MAE140 - Linear Circuits - Winter 18
Final, March 20, 2018

Instructions

(i) The exam is open book. You may use your class notes and textbook. You may use a hand calculator with no communication capabilities.

(ii) You have 180 minutes

(iii) Do not forget to write your **name** and **student number**

(iv) On the questions for which the answers are given, please provide detailed derivations

(v) The exam has 6 questions for a total of 60 points and 3 bonus points

Good luck!

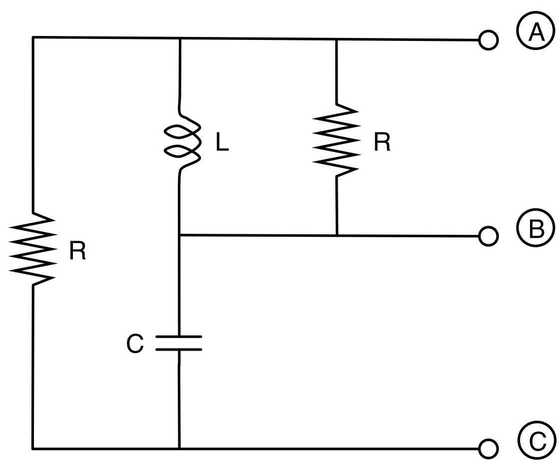

Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

All impedances should be given as a **ratio of two polynomials**.

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 1 into the s-domain.

Part II: [3 points] For the circuit you obtained in Part I, find the equivalent impedance as seen from terminals (A) and (B).

Part III: [3 points] For the circuit you obtained in Part I, find the equivalent impedance as seen from terminals (A) and (C).

Part IV: [3 points] For the circuit you obtained in Part I, find the equivalent impedance as seen from terminals (B) and (C).

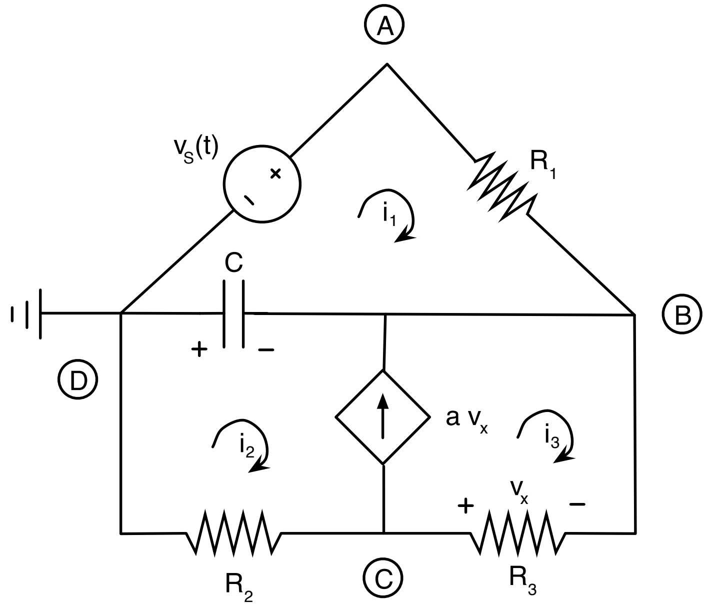

Figure 2: Nodal and Mesh Analysis Circuit for Question 2. a is a positive constant.

2. **Nodal and Mesh Analysis**

Part I: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its node-voltage equations. Use the reference node and other labels as shown in the figure. Do not assume zero initial conditions. Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

Part II: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its mesh-current equations. Use the mesh currents shown in the figure. Do not assume zero initial conditions. Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

Part III: [1 bonus point] Express the transform of the capacitor voltage using your unknown variables of Part I. Also, express the transform of the capacitor voltage using your unknown variables of Part II.

