_1 I_1(s) + \left(-\frac{V_C(0)}{s}\right) + \frac{1}{sC}(I_1(s) - I_2(s)) - V_s(s) = 0$$ [+1 point]

Finally, we take care of the dependent source

$$V_x(s) = R_3(-I_3(s))$$ [+1 point]

These are 4 eqs in 4 unknowns.

$\boxed{\text{Part III}}$

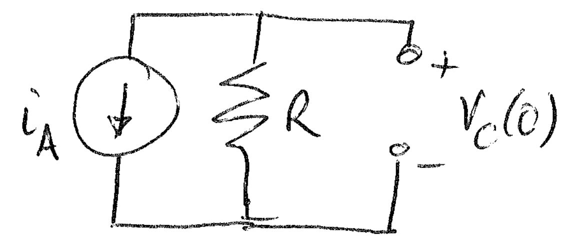
$V_C(s) = 0 - V_B(s)$ (for Part I) bonus [+0.5 point]

$$V_C(s) = \frac{1}{sC}\left(I_2(s) - I_1(s)\right) + \frac{V_C(0)}{s}$$ (for Part II) bonus [+0.5 point]

Part I

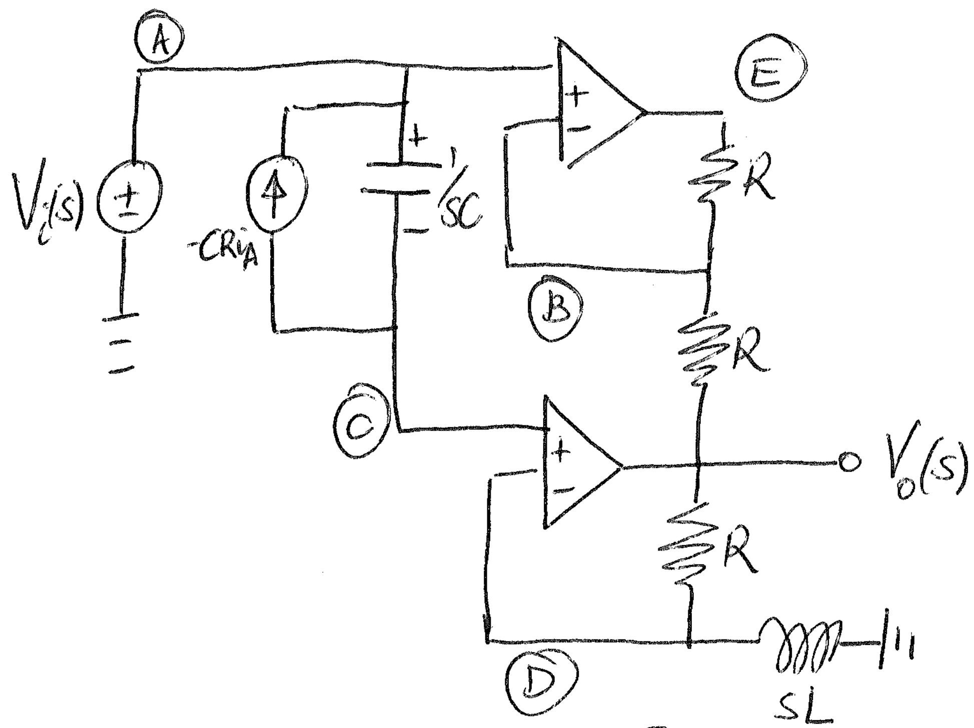
(7)

3_ We already are told that $i_L(0) = 0$.
For the capacitor, we know it has been subjected to DC excitation for a very long time. Hence it behaves like an open circuit, and hence [+1 point]



It follows that $V_c(0) = -Ri_A$. [+1 point]

Part II We use a current source to take care of the initial condition of the capacitor, as instructed.



[+0.5 point]
(for plot)

Since we do not recognize any of the basic building blocks, we use nodal analysis.

