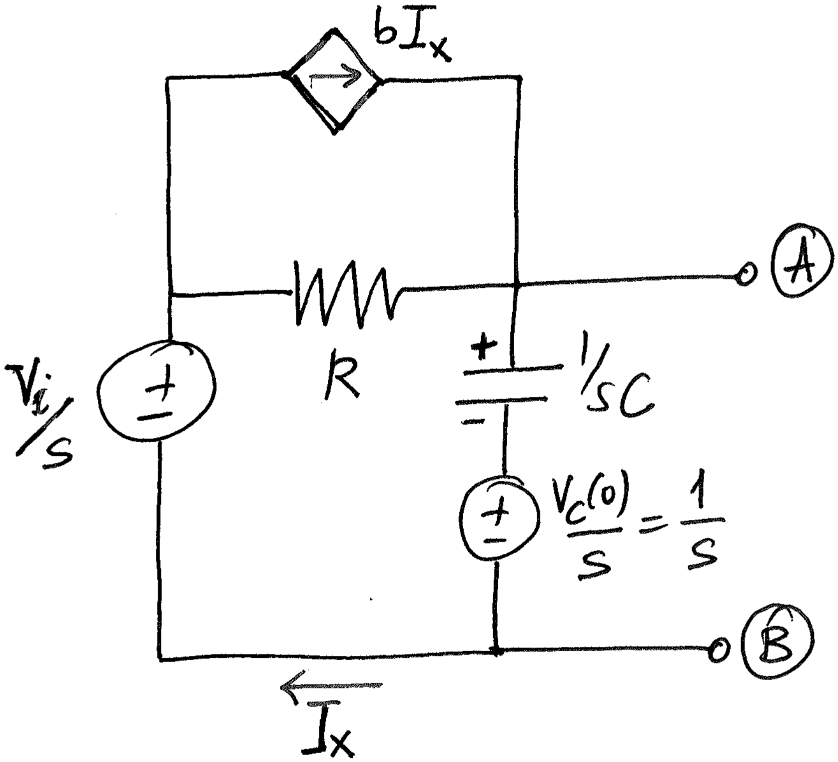
1.– Part I

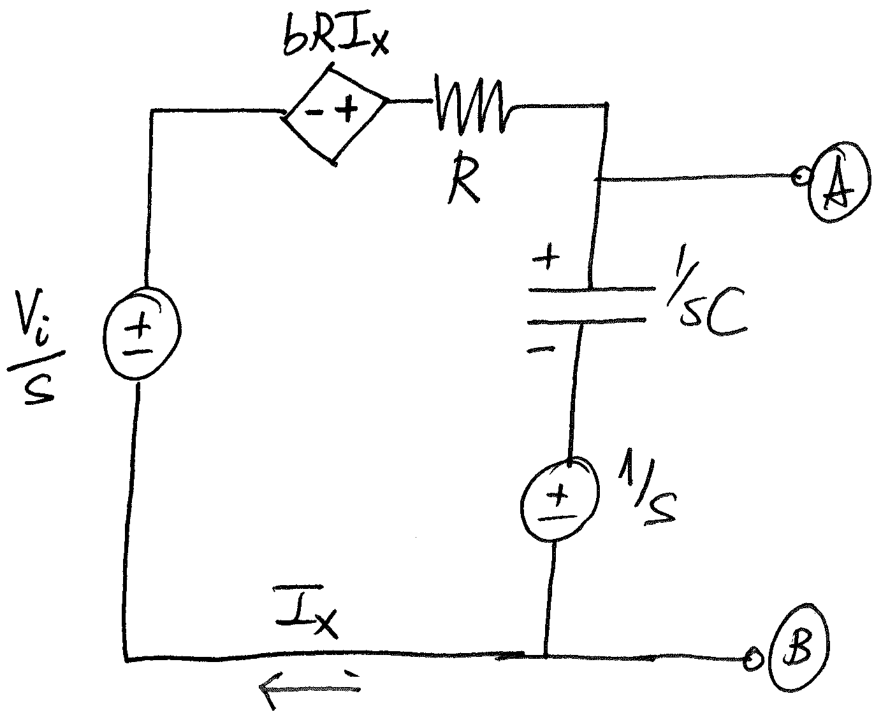


[+1 point for correct initial condition]

[+1 point for correct overall circuit]

Part II

We transform the current source in parallel w/ the resistor to obtain



We use voltage division to get the voltage drop across the impedance $1/Cs$

$$V_{1/Cs} = \frac{1/sC}{1/sC + R} \cdot \left(\frac{V_i}{s} + bRI_x - \frac{1}{s} \right)$$

Additionally,

$$I_x = \frac{1}{\frac{1}{sC} + R} \cdot \left(\frac{V_i}{s} + bRI_x - \frac{1}{s} \right) =$$

Therefore,
$$= \frac{C\cancel{s}}{1+RCs} \frac{1}{\cancel{s}} \left(V_i - 1 + bRI_x s \right)$$

[+1 point]

$$I_x \underbrace{\left(1 - \frac{C}{1+RCs} bRs \right)}_{\frac{1+RCs-bRCs}{1+RCs} = \frac{1+(1-b)RCs}{1+RCs}} = \frac{C}{1+RCs} (V_i - 1)$$

$$I_x = \frac{C}{1+(1-b)RCs} (V_i - 1)$$

[+1 point]

Therefore,

$$V_{1/Cs} = \frac{1}{1+RCs}\left(\frac{1}{s}(V_i-1) + \frac{bRC}{1+(1-b)RCs}(V_i-1)\right)$$

$$= \frac{1}{1+RCs}\,\frac{(V_i-1)}{s}\left(1 + \frac{bRC}{\frac{1}{s}+(1-b)RC}\right)$$

$$= \frac{1}{1+RCs}\,\frac{V_i-1}{\cancel{s}}\left(\frac{1}{s} + (1-\cancel{b})RC + \cancel{b}RC\right)\frac{\cancel{s}}{1+(1-b)RCs}$$

$$= \frac{1}{\cancel{1+RCs}}(V_i-1)\,\frac{1}{1+(1-b)RCs}\,\cancel{\left(\frac{1+RCs}{s}\right)}$$

$$= \frac{1}{1+(1-b)RCs}\,\frac{1}{s}\,(V_i-1)$$ [+1 point]