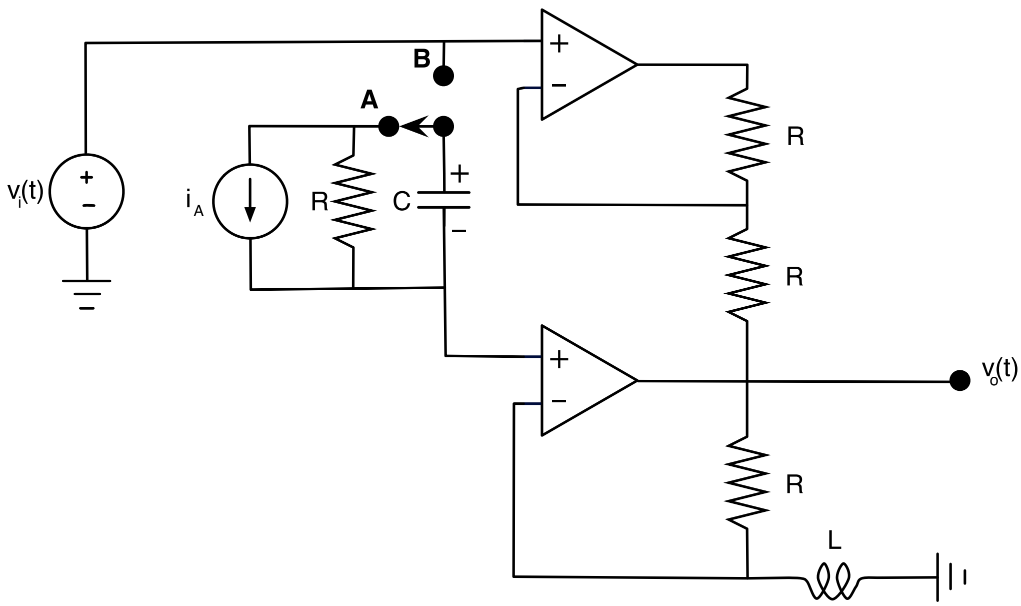

Figure 3: RCL circuit for Laplace Analysis for Question 3.

3. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 3. The value i_A of the current source is constant. The initial condition of the inductor is zero. The switch is kept in position **A** for a very long time. At $t = 0$, it is moved to position **B**. Show that the initial condition for the capacitor is given by

$$v_C(0^-) = -Ri_A.$$

[Show your work]

Part II: [4 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the capacitor in which the initial condition appears as a current source. Use nodal analysis to express the output response transform $V_o(s)$ as a function of $V_i(s)$ and i_A.

Part III: [2 points] Use partial fractions and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $i_A = 1\,\text{A}$, $v_i(t) = \cos(3t)u(t)\,V$, $C = 10\,m\text{F}$, $L = 100m\text{H}$, and $R = 1\,\text{Ohms}$ is

$$v_o(t) = \Big(\cos(3t) + \frac{10}{3}\sin(3t) + 1 + 10t\Big)u(t).$$

Part IV: [2 points] Decompose the output voltage of Part III as (i) the sum of the natural and forced response, and (ii) the sum of the zero-state and zero-input response.

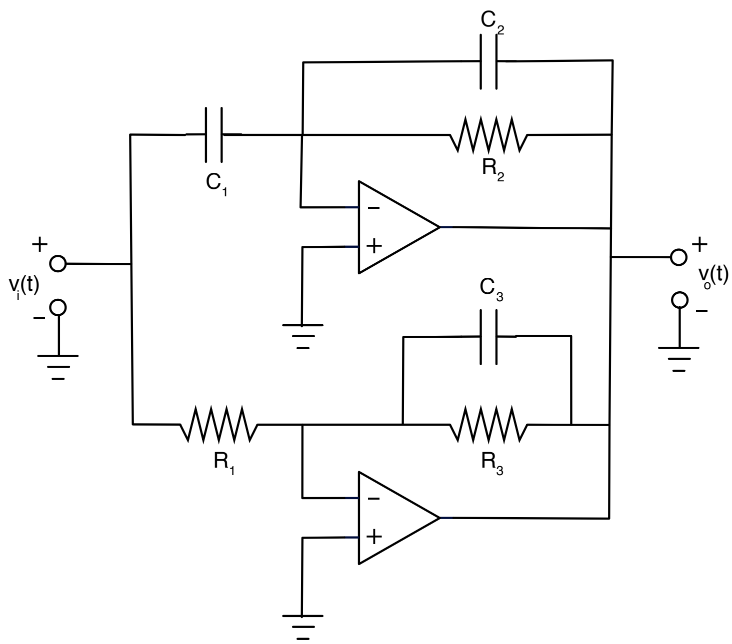

Figure 4: Frequency Response Analysis for Question 4.

4. **Frequency Response Analysis**

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 4 into the s-domain.

