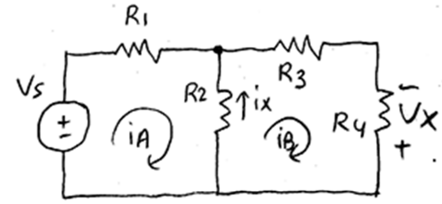
Homework 4 Solution



3-13 Formulate mesh current, form $A\vec{x} = \vec{b}$

mesh A → $-V_s + (R_1+R_2)\, i_A - R_2 i_B = 0$

mesh B → $-R_2 i_A + (R_2+R_3+R_4)\, i_B = 0$

$$\boxed{\begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3+R_4 \end{bmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} V_s \\ 0 \end{pmatrix}}$$

a) Solve for i_A, i_B. We can solve this using Matlab too.

$-R_2 i_A + (R_2+R_3+R_4)\, i_B = 0$

$$i_B = \frac{R_2}{R_2+R_3+R_4}\, i_A$$

$(R_1+R_2)\, i_A - R_2 i_B = V_s$

$$\left[R_1+R_2 - \frac{R_2^2}{R_2+R_3+R_4}\right] i_A = V_s$$

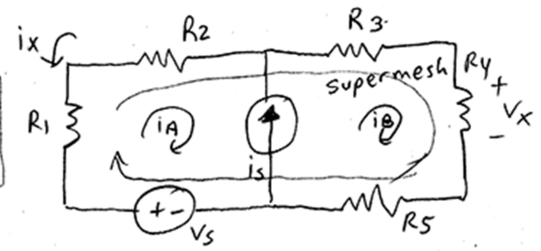
$$\boxed{i_A = \frac{R_2+R_3+R_4}{R_1(R_2+R_3+R_4) + R_2(R_3+R_4)}\, V_s}$$

$$\boxed{i_B = \frac{R_2}{R_1(R_2+R_3+R_4) + R_2(R_3+R_4)}\, V_s}$$

b) Solve V_x, i_x

$$\boxed{V_x = -R_4 i_B = \frac{-R_2 R_4}{R_1(R_2+R_3+R_4) + R_2(R_3+R_4)}\, V_s}$$

$$\boxed{i_x = i_B - i_A = \frac{-(R_3+R_4)}{R_1(R_2+R_3+R_4) + R_2(R_3+R_4)}\, V_s}$$



3-18 a) Formulate mesh current

$$\boxed{\begin{aligned} &\text{Supermesh} \rightarrow & -V_{s1} + (R_1+R_2)\, i_A + (R_3+R_4+R_5)\, i_B = 0 \\ &i_s \rightarrow & i_B - i_A = i_s \end{aligned}}$$

b) Using $V_s = 12V$, $i_s = 10mA$, $R_1 = 2.7k\Omega$, $R_2 = 1.5k\Omega$, $R_3 = 680\Omega$, $R_4 = 2.2k\Omega$, $R_5 = 3.3k\Omega$, solve for V_x, i_x.

$$\begin{bmatrix} 4200\Omega & 6180\Omega \\ -1 & 1 \end{bmatrix} \begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} 12V \\ 0.01A \end{pmatrix} \rightarrow \boxed{\begin{pmatrix} i_A \\ i_B \end{pmatrix} = \begin{pmatrix} -4.80\ mA \\ 5.20\ mA \end{pmatrix}}$$

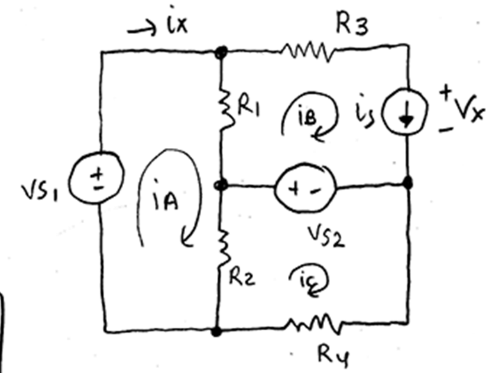
$$\boxed{\begin{aligned} V_x &= R_4 \cdot i_B = 11.44V \\ i_x &= -i_A = 4.80\ mA \end{aligned}}$$

c) Find total power dissipated in the circuit. → Sum of all positive power.