The voltage source represents a problem that we can easily take care of,

$V_A(s) = V_i(s)$ [+0.5 point]

(method 2)

Also, we don't write KCL eqs. for output nodes of the op-amps.
Ideal-op-amps conditions imply $V_A(s) = V_B(s)$ and $V_C(s) = V_D(s)$
[+0.5 point] [+0.5 point]

We then need to write KCL eqs for nodes B, C, and D.

KCL @ B: $\frac{1}{R}(V_B(s)-V_E(s))+\frac{1}{R}(V_B(s)-V_0(s))=0$ [+0.5 point]

KCL @ C: $sC(V_C(s)-V_A(s))=CRi_A$ [+0.5 point]

KCL @ D: $\frac{1}{R}(V_D(s)-V_0(s))+\frac{1}{sL}V_D(s)=0$ [+0.5 point]

From last equation, we get

$$V_0(s)=R\left(\frac{1}{R}+\frac{1}{sL}\right)V_D(s)=\frac{R+sL}{sL}V_D(s)$$

We also know that

$$V_D(s)=V_C(s)$$

From the second equation above,

$$V_C(s)=V_A(s)+\frac{Ri_A}{s}=V_i(s)+\frac{Ri_A}{s}$$

Therefore

$$V_0(s)=\frac{R+sL}{sL}\left(V_i(s)+\frac{Ri_A}{s}\right)$$ [+0.5 point]

$\boxed{\text{Part III}}$

$Ri_A=1$; $V_i(s)=\frac{s}{s^2+9}$; $R=1\Omega$; $L=10^{-1}H$

Therefore

$$V_0(s)=\frac{1+s10^{-1}}{s10^{-1}}\left(\frac{s}{s^2+9}+\frac{1}{s}\right)=$$ [+1 point]

$$= \frac{10+s}{s}\left(\frac{s}{s^2+9}+\frac{1}{s}\right) = \frac{s+10}{s^2+9} + \frac{10+s}{s^2} =$$

$$= \frac{s}{s^2+9} + \frac{10}{3}\cdot\frac{3}{s^2+9} + \frac{1}{s} + \frac{10}{s^2}$$ [+1 point]

We use inverse Laplace transform, w/ our knowledge of the table of basic transforms, to get

$$V_o(t) = \left(\cos(3t) + \frac{10}{3}\sin 3t + 1 + 10t\right)u(t)$$

$\boxed{\text{Part IV}}$

(i) the poles of the input are $\pm 3j$. Therefore the forced response is

$$\left(\cos 3t + \frac{10}{3}\sin 3t\right)u(t)$$ [+0.5 point]

and the natural response is

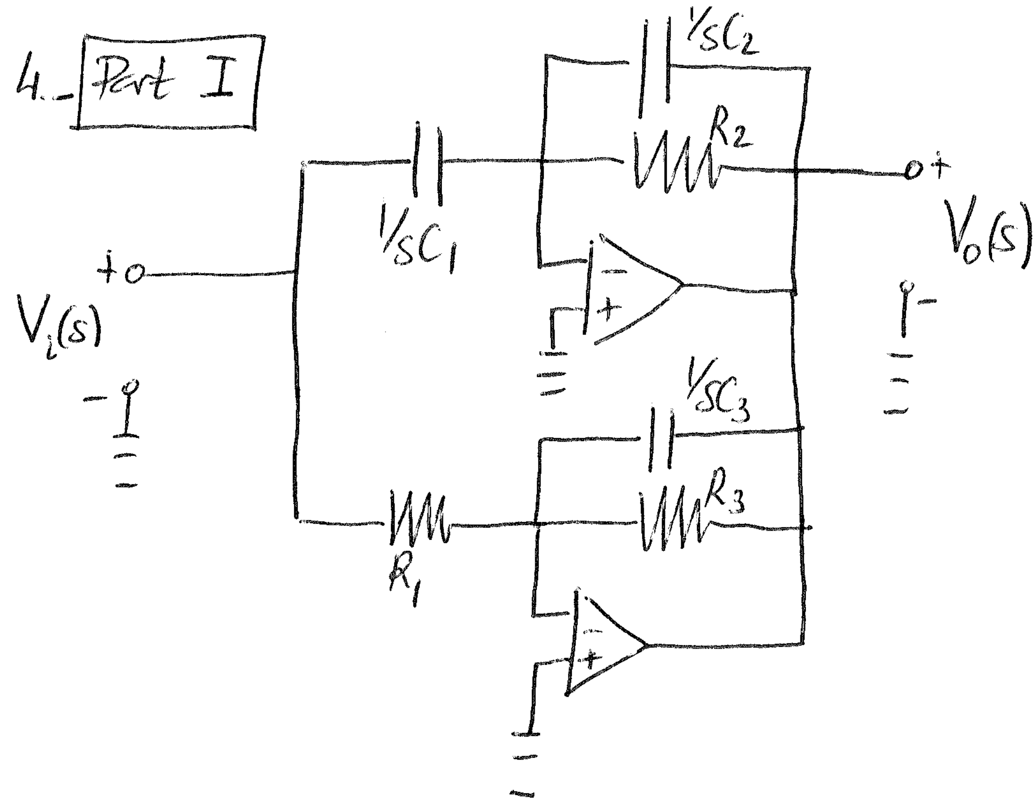
$$(1+10t)\,u(t)$$ [+0.5 point]

