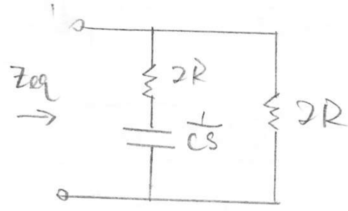
HW 7 Solution

10.3 a) S - domain circuits w/ zero IC



$$Z_{eq}(s) = \left(2R + \frac{1}{Cs}\right) \parallel 2R$$

$$= \frac{1}{\frac{1}{2R} + \frac{1}{2R + \frac{1}{Cs}}} = \frac{2R + \frac{1}{Cs}}{\left(2R + \frac{1}{Cs}\right)\frac{1}{2R} + 1}$$

$$= \frac{2R + \frac{1}{Cs}}{2 + \frac{1}{2RCs}} = \boxed{\frac{Rs + \frac{1}{2C}}{s + \frac{1}{4RC}}} = R\,\frac{s + \frac{1}{2RC}}{s + \frac{1}{4RC}}$$

$\boxed{\text{pole}: \ s = -\frac{1}{4RC}}$ $\boxed{\text{zero}: \ s = -\frac{1}{2RC}}$

b)

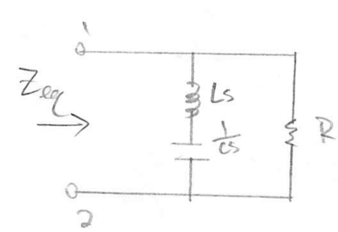
$$-\frac{1}{4RC} = -22 \times 10^3 \text{ rad/s} \Rightarrow RC = 1.1364 \times 10^{-5}$$

Let $R = 10\ \Omega$, $C = 1.1364 \times 10^{-6}\ F = 1.1364\ \mu F$

Resulting zero:

$$s = -\frac{1}{2RC} = 2 \cdot s_{pole} = \boxed{-44 \text{ krad/s}}$$

10.7 a) S- domain circuits w/ zero initial condition



$$Z_{eq} = \left(Ls + \frac{1}{Cs}\right) \parallel R$$

$$= \frac{1}{\frac{1}{R} + \frac{1}{Ls + \frac{1}{Cs}}} = \frac{Ls + \frac{1}{Cs}}{\frac{L}{R}s + \frac{1}{RCs} + 1}$$

$$= \frac{Ls^2 + \frac{1}{C}}{\frac{L}{R}s^2 + s + \frac{1}{RC}} = \boxed{R\,\frac{s^2 + \frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}}$$

$$\boxed{\begin{array}{l} \text{zeros}: \ s = \pm\frac{1}{\sqrt{LC}}\,j \\ \text{poles}: \ s = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC} \end{array}}$$

b) zeros : $s = \pm \frac{1}{\sqrt{LC}} j$ w/ $C = 0.1\,\mu F = 10^{-7} F$

$10j$ krad/s $= \frac{1}{\sqrt{L(0.1\mu F)}}$ $\Rightarrow$ $\boxed{L = 0.1\ H}$

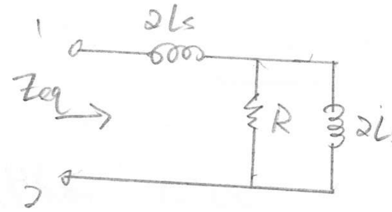
c) pole : $s = \frac{-RC \pm \sqrt{R^2C^2 - 4LC}}{2LC}$ w/ $R = 10^3\,\Omega$, $C = 10^{-7} F$, $L = 0.1\ H$

$$S = \frac{-(10^3\Omega)(10^{-7}F) \pm \sqrt{(10^3\Omega)^2(10^{-7}F)^2 - 4(0.1H)(10^{-7}F)}}{2 \cdot (0.1H)(10^{-7}F)}$$

$= -5000 \pm 8660j$ rad/s $= \boxed{-5 \pm 8.66j \text{ krad/s}}$

10.9

a) S-domain circuits w/ zero initial condition ;