Resistors always dissipate power. Also note that current is entering the Vs positive terminal.
↳ Vs is absorbing power.

$$P_{diss} = (R_1+R_2)\, i_A^2 + (R_3+R_4+R_5)\, i_B^2 - i_A V_s \rightarrow \boxed{P_{diss} = 0.32\ W}$$

3-20 a) Formulate mesh current, b) solve v_x, i_x symbolically, c) Solve v_x, i_x
d) Find power supplied by V_{S_1}.



a) mesh A → $-V_{S_1} + (R_1+R_2)i_A - i_B R_1 - i_C R_2 = 0$
mesh B → $i_B = i_S$
mesh C → $V_{S_2} + (R_2+R_4)i_C - R_2 i_A = 0$

↳ $$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 \\ 0 & 1 & 0 \\ -R_2 & 0 & R_2+R_4 \end{bmatrix}\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} V_{S_1} \\ i_S \\ -V_{S_2} \end{bmatrix}$$

b) Solve with Matlab

↳ $$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} \dfrac{V_{S_1}(R_2+R_4) - V_{S_2}R_2 + (R_2+R_4)R_1 i_S}{R_1R_2 + R_1R_4 + R_2R_4} \\ i_S \\ \dfrac{R_2 V_{S_1} - (R_1+R_2)V_{S_2} + R_1R_2 i_S}{R_1R_2 + R_1R_4 + R_2R_4} \end{bmatrix}$$

KVL on mesh B → $-V_{S_2} + (i_B - i_A)R_1 + i_B R_3 + V_x = 0$

$$\boxed{\begin{aligned} V_x &= V_{S_2} - (R_1+R_3)i_B + R_1 i_A \\ i_x &= i_A \end{aligned}}$$

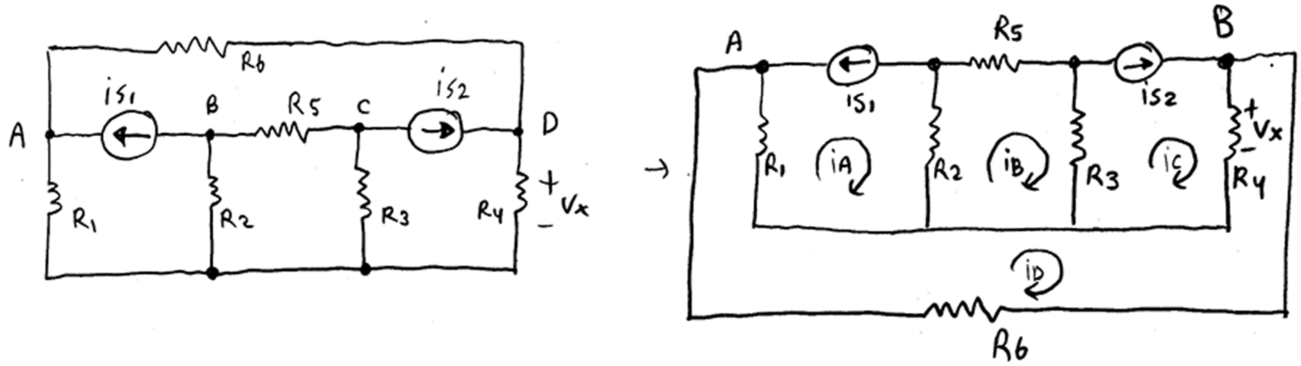
c) Solve using Matlab with $V_{S_1} = 15V$, $V_{S_2} = 5V$, $i_S = 2.5mA$, $R_1 = R_2 = 8.2k\Omega$
$R_3 = 2.2k\Omega$, $R_4 = 3.3k\Omega$

$$\boxed{\begin{pmatrix} i_A \\ i_B \\ i_C \end{pmatrix} = \begin{pmatrix} 3.026\ mA \\ 2.500\ mA \\ 1.723\ mA \end{pmatrix}} \rightarrow \boxed{\begin{aligned} V_x &= 3.814\ V \\ i_x &= 3.026\ mA \end{aligned}}$$

d) $P_{V_{S_1}} = -V_{S_1} \cdot i_A \rightarrow \boxed{P_{V_{S_1}} = -45.4\ mW}$
$= -15V \cdot (3.026mA)$