(ii) in the response transform, the terms $\frac{s}{s^2+9}$ and $\frac{10}{s^2+9}$ come from the input source and the terms $\frac{1}{s}+\frac{10}{s^2}$ come from the initial condition. Therefore,

zs is $\left(\cos 3t + \frac{10}{3}\sin 3t\right)u(t)$ [+0.5 point]

zi is $(1+10t)\,u(t)$ [+0.5 point]

4.- Part I



[+1 point]

Part II

We recognize this circuit as two inverting op-amps in parallel. The inverting op-amp at the top has transfer function [+1 point]

$$T_1(s) = -\frac{R_2 \,\|\, \frac{1}{sC_2}}{\frac{1}{sC_1}} = -sC_1 \cdot \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} =$$

$$= \frac{-sC_1 R_2}{R_2 C_2 s + 1}$$ [+1 point]

The inverting op-amp at the bottom has transfer function

$$T_2(s) = -\frac{R_3 \,\|\, \frac{1}{sC_3}}{R_1} = -\frac{1}{R_1} \cdot \frac{R_3 \frac{1}{sC_3}}{R_3 + \frac{1}{sC_3}} = -\frac{R_3}{R_1(R_3 C_3 s + 1)}$$

[+1 point]

Therefore, $T(s) = T_1(s) + T_2(s)$, as stated.

$\boxed{\text{Part III}}$ We substitute the given values, and obtain

$$T(s) = -\frac{s\,10^{-5}\cdot 10^{2}}{10^{-5}10^{2}s+1} - 1\cdot\frac{1}{10^{2}10^{-4}s+1} =$$

$$= -\frac{s}{s+1000} - \frac{100}{s+100}$$

$$T(j\omega) = -\frac{j\omega}{j\omega+1000} - \frac{100}{j\omega+100} = \frac{-(100j\omega-\omega^2+100j\omega+10^5)}{-\omega^2+10^5+1100\omega j} =$$

$$= -\frac{(10^5-\omega^2)+200\omega j}{(10^5-\omega^2)+1100\omega j}$$

$$|T(j\omega)| = \frac{\sqrt{(10^5-\omega^2)^2+200^2\omega^2}}{\sqrt{(10^5-\omega^2)^2+1100^2\omega^2}}$$ [+0.5 point]

$$\angle T(j\omega) = \pi + \arctan\frac{200\omega}{10^5-\omega^2} - \arctan\frac{1100\omega}{10^5-\omega^2}$$ [+0.5 point]