Finally,

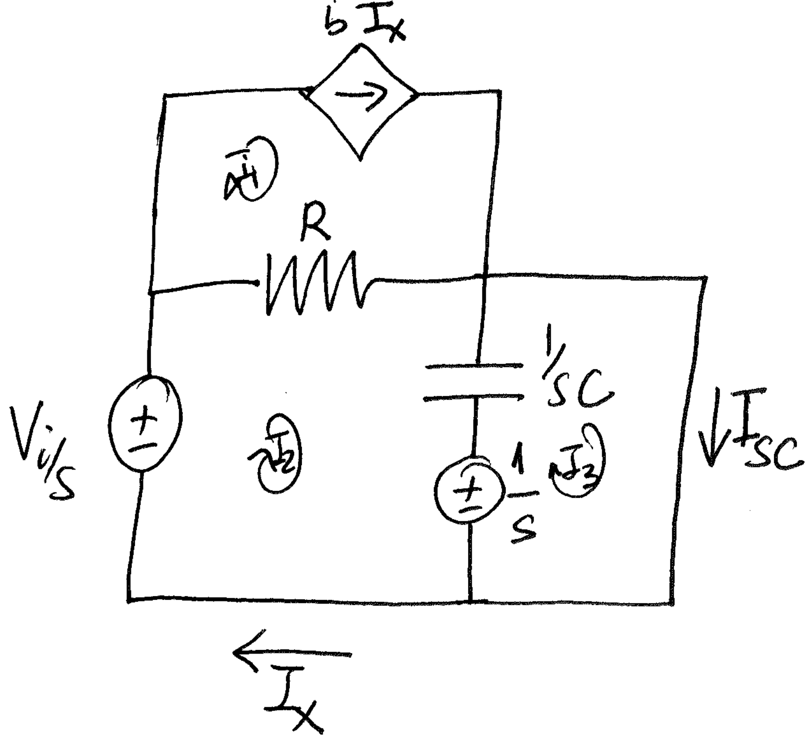
$$V_{AB} = V_{1/sC} + \frac{1}{s} = \frac{V_i + (1-b)RCs}{s(1+(1-b)RCs)}.$$

[+1 point]

Part III

Because we have dependent sources, we cannot turn off sources // use the lookback method. [+1 point]

Instead, we short-circuit the circuit and find i_{sc}.



[+1 point]

We use mesh current analysis;

method 2: $I_1(s) = bI_x(s)$

KVL @ mesh 2: $R(I_2(s) - I_1(s)) + \frac{1}{SC}(I_2(s) - I_3(s)) + \frac{1}{S} - \frac{V_i}{S} = 0$

KVL @ mesh 3: $\frac{1}{SC}(I_3(s) - I_2(s)) - \frac{1}{S} = 0$

Additionally, $I_2(s) = I_x(s)$ & $I_{SC}(s) = I_3(s)$

Solving for $I_1(s)$, $I_2(s)$, $I_3(s)$, $I_x(s)$, we obtain

$$I_{SC}(s) = I_3(s) = \frac{CRs(1-b) + V_i}{(1-b)Rs} \qquad [+1 \text{ point}]$$

Therefore

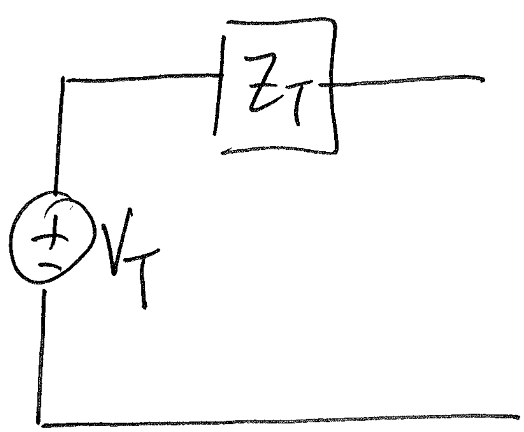
$$Z_T = \frac{V_{OT}(s)}{I_{SC}(s)} = \frac{\cancel{V_i + (1-b)RCs}}{s(1+(1-b)RCs)} \cdot \frac{(1-b)Rs}{\cancel{(1-b)RCs + V_i}}$$

$$= \frac{(1-b)Rs}{s(1+(1-b)RCs)} \qquad [+1 \text{ point}]$$

We also know

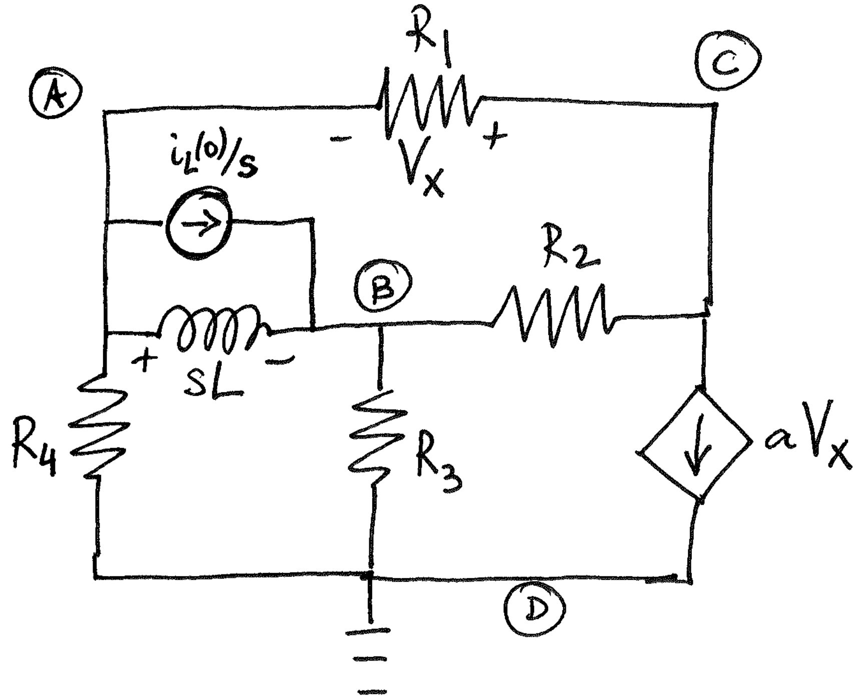
$V_T = V_{AB}$ from Part II

Hence



2.- Part I

Since we cannot assume zero initial conditions, we need to worry about the initial condition of the inductor. Since we are gonna use nodal analysis, we employ a current source to take care of it. The circuit in the s-domain looks like