3-21 a) Avoid supermesh, b) formulate mesh current, c) Solve V_x using $R_1 = R_2 = R_3 = R_4 = 1k\Omega$, $R_5 = R_6 = 10k\Omega$, $i_{s1} = 100\,mA$, $i_{s2} = 50\,mA$.

a)



b) mesh A $\rightarrow$ $i_A = -i_{s1}$

mesh B $\rightarrow$ $(R_2 + R_3 + R_5)\, i_B - R_2 i_A - R_3 i_C = 0$

mesh C $\rightarrow$ $i_C = i_{s2}$

mesh D $\rightarrow$ $(R_1 + R_4 + R_6)\, i_D - R_1 i_A - R_4 i_C = 0$

$$\hookrightarrow \boxed{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -R_2 & R_2+R_3+R_5 & -R_3 & 0 \\ 0 & 0 & 1 & 0 \\ -R_1 & 0 & -R_4 & R_1+R_4+R_6 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \end{bmatrix} = \begin{bmatrix} -i_{s1} \\ 0 \\ i_{s2} \\ 0 \end{bmatrix}}$$

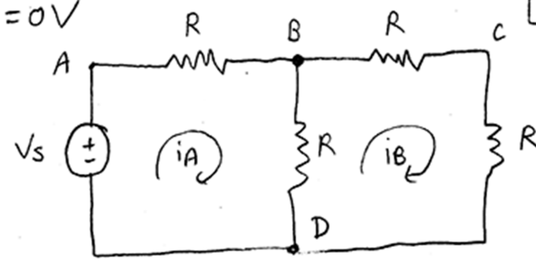
We have:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1k\Omega & 12k\Omega & -1k\Omega & 0 \\ 0 & 0 & 1 & 0 \\ -1k\Omega & 0 & -1k\Omega & 12k\Omega \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \end{bmatrix} = \begin{bmatrix} -100\,mA \\ 0 \\ 50\,mA \\ 0 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \end{bmatrix} = \begin{bmatrix} -100\,mA \\ -4.1667\,mA \\ 50\,mA \\ -4.1667\,mA \end{bmatrix}$$

c) $V_x = R_4 (i_C - i_D) = 1k\Omega\,(50\,mA + 4.1667\,mA) \rightarrow \boxed{V_x = 54.1667\,V}$

3-24 All $R = 1k\Omega$, $V_s = 12V$, $V_c = -2.4V$, $V_B = 0V$
Find V_A, V_D, mesh current i_A, i_B



Node voltages

node B $\rightarrow \left(\frac{1}{R}+\frac{1}{R}+\frac{1}{R}\right)\cancelto{0}{V_B} - \frac{V_A}{R} - \frac{V_C}{R} - \frac{V_D}{R} = 0$

$$V_A + V_D = -V_C = 2.4V$$

$$V_A - V_D = V_s = 12V \rightarrow \boxed{\begin{aligned} V_A &= 7.2V \\ V_D &= -4.8V \end{aligned}}$$

mesh current

mesh A $\rightarrow$ $-V_s + 2Ri_A - Ri_B = 0$

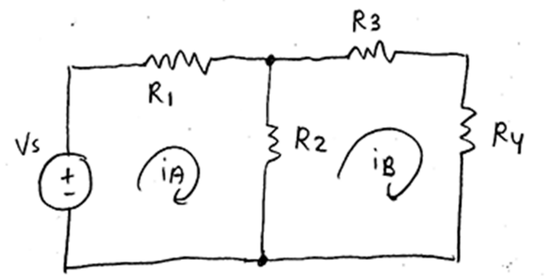
mesh B $\rightarrow$ $-Ri_A + 3Ri_B = 0$

$$i_A = 3i_B$$

$\rightarrow$ $-V_s + 5Ri_B = 0 \rightarrow i_B = \frac{V_s}{5R} = \frac{12V}{5k\Omega}$

$$\boxed{\begin{aligned} i_B &= 2.4mA \\ i_A &= 7.2mA \end{aligned}}$$

3-25 a) Mesh current to solve i_A, b) Compute V_s/i_A, c) Find equivalent resistance
Solve symbolically.



a) Same problem as 3-24.