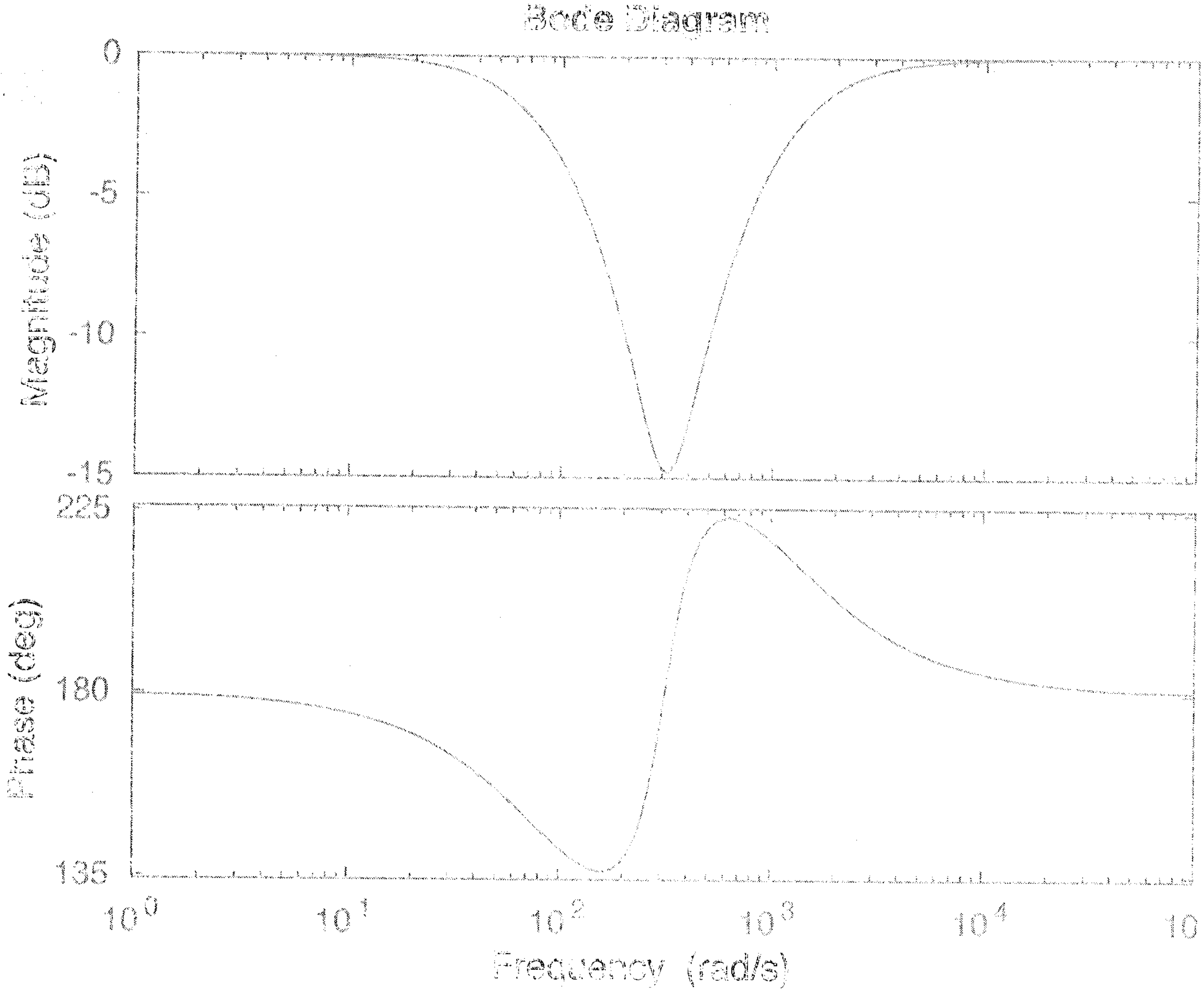
(where we have carefully taken care of the negative sign in the numerator)

DC gain $|T(j0)| = 1$ [+0.5 point] $\angle T(j0) = 0+\pi = \pi$ [+0.5 point]

∞-freq gain $|T(j\infty)| = 1$ [+0.5 point] $\angle T(j\infty) = \pi$ [+0.5 point]

From what we've seen in class, we know this is a bandstop filter. $T_1(s)$ is a high-pass filter w/ a cutoff freq of $\alpha_1 = 1000$ and $T_2(s)$ is a low-pass filter w/ cutoff-freq of $\alpha_2 = 100$. Since $\alpha_2 \ll \alpha_1$, this yields a bandstop filter, with cut-off frequencies α_2 and α_1.