[+1 point]
(1/2 correct drawing, 1/2 correct initial condition)

Node D is ground, so we don't write KCL for it.

KCL @ A

$$G_1(V_A - V_C) + G_4(V_A) + \frac{1}{sL}(V_A - V_B) = -\frac{i_L(0)}{s}$$ [+1 point]

KCL @ B

$$\frac{1}{sL}(V_B - V_A) + G_3 V_B + G_2(V_B - V_C) = \frac{i_L(0)}{s}$$ [+1 point]

KCL @ C

$$G_1(V_C - V_A) + G_2(V_C - V_B) = -aV_x \qquad \left(G_i = \frac{1}{R_i}\right)$$

[+1 point]

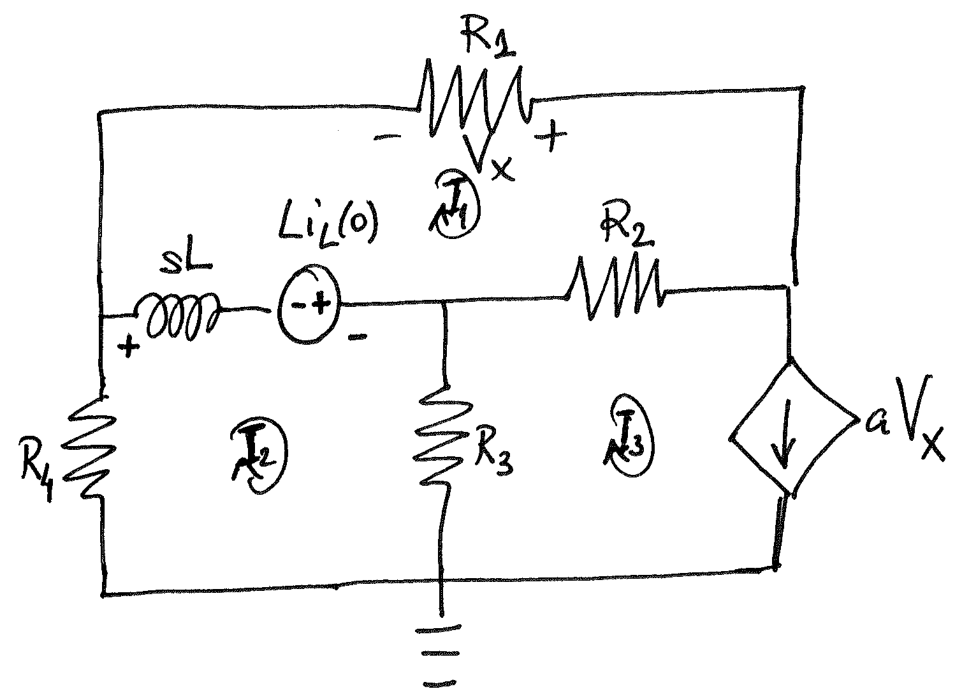
We need to properly account for the dependent source. Therefore

$$V_x = V_C - V_A \qquad [+1 \text{ point}]$$

So we have 4 eqs in 4 unknowns ($V_A(s), V_B(s), V_C(s), V_X(s)$).

Part II

As in Part I, we need to worry about the initial condition of the inductor, and we use now a voltage source for it (since we are gonna employ mesh current analysis). The circuit in the s-domain looks like



[+1 point]
(1/2 correct drawing, 1/2 correct initial condition)

We have a current source, which is a problem for mesh analysis. Fortunately, it just belongs to 1 mesh, so we employ method 2 to state

$$I_3(s) = a V_x(s) \qquad [+1 \text{ point}]$$

Next, we write KVL eqs for meshes 1 & 2.

KVL @ mesh 1,

$$R_1 I_1 + R_2(I_1 - I_3) + sL(I_1 - I_2) = -Li_L(0)$$

[+1 point]

KVL @ mesh 2,

$$sL(I_2 - I_1) + R_3(I_2 - I_3) + R_4 I_2 = Li_L(0)$$

[+1 point]

Finally, we need to account for the dependent source,

$$V_x(s) = -R_1 I_1(s)$$

[+1 point]

This gives us a total of 4 eqs in 4 unknowns $(I_1(s), I_2(s), I_3(s), V_x(s))$.

Part III

With unknowns of Part I, we have

$$V_L(s) = V_A(s) - V_B(s)$$

[+1 bonus point]

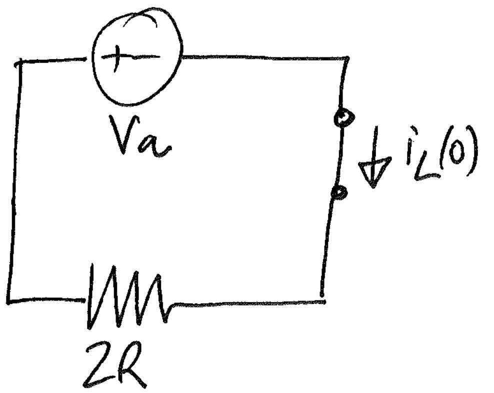
With unknowns of Part II, we have

$$V_L(s) = sL(I_2(s) - I_1(s)) - Li_L(0)$$

[+1 bonus point]