Part II: [3 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ is given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = -\frac{R_2 C_1 s}{R_2 C_2 s + 1} - \frac{R_3}{R_1}\frac{1}{R_3 C_3 s + 1}$$

[Show your work]

Part III [5 points] Let $R_1 = R_2 = R_3 = 100$Ohms, $C_1 = C_2 = 10\,\mu$F and $C_3 = 100\,\mu$F. Compute the gain and phase functions of $T(s)$. What are the DC gain and the ∞-freq gain? What are the corresponding values of the phase function? What are the cut-off frequencies? Sketch plots for the gain and phase functions. What type of filter is this one?
[Explain your answer]

Part IV [1 point] Using what you know about frequency response, compute the steady-state response $v_o^{SS}(t)$ of this circuit when $v_i(t) = \cos(500t + \frac{\pi}{2})$ using the same values of R_1, R_2, R_3, C_1, C_2, and C_3 as in Part III.

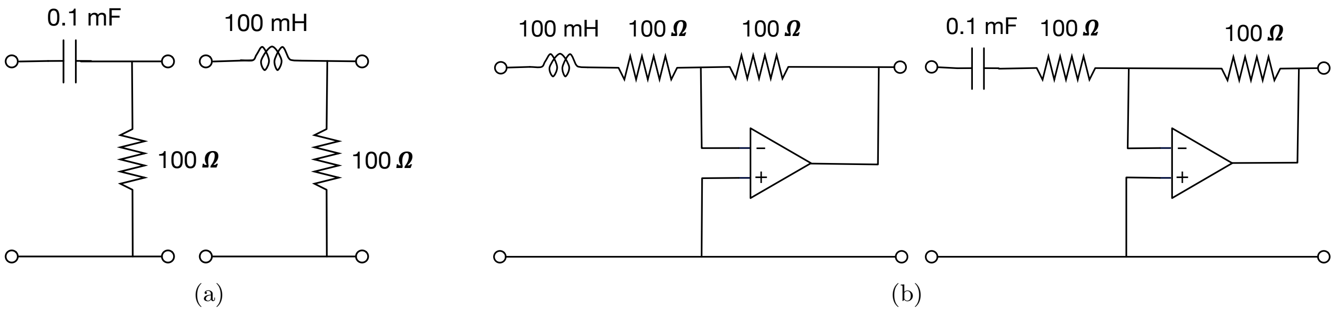

Figure 5: Circuits for Question 5.

5. **Loading and the Chain Rule**

A former instructor of MAE140 was given the task of designing a circuit with the following transfer function

$$T(s) = \frac{1000s}{s^2 + 1100s + 10^5}$$

Part I: [2 points] He first decomposed the transfer function as follows

$$T(s) = \Big(\frac{s}{s+100}\Big)\Big(\frac{1000}{s+1000}\Big)$$

and came up with a design that combines in series two voltage dividers, see Figure 5(a). Compute the transfer function of each voltage divider and show that their product is equal to $T(s)$.

Part II: [3 points] When he connected the two stages in series and used the input $v_i(t) = \cos(500t)$, he was surprised to observe that the steady-state output was not $v_o^{SS}(t) = \sqrt{\frac{10}{13}}\cos(500t - 0.2662)$, as he was expecting. Can you explain why he was expecting that response and why he did not get it? Properly justify your answer.

Part III: [1 point] Could you fix the design provided by the instructor, still employing his two voltage dividers and possibly using one op-amp, so that he gets the steady-state output he was aiming for? Explain how.

Part IV: [3 points] The instructor could not figure out why his design with voltage dividers was not working, so he abandoned it. He decomposed again the transfer function, this time as follows

$$T(s) = \Big(\frac{-1000}{s+1000}\Big)\Big(\frac{-s}{s+100}\Big)$$

and came up with a design that combines in series the two stages in Figure 5(b). Compute the transfer function of each stage and show that their product is equal to $T(s)$.

Part V: [1 point] If he connects in series the two stages in Figure 5(b) and uses the input $v_i(t) = \cos(500t)$, would he get the steady-state output he was looking for? Why? Properly justify your answer.

Part VI: [2 bonus points] If the instructor were to connect a source circuit whose Thévenin equivalent is $v_T(t) = v_i(t)$ and $R_T = 100$Ohms to his design with op-amps in Part IV, should he expect to get the steady-state output he was looking for too?

6. **Design** [10 points]

Provide an alternative design solution to the ones proposed by the instructor in Question 5. The goal is to design a circuit whose transfer function is

$$T(s) = \frac{1000s}{s^2 + 1100s + 10^5}.$$

You can only use 1 op-amp and components with the same values as those used in the instructor's design (i.e., 100Ω-resistors, $100\,m$H-inductors, and $0.1\,m$F-capacitors). Your design should be based on connecting two voltage dividers and one non-inverting op-amp. Make sure you properly justify that the chain rule applies.