The Bode plot looks like this [+1 point]



[+1 point]

Part IV

We know that

$$V_{oss}(t) = |T(j500)| \cos\left(500t + \frac{\pi}{2} + \angle T(j500)\right)$$

[+0.5 point]

Since

$$|T(j500)| \simeq 0.316$$

$$\angle T(j500) \simeq 3.858$$

[+0.5 point]

We conclude

$$V_{oss}(t) = 0.316 \cos\left(500t + \frac{\pi}{2} + 3.858\right)$$

5.- $\boxed{\text{Part I}}$

The first voltage divider has transfer function

$$T_1(s) = \frac{100}{100 + \frac{1}{10^{-4} s}} = \frac{1}{1 + 100\left(\frac{1}{s}\right)} = \frac{s}{s+100}$$ [+1 point]

The second voltage divider has transfer function

$$T_2(s) = \frac{100}{100 + s\,10^{-1}} = \frac{1000}{1000 + s}$$ [+1 point]

Their product is $T(s)$

$\boxed{\text{Part II}}$

If the transfer function of the connection in series was $T(s)$, then one would expect to get

$$V_{2ss}(t) = |T(j500)| \cdot \cos(500t + \angle T(j500))$$ [+1 point]

Since $T(j\omega) = \dfrac{j\omega}{j\omega + 100} \cdot \dfrac{1000}{1000 + j\omega}$

$$|T(j\omega)| = \frac{\omega}{\sqrt{10^4 + \omega^2}} \cdot \frac{10^3}{\sqrt{10^6 + \omega^2}}$$

$$\angle T(j\omega) = \frac{\pi}{2} - \arctan\frac{\omega}{100} + 0 - \arctan\frac{\omega}{1000}$$

$|T(j500)| \simeq 0.877$

$\angle T(j500) \simeq -0.2662$ [+1 point]

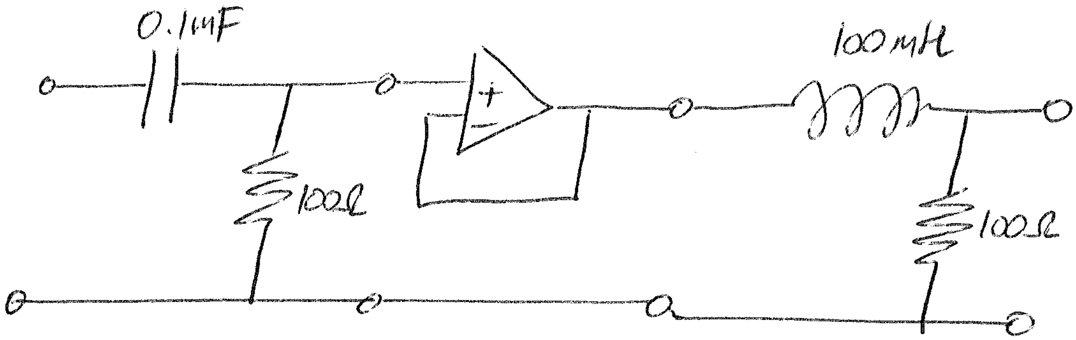
That explains why the instructor was expecting that steady-state response

The reason why he did not get it is loading. [+1 point]

No matter in which order one connects the two voltage dividers, the second stage always loads the first one.