3.- Part I

Under DC excitation, the inductor behaves as a short circuit, hence

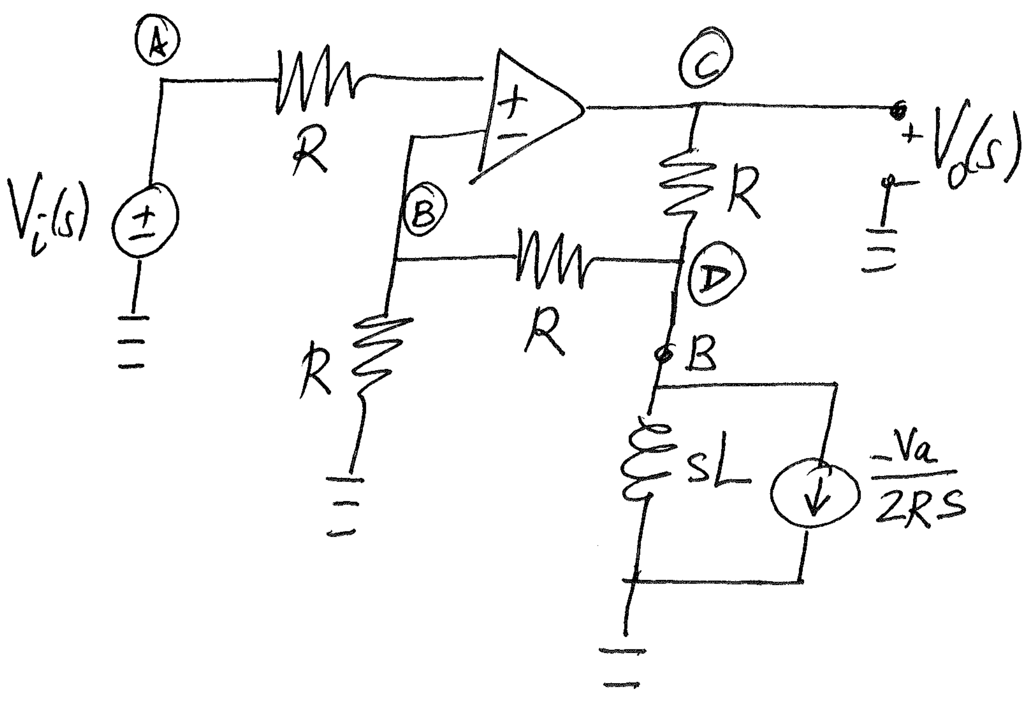


[+1 point]

By Ohm's law, $i_L(0) = -\frac{V_a}{2R}$. [+1 point]

Part II

Switch is at (B), hence



[+1 point]

As instructed, we use nodal analysis.

Method 2 tells us that $V_A = V_i(s)$. [+ 0.5 point]

Also, C is the output node for an Op-amp, so we don't write KCL for it.

KCL @ B

$$G V_B + G(V_B - V_D) = 0$$ [+0.5 point]

KCL @ D

$$G(V_D - V_C) + G(V_D - V_B) + \frac{1}{sL}(V_D - 0) = \frac{V_a}{2Rs}$$

[+0.5 point]

Because of ideal op-amp,

$$V_A(s) = V_B(s)$$ [+0.5 point]

Solving for V_A, V_B, V_C, V_D, we obtain

$$V_o(s) = V_C(s) = + \frac{-L V_a + 4R V_i(s) + 6Ls V_i(s)}{2Ls}$$

$$= \frac{2R + 3Ls}{Ls} V_i(s) - \frac{V_a}{2s}$$ [+1 point]

Part III

With the values given, we have $V_i(s) = \dfrac{1}{s+10}$

and then

$$V_o(s) = \frac{20+0.3s}{0.1s}\,\frac{1}{s+10} - \frac{1}{2s} =$$

$$= \frac{200+3s}{s(s+10)} - \frac{1}{2s}$$

Using partial fractions

$$V_o(s) = \frac{A}{s} + \frac{B}{s+10}$$ [+1 point]

After some computations,

$$A = 19.5$$
$$B = -17$$ [+1 point]

Using the inverse Laplace transform,

$$V_o(t) = (19.5 - 17e^{-10t})\,u(t)$$

Part IV

(i) forced pole is at $s=-10$. Hence

$$v_o(t) = \underbrace{19.5\,u(t)}_{\text{natural}} - \underbrace{17e^{-10t}u(t)}_{\text{forced}}$$ [+1 point]

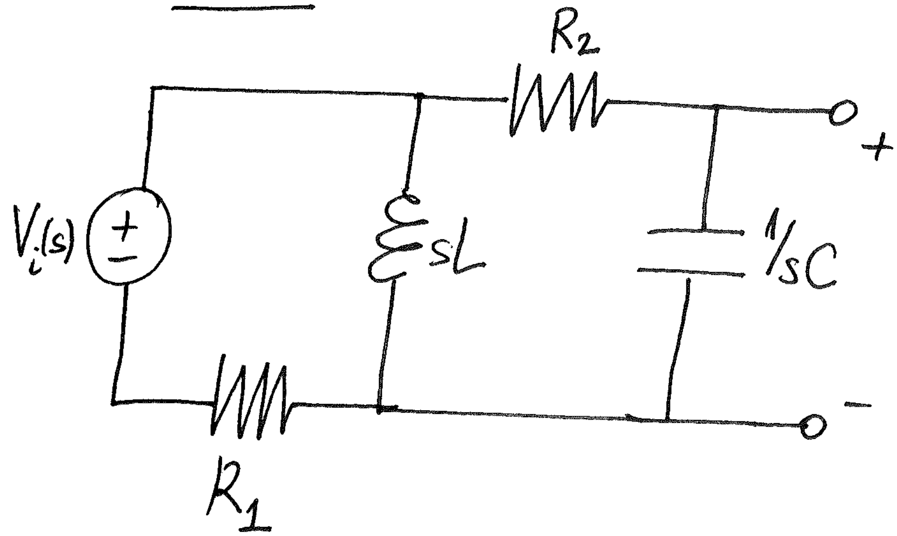
(ii) zero-state,

$$V_{zs}(s) = \frac{200+3s}{s(s+10)} \quad ; \quad v_{zs}(t) = 20u(t) - 17e^{-10t}u(t)$$ [+0.5 point]

zero-input

$$V_{zi}(s) = -\frac{1}{2s} \quad ; \quad v_{zi}(t) = -0.5\,u(t)$$ [+0.5 point]

