MAE140 - Linear Circuits - Winter 19
Final, March 19, 2019

Instructions

(i) The exam is open book. You may use your class notes and textbook. You may use a hand calculator with no communication capabilities.

(ii) You have 180 minutes

(iii) Do not forget to write your **name** and **student number**

(iv) On the questions for which the answers are given, please provide detailed derivations

(v) The exam has 6 questions for a total of 60 points and 2 bonus points

Good luck!

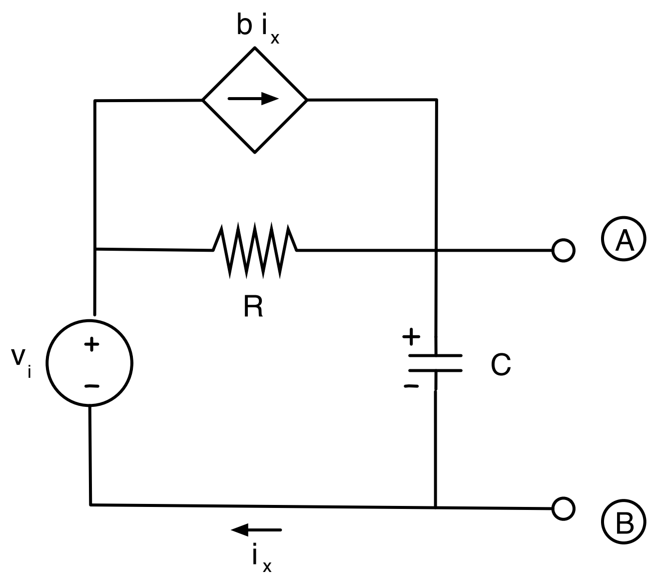

Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

   Here, v_i is a constant voltage source.

   Part I: [2 points] Assuming $v_C(0) = 1V$, transform the circuit in Figure 1 into the s-domain, using a voltage source to account for the initial condition of the capacitor.

   Part II: [4 points] For the circuit you obtained in Part I, find the open-circuit voltage transform as seen from terminals A-B. The answer should be given as a ratio of two polynomials.

   Part III: [4 points] For the circuit you obtained in Part I, find the Thévenin equivalent in the s-domain as seen from terminals A-B (the impedance should be given as a ratio of two polynomials).

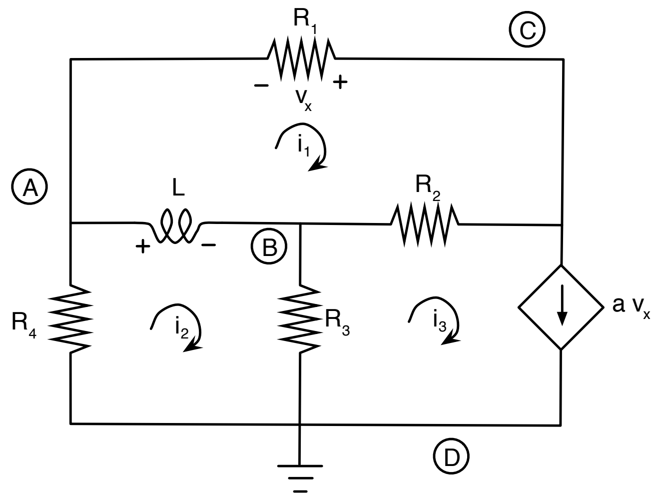

Figure 2: Nodal and Mesh Analysis Circuit for Question 2. a is a positive constant.

2. **Nodal and Mesh Analysis**

   Part I: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its node-voltage equations. Use the reference node and other labels as shown in the figure. Do not assume zero initial conditions. Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

   Part II: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its mesh-current equations. Use the mesh currents shown in the figure. Do not assume zero initial conditions. Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

   Part III: [2 bonus points] Express the transform of the inductor voltage as a function of your unknowns in Part I. Do the same as a function of your unknowns in Part II.