Part III

Yes. Simply by inserting a voltage follower in-between the two voltage dividers we avoid loading,



[+1 point]

$$T(s) = T_1(s) \cdot 1 \cdot T_2(s)$$

(thanks to the ∞-input impedance & 0-output impedance of op amps)

Part IV

[+1 point]

Both stages are simple inverting-opamps. We now get

$$T_1(s) = -\frac{100}{100+0.1s} = \frac{-1000}{1000+s} \qquad [+1\ \text{point}]$$

$$T_2(s) = -\frac{100}{100+\frac{1}{10^{-4}s}} = \frac{-10^{-2}s}{10^{-2}s+1} = \frac{-s}{s+100} \qquad [+1\ \text{point}]$$

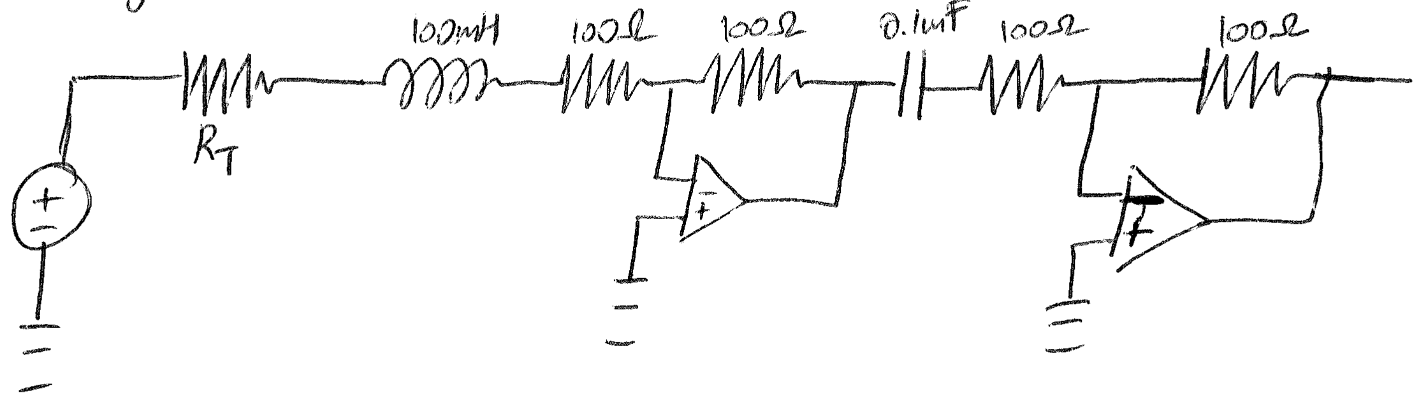
Their product is equal to T(s))

Part V

Yes, he would get that response. When the two stages are connected, there is no loading thanks to the 0-output impedance of the op-amp. [+1 point]

Part VI

No again. ✓ [+1 bonus point] The circuit would look like



Again, there is loading due to the presence of R_T and the fact that current flows in the inverting op-amp.

[+1 bonus point]

6.- With the instructions provided in the statement, we decompose $T(s)$ as follows

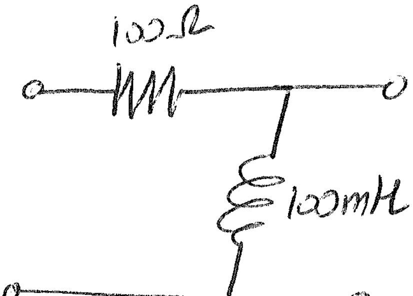
$$T(s) = \underbrace{\frac{s}{s+1000}}_{\text{voltage divider}} \cdot \underbrace{10}_{\text{non-inverting opamp}} \cdot \underbrace{\frac{100}{s+100}}_{\text{voltage divider}}$$

[+2 points]