4.- Part I



[+1 point]

(zero initial conditions means we do not need to add sources for the capacitor or the inductor)

Part II

To show this, we can use a number of techniques (source transformation, nodal analysis, ...). We settle for mesh analysis, labelling meshes as 1 & 2 from left to right.

$$sL\left(I_1(s)-I_2(s)\right)+R_1 I_1(s)=V_i(s) \qquad \text{[+1 point]}$$

$$\frac{1}{sC}I_2(s)+R_2 I_2(s)+sL\left(I_2(s)-I_1(s)\right)=0 \qquad \text{[+1 point]}$$

We solve for $I_1(s)$ in the second equation,

$$(sL)\,I_1(s)=\left(R_2+sL+\frac{1}{sC}\right)I_2(s)$$

$$I_1(s)=\frac{R_2+sL+\frac{1}{sC}}{sL}\,I_2(s)$$

Substituting into the first equation,

$$-sL\,I_2(s) + (R_1+sL)\,\frac{R_2+sL+\frac{1}{sC}}{sL}\,I_2(s) = V_i(s)$$

$$I_2(s) = \frac{sL}{-s^2L^2 + (R_1+sL)\left(R_2+sL+\frac{1}{sC}\right)}\,V_i(s)$$

Therefore,

[+1 point]

$$V_o(s) = \frac{1}{sC}\,I_2(s) = \frac{L/C}{-s^2L^2+(R_1+sL)\left(R_2+sL+\frac{1}{sC}\right)}\,V_i(s) =$$

$$= \frac{L/\cancel{C}\cdot s\cancel{C}}{R_1R_2Cs + R_1LCs^2 + R_1 + R_2LCs^2 + Ls}\,V_i(s) =$$

$$= \frac{sL}{(R_1+R_2)LCs^2 + (R_1R_2C+L)s + R_1}\,V_i(s) \quad \#$$

Part III

With the values given, the transfer function takes the form

$$T(s) = \frac{0.1\,s}{123.85 + 0.866941 s + 0.00123855 s^2} \approx$$

$$\approx \frac{80.75\,s}{s^2 + 700 s + 10^5} = \frac{80.75\,s}{(s+200)(s+500)}$$

Then,

$$T(j\omega) = \frac{80.75\, j\omega}{-\omega^2 + 700 j\omega + 10^5} = \frac{80.75\omega \cdot j}{(10^5 - \omega^2) + 700\omega j}$$

The gain function is

$$|T(j\omega)| = \frac{80.75\omega}{\sqrt{(10^5-\omega^2)^2 + 700^2\omega^2}} \qquad [+0.5 \text{ point}]$$

The phase function is

$$\measuredangle T(j\omega) = \frac{\pi}{2} - \arctan \frac{700\omega}{10^5 - \omega^2} \qquad [+0.5 \text{ point}]$$

The DC gain is

$$|T(j0)| = 0 \qquad [+0.5\ \text{point}]$$

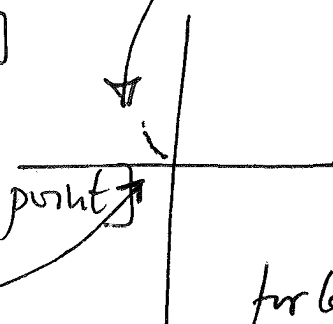
The ∞-freq gain is

$$|T(j\infty)| = 0 \qquad [+0.5\ \text{point}]$$

The corresponding values of the phase function are

$$\angle T(j0) = \frac{\pi}{2} \text{ rad} \qquad [+0.5\ \text{point}]$$

$$\angle T(j\infty) = \frac{\pi}{2} - \pi = -\frac{\pi}{2} \text{ rad} \qquad [+0.5\ \text{point}]$$



for large w, $(10^5 - w^2, 700w)$ sits in this gradant

To figure out the cut-off fregrencies and determine the type of filter (and given the DC and ∞-freq gains found above, which makes us think about a bandpass filter), we observe that

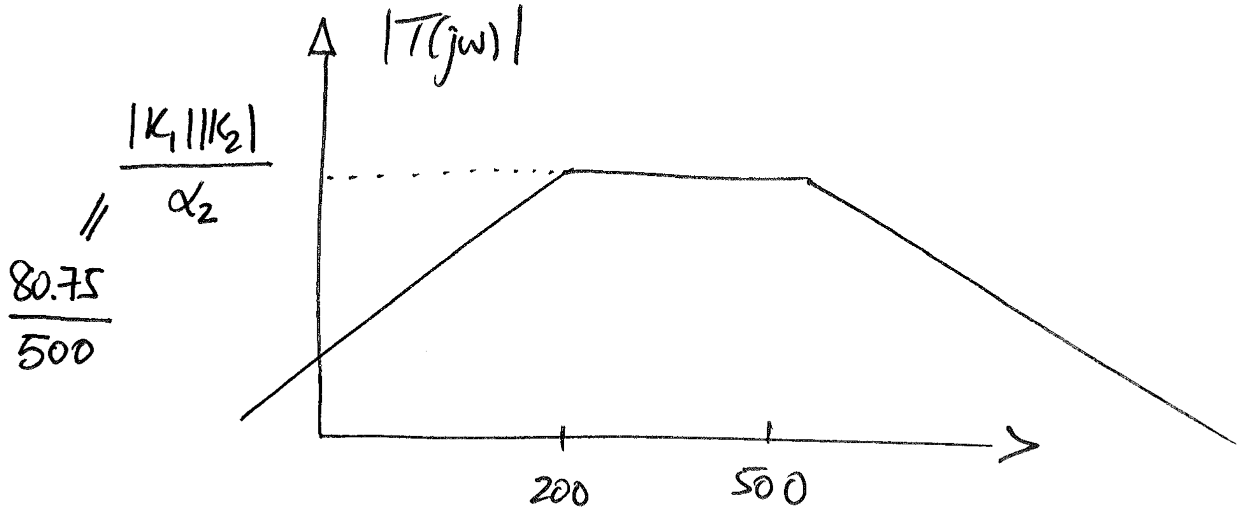
$$T(s) = \boxed{\frac{80.75\,s}{s+200}} \cdot \boxed{\frac{1}{s+500}}$$