$Z_{eq}(s) = 2Ls + (R \| 2Ls)$

$= 2Ls + \frac{1}{\frac{1}{R} + \frac{1}{2Ls}} = 2Ls + \frac{2LRs}{2Ls + R} = \frac{2Ls(2Ls+R) + 2LRs}{2Ls + R}$

$= \frac{4L^2s^2 + 4LRs}{2Ls + R} = \boxed{2L \frac{s^2 + \frac{R}{L}s}{s + \frac{R}{2L}}}$

pole : $s = -\frac{R}{2L}$, zeros : $s = 0, -\frac{R}{L}$

b) pole : $s = -\frac{R}{2L} = -15$ krad/s $= -15\,000$ rad/s

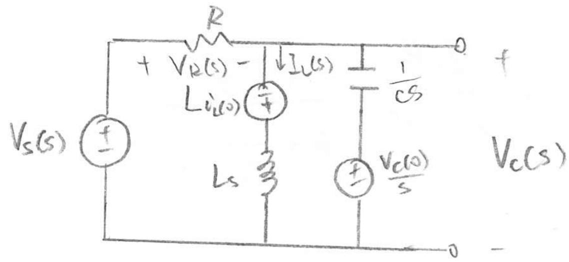
$\frac{R}{L} = 3 \times 10^4$ rad/s.

Let $\boxed{L = 0.1\ H}$. then $\boxed{R = 3\ k\Omega}$

zeros : $\boxed{s_1 = 0}$. $\boxed{s_2 = -\frac{R}{L} = -30 \text{ krad/s}}$

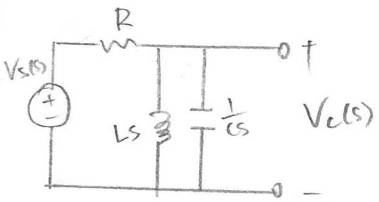
10.36.

S - domain circuits :



$V_c(0) = 0, \quad i_L(0) = I_0$

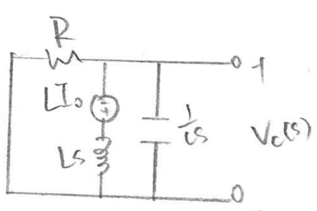
Zero state (no initial condition) :



$$V_c^{zs}(s) = V_s(s) \frac{Ls \parallel \frac{1}{Cs}}{R + Ls \parallel \frac{1}{Cs}} = V_s(s) \frac{\frac{1}{\frac{1}{Ls} + Cs}}{R + \frac{1}{\frac{1}{Ls} + Cs}}$$

$$= \frac{V_s(s)}{\frac{R}{L}\frac{1}{s} + CRs + 1} = \frac{s V_s(s)}{RCs^2 + s + \frac{R}{L}} = \boxed{\frac{1}{RC} \frac{s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}} V_s(s)}$$

Zero input (no input $V_s(s)$)

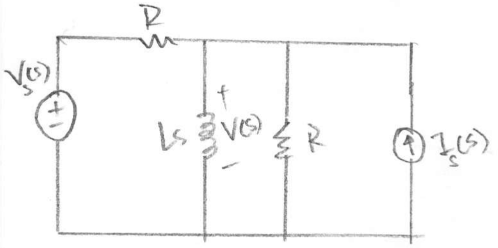


$$V_c^{zi}(s) = (-LI_0) \frac{R \parallel \frac{1}{Cs}}{Ls + R \parallel \frac{1}{Cs}} = (-LI_0) \frac{\frac{1}{\frac{1}{R} + Cs}}{Ls + \frac{1}{\frac{1}{R} + Cs}}$$

$$= -LI_0 \frac{1}{\frac{L}{R}s + LCs^2 + 1} = \boxed{-\frac{I_0}{C} \frac{1}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}}$$

10.47

S - domain circuit (no initial energy)

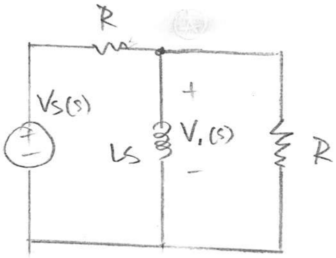


$$V_s(s) = \mathcal{L}\{V_A \sin \beta t\} = \frac{V_A \beta}{s^2 + \beta^2}$$

$$I_s(s) = \mathcal{L}\{I_B u(t)\} = \frac{I_B}{s}$$

Superposition :