We are constrained by the R, C, L values used by the instructor. Therefore, we write

$$\frac{s}{s+1000} = \frac{10^{-1}s}{10^{-1}s+100}$$

This we can do with

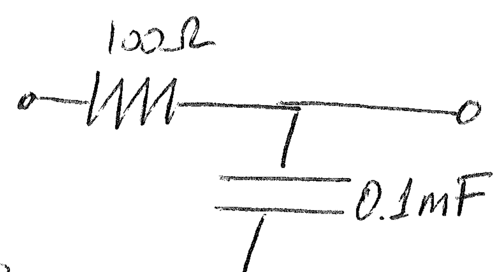


[+2 points]

Also

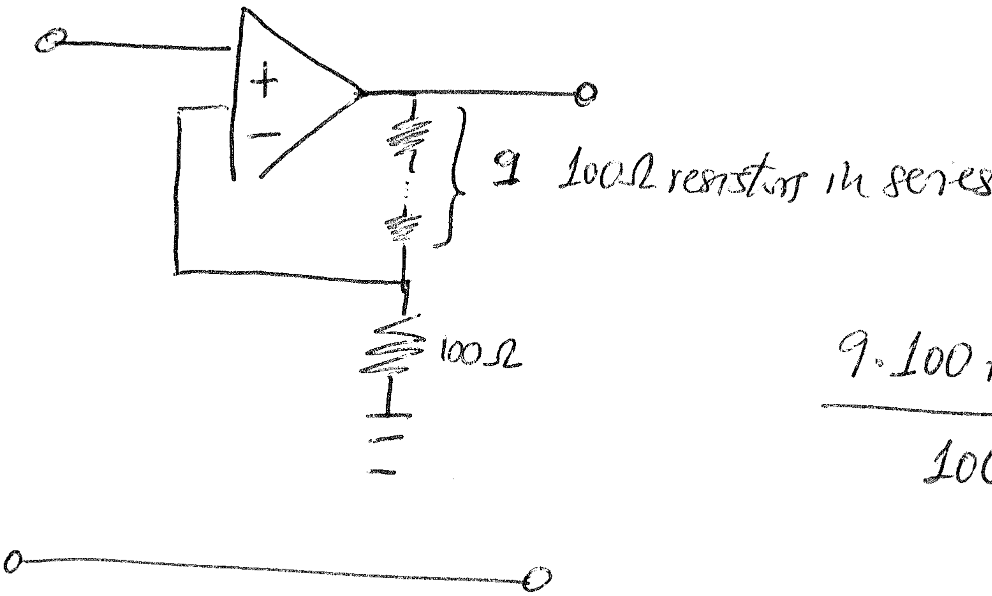
$$\frac{100}{s+100} = \frac{100/s}{1+100/s} = \frac{10^4/s}{100+10^4/s} = \frac{\frac{1}{10^{-4}s}}{100+\frac{1}{10^{-4}s}}$$

This we can do with



[+2 points]

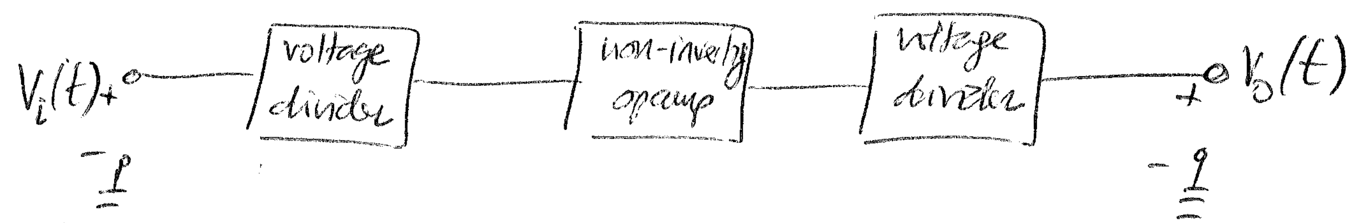
Finally, we can get a 10-gain with a non-inverting op-amp by stacking several 100Ω resistor in series



$$\frac{9 \cdot 100 + 100}{100} = 10$$

[+2 points]

Our final design is then



There is no loading because the non-inverting op-amp has 0-output impedance and ∞-input impedance.

[+2 points]