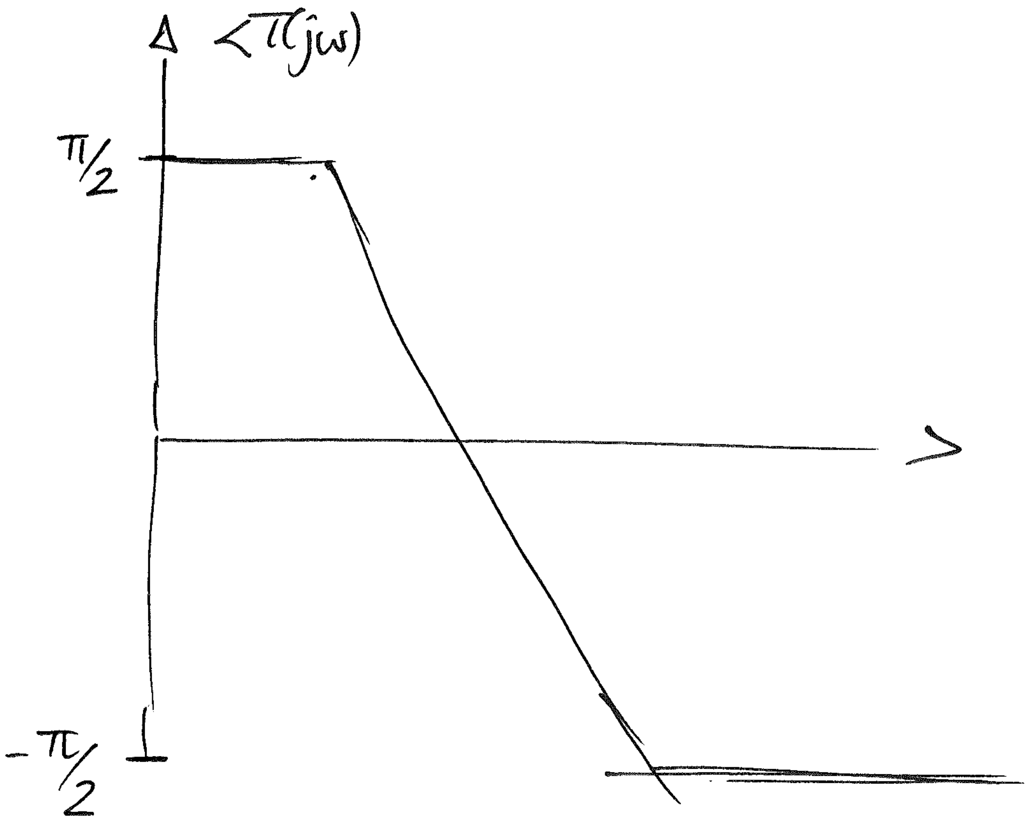
high-pass filter   low-pass filter

Cutoff freqs. are $\alpha_1 = 200$ rad/s and $\alpha_2 = 500$ rad/s

$$(K_1 = 80.75\ ;\ K_2 = 1) \qquad [+1\ \text{point}]$$

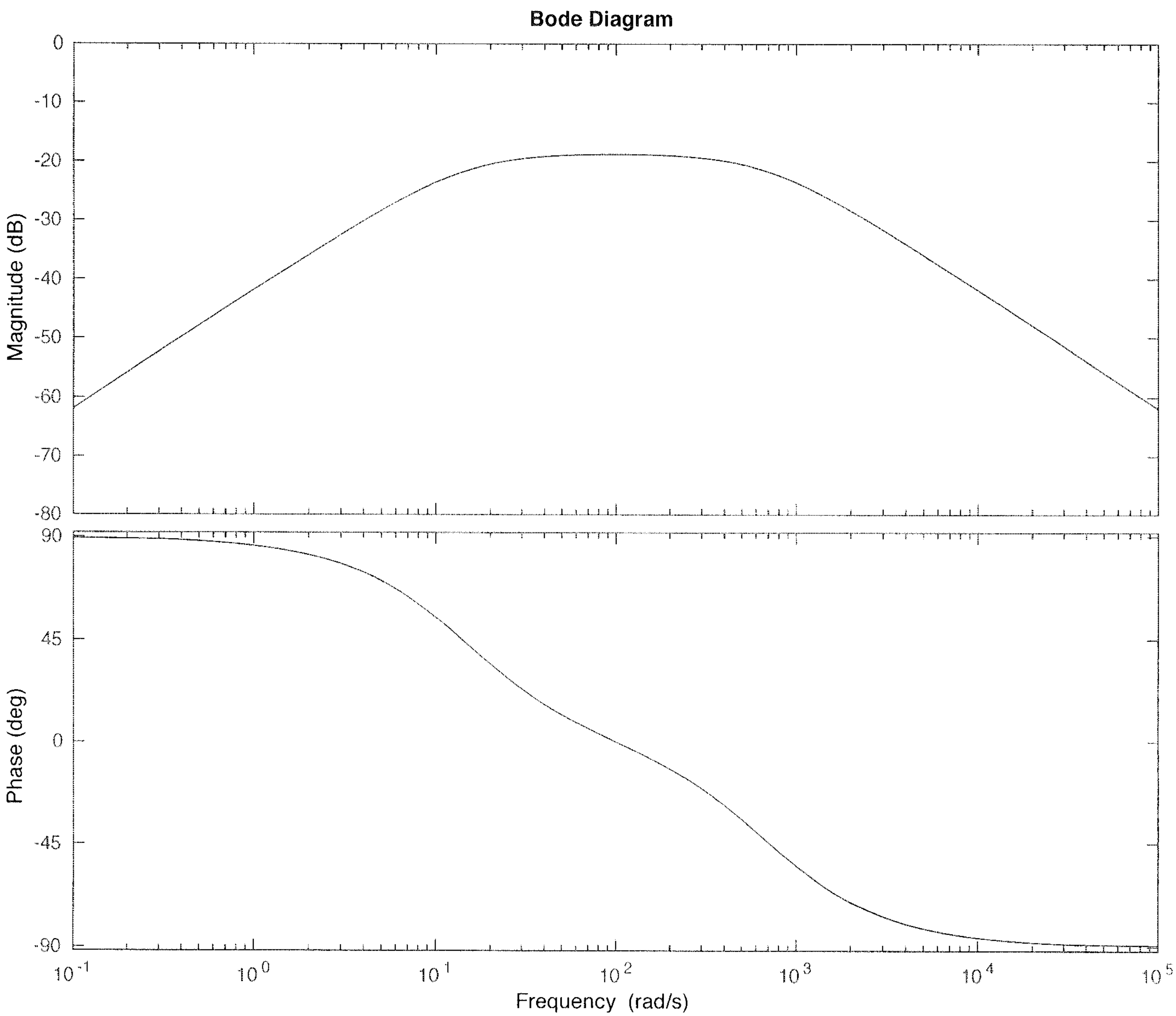
Here is a sketch, based on what we've learned in class