① Zero out $I_s(s)$, analyze effect from $V_s(s)$ on $V_1(s)$

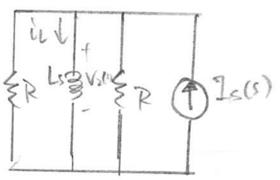


Voltage division

$$V_1(s) = V_s(s)\,\frac{Ls\|R}{R+Ls\|R} = \frac{V_A\beta}{s^2+\beta^2}\;\frac{\frac{1}{\frac{1}{Ls}+\frac{1}{R}}}{R+\frac{1}{\frac{1}{Ls}+\frac{1}{R}}}$$

$$= \frac{V_A\beta}{s^2+\beta^2}\cdot\frac{\frac{1}{2}s}{\frac{R}{2L}+s}$$

② Zero out $V_s(s)$ analyze effect from $I_s(s)$ on $V_2(s)$



Current division

$$i_L(s) = \frac{\frac{1}{Ls}}{\frac{1}{R}+\frac{1}{Ls}+\frac{1}{R}}\cdot I_s(s) = \frac{R}{2Ls+R}\,\frac{I_B}{s}$$

$$V_2(s) = Ls\cdot i_L(s) = \frac{LRI_s}{2Ls+R} = \frac{\frac{RI_s}{2}}{s+\frac{R}{2L}}$$

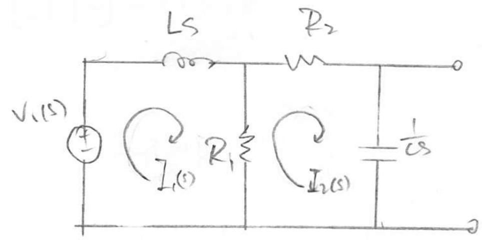
$$V_s(s) = V_1(s)+V_2(s) = \frac{V_A\beta}{s^2+\beta^2}\cdot\frac{\frac{1}{2}s}{\frac{R}{2L}+s} + \frac{\frac{RI_s}{2}}{s+\frac{R}{2L}} = \frac{1}{s+\frac{R}{2L}}\left(V_A\beta\frac{s/2}{s^2+\beta^2}+\frac{RI_s}{2}\right)$$

$$= \boxed{\frac{\frac{RI_s}{2}s^2+\frac{V_A\beta s}{2}+\frac{RI_s\beta^2}{2}}{\left(s+\frac{R}{2L}\right)\left(s^2+\beta^2\right)}}$$

natural pole : $-\frac{R}{2L}$, forced pole : $\pm\beta j$

10.51

a) s-domain w/ no initial condition



KVL at $\circlearrowright_{i_1(s)}$

$$\boxed{-V_1(s) + i_1\cdot Ls + R_1(i_1-i_2) = 0} \quad ①$$

KVL at $\circlearrowright_{i_2(s)}$

$$\boxed{R_2 i_2 + \frac{1}{Cs} i_2 + R_1(i_2-i_1) = 0} \quad ②$$

b) from ① $\rightarrow$ $i_1(s) = \dfrac{V_1(s)+R_1 i_2(s)}{Ls+R_1}$ plug into ②

$$\left(R_2+\frac{1}{Cs}+R_1\right) i_2 = \frac{R_1}{Ls+R_1}V_1(s) + \frac{R_1^2}{Ls+R_1}\, i_2(s)$$

$$i_2(s) = \frac{R_1}{(Ls+R_1)(R_2+R_1+\frac{1}{Cs})-R_1^2} V_1(s)$$

$$= \frac{R_1}{LR_2 s + LR_1 s + \frac{L}{C} + R_1R_2 + \frac{R_1}{Cs}} = \frac{R_1 Cs\, V(s)}{LC(R_1+R_2)s^2+(R_1R_2C+L)s+R_1}$$