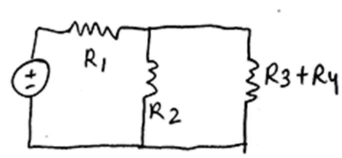
mesh A $\rightarrow$ $-V_s + (R_1+R_2)i_A - R_2 i_B = 0$

mesh B $\rightarrow$ $(R_2+R_3+R_4)i_B - R_2 i_A = 0$

$\hookrightarrow i_B = \frac{1}{R_2}\left((R_1+R_2)i_A - V_s\right)$

$$(R_2+R_3+R_4)\cdot\frac{1}{R_2}\left((R_1+R_2)i_A - V_s\right) - R_2 i_A = 0$$

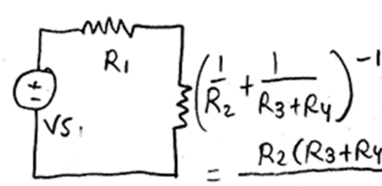
$$\left[(R_1+R_2)(R_2+R_3+R_4) - R_2^2\right] i_A = (R_2+R_3+R_4)V_s$$



b) $$\frac{V_s}{i_A} = \frac{R_1(R_2+R_3+R_4) + R_2(R_3+R_4)}{(R_2+R_3+R_4)}$$

$$\boxed{\frac{V_s}{i_A} = R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}}$$

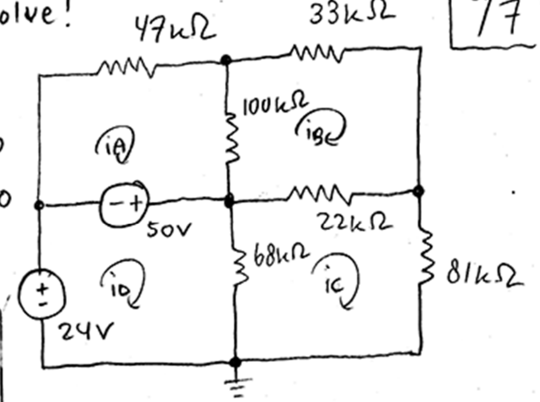
Same!



$$= \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$$

c) Shown on the right $\rightarrow$ $\boxed{R_{eq} = R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}}$

3-27 a) Formulate mesh current into $A\vec{x} = \vec{b}$ and solve!



mesh A → $(47k\Omega + 100k\Omega)i_A - (100k\Omega)i_B + 50V = 0$

mesh B → $(100k\Omega + 33k\Omega + 22k\Omega)i_B - (100k\Omega)i_A - (22k\Omega)i_D = 0$

mesh C → $(68k\Omega + 22k\Omega + 81k\Omega)i_C - (22k\Omega)i_B - (68k\Omega)i_D = 0$

mesh D → $-24V - 50V + 68k\Omega(i_D - i_C) = 0$

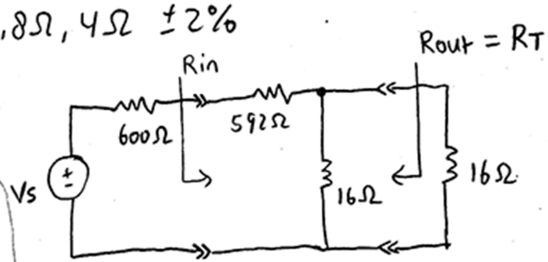
$$\begin{bmatrix} 147k\Omega & -100k\Omega & 0 & 0 \\ -100k\Omega & 155k\Omega & -22k\Omega & 0 \\ 0 & -22k\Omega & 171k\Omega & -68k\Omega \\ 0 & 0 & -68k\Omega & 68k\Omega \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_D \end{bmatrix} = \begin{bmatrix} -50V \\ 0 \\ 0 \\ 74V \end{bmatrix}$$

using Matlab → $\vec{x} = A^{-1}\vec{b}$

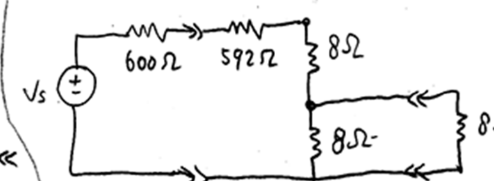
$$\begin{pmatrix} i_A \\ i_B \\ i_C \\ i_D \end{pmatrix} = \begin{pmatrix} -0.4907\ mA \\ -0.2213\ mA \\ 0.6712\ mA \\ 1.7594\ mA \end{pmatrix}$$