[+1 point]

This is a bandpass filter.



Bode Diagram
Magnitude (dB)
0
-10
-20
-30
-40
-50
-60
-70
-80
Phase (deg)
90
45
0
-45
-90
10 -1
10 0
10 1
10 2
10 3
10 4
10 5
Frequency (rad/s)

Part IV

We know that

$$V_o^{SS}(t) = |T(j300)| \cdot \cos\left(300t + \frac{\pi}{4} + \angle T(j300)\right)$$

[+0.5 point]

Since

$$|T(j300)| = 0.115277$$

$$\angle T(j300) = 0.04758 \text{ rad}$$

[+0.5 point]

Therefore,

$$V_o^{SS}(t) = 0.115277 \cos\left(300t + \frac{\pi}{4} + 0.04758\right).$$

5.- Part I

We set

$$\frac{s^2+201s+500}{s^2+700s+10^5} = \frac{k_1 s}{s+\alpha} + \frac{k_2}{s+\beta}$$

Note that $s^2+700s+10^5 = (s+200)(s+500)$ [+1 point]

Therefore we set $\alpha = 500$, $\beta = 200$, and we have [+1 point]

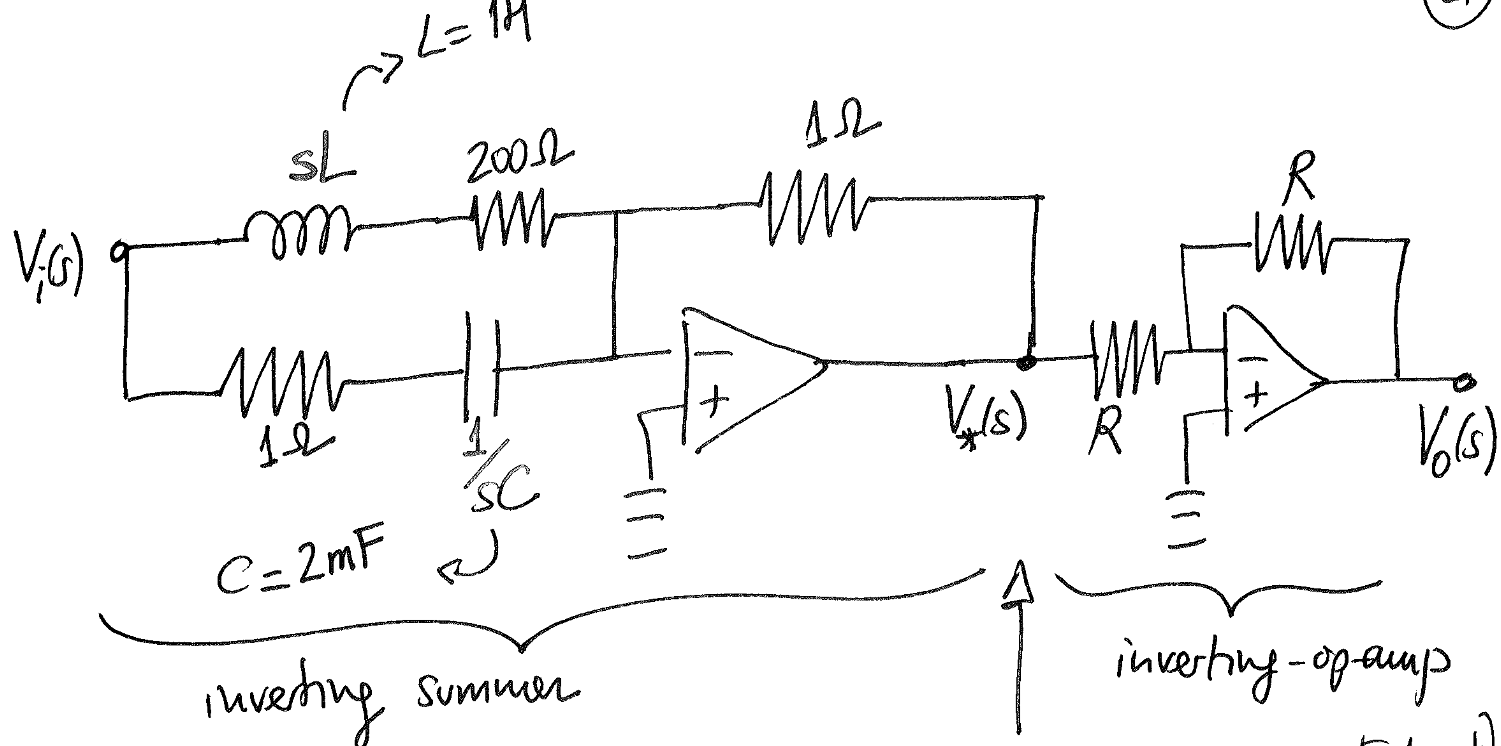
$$\frac{K_1 s}{s+500} + \frac{k_2}{s+200} = \frac{K_1 s^2 + 200k_1 s + k_2 s + 500k_2}{(s+500)(s+200)}$$