c) pole: $s = \dfrac{-(R_1R_2C+L)\pm\sqrt{(R_1R_2C+L)^2-4LCR_1(R_1+R_2)}}{2LC(R_1+R_2)}$

zero: $s=0$

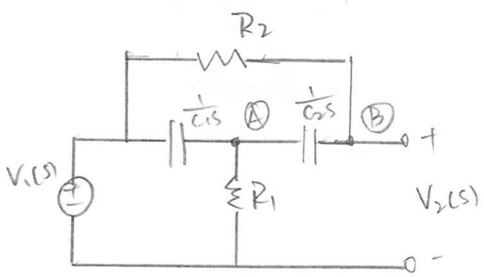
d) $V_1(s) = \mathcal{L}\{v_1(t)\} = \dfrac{20}{s}$

$$i_2(s) = \frac{10^{-3}s}{1.5\times10^{-3}s^2+2s+2\times10^3}\cdot\frac{20}{s} = \frac{13.33}{s^2+1.33\times10^3 s+1.33\times10^6} = \frac{13.33}{(s+666.66)^2+941.03^2}$$

$$i_2(t) = \mathcal{L}^{-1}\{i_2(s)\} = \boxed{0.014\, e^{-666.66t}\sin(941.03t)\, u(t)}$$

10.56

a) s domain circuits w/ no initial energy



KCL at A:

$$\boxed{\frac{V_A-V_1}{\frac{1}{C_1 s}} + \frac{V_A-V_B}{\frac{1}{C_2 s}} + \frac{V_A}{R_1} = 0} \quad ①$$

KCL at B:

$$\boxed{\frac{V_B-V_1}{R_2} + \frac{V_B-V_A}{\frac{1}{C_2 s}} = 0} \quad ②$$

$V_2(s) = V_B(s)$

b) from ② $V_A(s) = \frac{1}{C_2 s}\left(\frac{V_B - V_1}{R_2} + \frac{V_B}{\frac{1}{C_2 s}}\right)$ and plug into ①

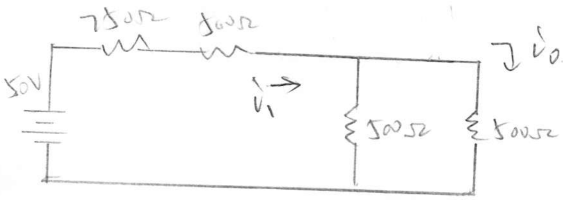
$$\frac{1}{C_2 s}\left(\left(\frac{1}{R_2}+C_2 s\right)V_2 - \frac{1}{R_2}V_1\right)\left(C_1 s + C_2 s + \frac{1}{R_1}\right) = C_1 s V_1 + C_2 s V_2$$

$$\left[\frac{(\frac{1}{R_2}+C_2 s)(C_1 s + C_2 s + \frac{1}{R_1})}{C_2 s} - C_2 s\right] V_2 = \left[C_1 s + \frac{C_1 s + C_2 s + \frac{1}{R_1}}{R_2 C_2 s}\right] V_1$$

$$\boxed{\frac{V_2(s)}{V_1(s)} = \frac{R_1C_1R_2C_2 s^2 + (R_1C_1 + R_1C_2)s + 1}{R_1C_1R_2C_2 s^2 + (R_1C_1+R_1C_2+R_2C_2)s+1}}$$

10.70

Initial condition as $t \to 0^-$

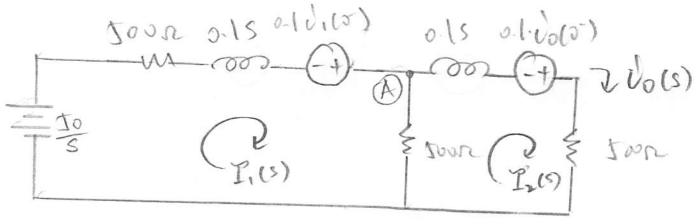


$$i_1(0^-) = \frac{50\ V}{750Ω + 500Ω + (500 \| 500)} = \frac{50\ V}{1500\ Ω} = \frac{1}{30}\ A$$

$$= 0.0333\ A$$

$$i_o(0^-) = \frac{1}{2} i_1 = 0.0167\ A = \frac{1}{60}\ A$$

Then, close the switch and transform into s-domain



Mesh Current :