3-97 Check if $R_{in} = 600\Omega \pm 2\%$ and $R_{out} = 16\Omega, 8\Omega, 4\Omega \pm 2\%$

Essentially, we have the following circuits →



or



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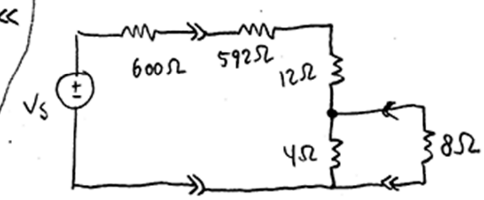
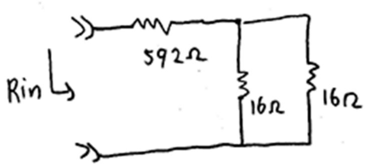


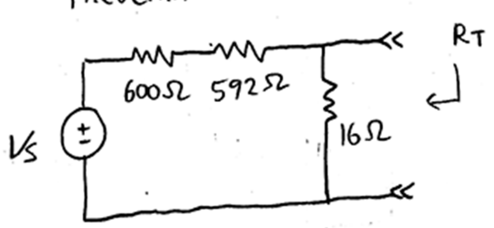
→ 16Ω case



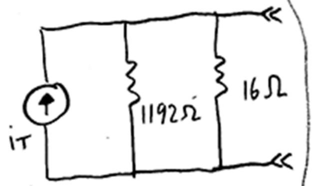
$R_{in} = 592\Omega + \left(\frac{1}{16\Omega} + \frac{1}{16\Omega}\right)^{-1}$

$\boxed{R_{in} = 600\Omega}$

Thevenin Resistance = Rout



→



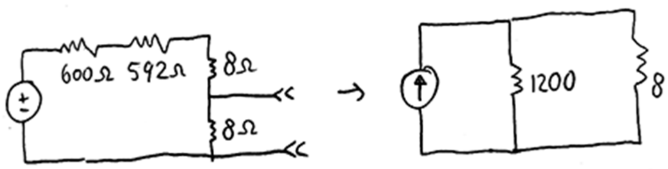
$R_T = \left(\frac{1}{1192\Omega} + \frac{1}{16\Omega}\right)^{-1}$

$\boxed{R_T = 15.788\Omega}$ ← Rout

8Ω case

$$R_{in} = 592\Omega + 8\Omega + \left(\frac{1}{8\Omega} + \frac{1}{8\Omega}\right)^{-1}$$

$$\boxed{R_{in} = 604\,\Omega}$$



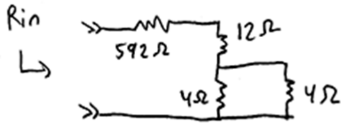
$$R_{out} = \left(\frac{1}{1200\Omega} + \frac{1}{8\Omega}\right)^{-1}$$

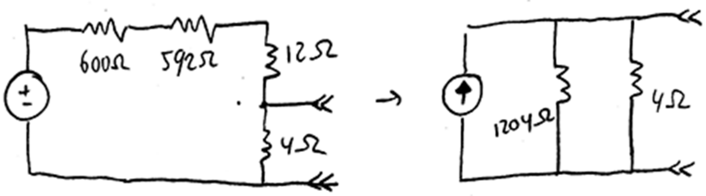
$$\boxed{R_{out} = 7.947\,\Omega}$$

4Ω Case

$$R_{in} = 592\Omega + 12\Omega + \left(\frac{1}{4\Omega} + \frac{1}{4\Omega}\right)^{-1}$$

$$\boxed{R_{in} = 606\,\Omega}$$





$$R_{out} = \left(\frac{1}{1204\Omega} + \frac{1}{4\Omega}\right)^{-1}$$

$$\boxed{R_{out} = 3.987\,\Omega}$$

So, we have :

case	Rin	Rout
16Ω	600Ω	15.788Ω
8Ω	604Ω	7.947Ω
4Ω	606Ω	3.987Ω

Target range :