Therefore,

$$\left.\begin{array}{l} k_1 = 1 \\ 201 = 200k_1 + k_2 \\ 500 = 500k_2 \end{array}\right] \Rightarrow \begin{array}{l} k_1 = 1 \\ k_2 = 1 \end{array}$$ [+1 point]

Part II

There are many possible designs based on the decomposition of Part I.
One could do for instance an inverting summer followed by a gain of -1. [+1 point]



no loading because of 0 output impedance

[+1 point]

$$\frac{V_*(s)}{V_i(s)} = -\frac{1}{s+200} - \frac{1}{1+\frac{1}{s\,2\cdot 10^{-3}}} = -\left(\frac{1}{s+200} + \frac{s}{s+\frac{1}{2\cdot 10^{-3}}}\right)$$

[+1 point]

$$\frac{V_0(s)}{V_*(s)} = -1$$

[+1 point]

$$\frac{V_0(s)}{V_i(s)} = -\left(\frac{1}{s+200} + \frac{s}{s+500}\right)\cdot(-1) =$$

$$= \frac{1}{s+200} + \frac{s}{s+500}$$

[+1 point]

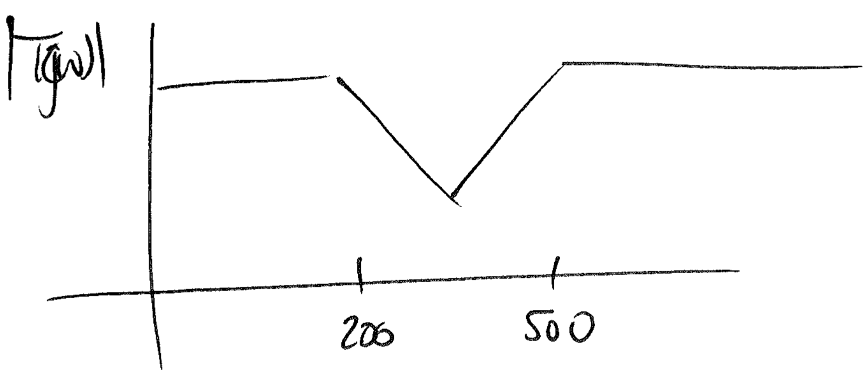
Part III

From what we have seen in class, we know this is a bandstop filter

$$\underbrace{\frac{s}{s+500}}_{\text{high pass}} + \underbrace{\frac{1}{s+200}}_{\text{low-pass}}$$

[+2 points]

$200 < 500$



6. — Part I

Both are voltage dividers, so

$$T_1(s) = \frac{sL}{R_1 + sL}$$ [+1 point]

$$T_2(s) = \frac{1/sC}{\frac{1}{sC} + R_2} = \frac{1}{1 + R_2 C s}$$ [+1 point]

Part II

$$\tilde{T}(s) = T_1(s)\,T_2(s) = \frac{sL}{R_1 + sL} \cdot \frac{1}{1 + R_2 C s} = \frac{sL}{R_2 C L s^2 + (R_1 R_2 C + L)s + R_1}$$

[+1 point]

Part III

In fact, $T(s)$ & $\tilde{T}(s)$ are different (the coefficient of the s^2 term in the denominator differs).

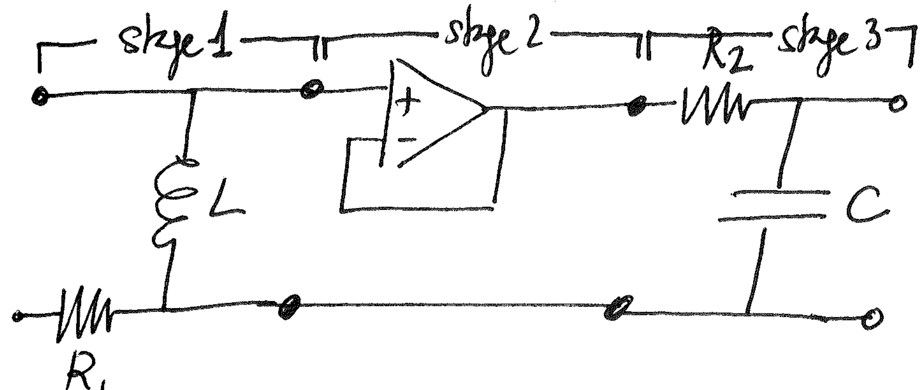
The answer in Q4, Part II is the correct one. [+1 point]

The reason is loading. The instructor incorrectly assumed that, when connecting the 2 stages in Figure 5, the transfer function was the product. [+1 point]

This is only the case if the second stage does not load the first one. When you put 2 voltage dividers in series, the second one loads the first one, invalidating the instructor's thinking. [+2 points]

Part IV

We could do it with a voltage follower, like this



[+1 point]

$$\underbrace{}_{T_1(s)} \cdot \underbrace{}_{1} \cdot \underbrace{}_{T_2(s)} = \tilde{T}(s)$$

In this circuit, there is no loading, since the voltage follower does not load the voltage divider (because of ∞ input impedance of op-amp) and the second voltage divider does not load the voltage follower (because of 0 output impedance of op-amp). [+1 point]

The order matters, meaning that

stage 1 & stage 3 can be interchanged

stage 2 <u>must</u> be in the middle.

[+1 point]