I_1 : $-\frac{50}{s} + (500 + 0.1s) I_1(s) - 0.1 i_1(0^-) + 500(I_1 - I_2) = 0$

I_2 : $(0.1s + 500) I_2 - 0.1 i_o(0^-) + 500(I_2 - I_1) = 0$ w/ $I_2(s) = i_o(s)$

Solve for I_2 :

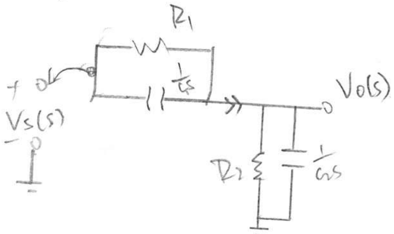
$$i_o(s) = I_2(s) = \frac{\frac{\frac{1}{600}}{0.1s+1000} + \frac{\frac{500}{300} + \frac{25000}{s}}{(0.1s+1000)^2}}{1 - \frac{500^2}{(0.1s+1000)^2}} = \frac{s^2 + 20000s + 25000 \times 6000}{6000\, s\left(\frac{1}{100}s^2 + 200s + 10^6 - 500^2\right)}$$

$$i_o(s) = \frac{-\frac{1}{40}}{s+5000} + \frac{\frac{1}{30}}{s} + \frac{1/120}{s+15000}$$

$$i_o(t) = \mathcal{L}^{-1}\{i_o(s)\} = \left(-\frac{1}{40}e^{-5000t} + \frac{1}{30} + \frac{1}{120}e^{-15000t}\right) u(t)$$

10.79

a) Assume zero initial condition, s-domain circuits



Voltage division

$$V_o(s) = \frac{R_2 \| \frac{1}{C_2 s}}{(R_1 \| \frac{1}{C_1 s}) + (R_2 \| \frac{1}{C_2 s})} V_s(s)$$

$$\boxed{V_o(s) = \frac{R_1 R_2 C_1 s + R_2}{R_1 R_2 (C_1 + C_2) s + R_1 + R_2} V_s(s)}$$

b) let $\frac{V_o}{V_s} = \frac{1}{2}$, then, w/ $R_2 = 10\,M\Omega$, $C_2 = 5\,pF$

$$2R_1R_2C_1 s + 2R_2 = R_1R_2(C_1+C_2)s + (R_1+R_2)$$

$$\Rightarrow \begin{cases} 2R_1R_2C_1 = R_1R_2(C_1+C_2) \\ R_1 + R_2 = 2R_2 \end{cases} \Rightarrow \begin{cases} R_1 = 10\,M\Omega \\ C_1 = 5\,pF \end{cases}$$

10.83

The first op-Amp is a buffer to protect the source and the second op Amp act as inverting. thus, ① when $\alpha \in \mathbb{R}$

$$\frac{V_o(s)}{V_s(s)} = -\frac{R_2}{R_1 \| \frac{1}{Cs}} = -\frac{R_2}{\frac{1}{\frac{1}{R_1}+Cs}} = -CR_2\left(\frac{1}{R_1C}+s\right) \Rightarrow V_o(s) = -\left(s+\frac{1}{R_1C}\right)\cdot CR_2 \frac{K}{(s+\alpha)}$$

the pole $s = -\frac{1}{R_1C}$ got cancelled and then, $V_o(s) = -CR_2K = -\frac{1}{\alpha R_1}R_2K = -\frac{K}{\alpha}\frac{R_2}{R_1}$

$\frac{R_2}{R_1}$ could be used to adjust gain $\therefore$ claim is correct when α is real

② when $\alpha \in \mathbb{C}$.

There are no combination of $R_1, C \in \mathbb{R}$ such that $\frac{1}{R_1 C} \in \mathbb{C}$

$\therefore$ claim is not correct $\forall \alpha \in \mathbb{C}$

③ $\alpha = 0$.

$\frac{1}{R_1 C} = 0$ either $R_1 \to \infty$, or $C \to \infty$ would do.