$R_{in} = 600\Omega \pm 2\% \in [588\Omega, 612\Omega]$

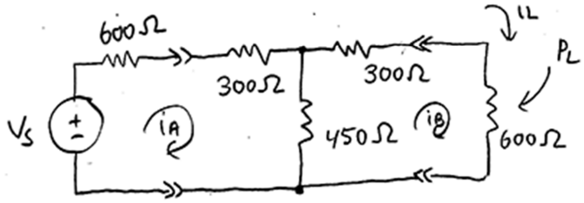
R_{out} : $16\Omega \pm 2\% \in [15.68\Omega, 16.32\Omega]$

$8\Omega \pm 2\% \in [7.84\Omega, 8.16\Omega]$

$4\Omega \pm 2\% \in [3.92\Omega, 4.08\Omega]$

Their claim is proven correct!

3-98 Find the power with or without the attenuator! Express the fraction in dB!



Find i_L (load current)

mesh A → $-V_s + (600\Omega + 300\Omega + 450\Omega)\, i_A - (450\Omega)\, i_B = 0$

mesh B → $(450\Omega + 300\Omega + 600\Omega)\, i_B - (450\Omega)\, i_A = 0$

$i_L = i_B$

$(1350\Omega)\, i_A - (450\Omega)\, i_B = V_s$

$-(450\Omega)\, i_A + (1350\Omega)\, i_B = 0 \rightarrow i_A = 3 i_B$

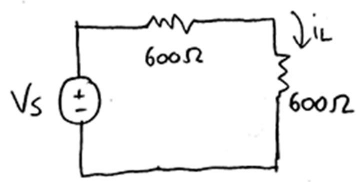
$(1350\Omega)\, 3 i_B - (450\Omega) i_B = V_s$

$$\boxed{i_B = \frac{V_s}{3600\,\Omega}}$$

$\hookrightarrow i_L = \dfrac{V_s}{3600\Omega} \rightarrow P_L = R_L i_L^2 = (600\Omega)\left(\dfrac{V_s}{3600\Omega}\right)^2$

$$\boxed{P_{L1} = \frac{1}{21600\Omega} V_s^2}$$

→ without the attenuator



$i_L = \dfrac{V_s}{1200\Omega} \rightarrow P_L = R_L i_L^2 = (600\Omega)\left(\dfrac{V_s}{1200\Omega}\right)^2$

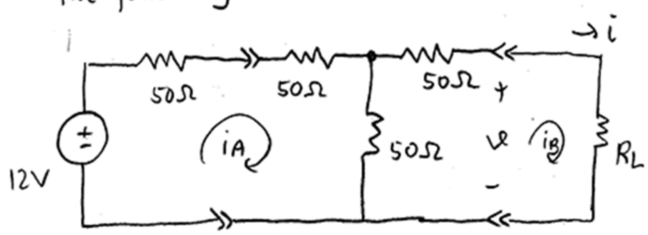
$$\boxed{P_{L2} = \frac{1}{2400\Omega} V_s^2}$$

The attenuator reduction $\rightarrow G = \dfrac{P_{L1}}{P_{L2}} = \dfrac{2400}{21600} = \dfrac{1}{9}$

$\hookrightarrow 10 \cdot \log_{10}(G) = -9.54\text{ dB}$

3-100 Typo in the problem. We want to solve $v \leq 4V$ and $i \leq 50mA$

The following circuit should work:



mesh A → $-12V + (150\Omega) i_A - (50\Omega) i_B = 0$

mesh B → $(-50\Omega) i_A + (100\Omega + R_L) i_B = 0$

$i_A = \dfrac{100\Omega + R_L}{50\Omega} i_B$

$\hookrightarrow (150\Omega)\dfrac{100\Omega + R_L}{50\Omega} i_B - (50\Omega) i_B = 12V$

$(250\Omega + 3R_L)\, i_B = 12V$

$$\boxed{i_B = \frac{12V}{250\Omega + 3R_L} = i}$$

$i = i_B$

Then, $\boxed{v = R_L i = \dfrac{R_L}{250\Omega + 3R_L} 12V}$

So, we have the following v and i range.

$\lim_{R_L \to 0} v = 0V$	$\lim_{R_L \to \infty} v = 4V$
$\lim_{R_L \to 0} i = 48mA$	$\lim_{R_L \to \infty} i = 0mA$

→ Shown that for any R_L, $v \leq 4V$ and $i \leq 50mA$