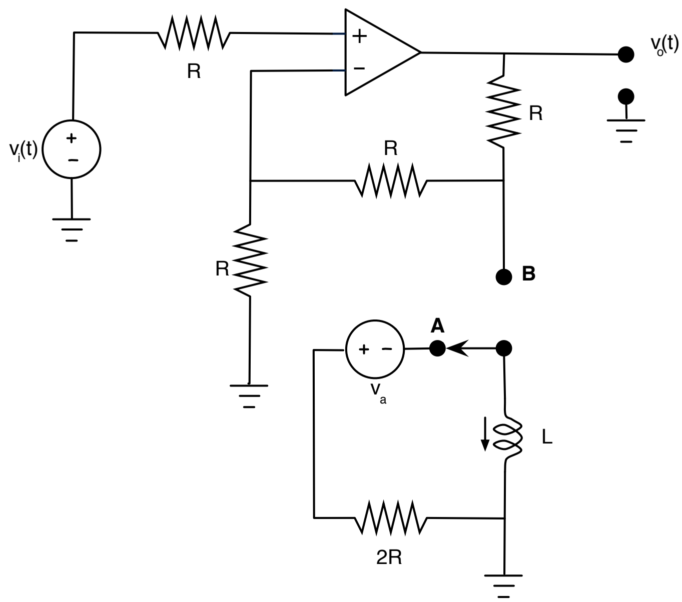

Figure 3: RCL circuit for Laplace Analysis for Question 3.

3. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 3. The value v_a of the voltage source is constant. The switch is kept in position **A** for a very long time. At $t = 0$, it is moved to position **B**. Show that the initial condition for the inductor is given by

$$i_L(0^-) = -\frac{v_a}{2R}.$$

[Show your work]

Part II: [4 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the inductor in which the initial condition appears as a current source. Use nodal analysis to express the output response transform $V_o(s)$ as a function of $V_i(s)$ and v_a.

Part III: [2 points] Use partial fractions and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $v_A = 1\,\mathrm{V}$, $v_i(t) = e^{-10t}u(t)\,V$, $L = 100m\mathrm{H}$, and $R = 10\,\mathrm{Ohms}$ is

$$v_o(t) = \left(19.5 - 17e^{-10t}\right)u(t).$$

Part IV: [2 points] Decompose the output voltage of Part III as (i) the sum of the natural and forced response, and (ii) the sum of the zero-state and zero-input response.

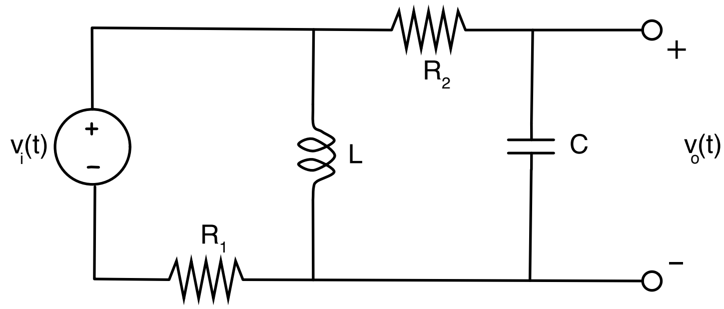

Figure 4: Frequency Response Analysis for Question 4.

4. **Frequency Response Analysis**

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 4 into the s-domain.

Part II: [3 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ is given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{Ls}{(R_1+R_2)CLs^2 + (R_1R_2C + L)s + R_1}.$$

[Show your work]

Part III [5 points] Let $R_1 = R_2 = 123.85$Ohms, $C = 50\,\mu$F and $L = 100$mH. Show that with these values, the transfer function takes the form

$$T(s) = \frac{80.75s}{s^2 + 700s + 10^5}$$

Compute the gain and phase functions of $T(s)$. What are the DC gain and the ∞-freq gain? What are the corresponding values of the phase function? What are the cut-off frequencies? Sketch plots for the gain and phase functions. What type of filter is this one?
[Explain your answer]

Part IV [1 point] Using what you know about frequency response, compute the steady-state response $v_o^{SS}(t)$ of this circuit when $v_i(t) = \cos(300t + \frac{\pi}{4})$ using the same values of R_1, R_2, C, and L as in Part III.

5. **OpAmp Design**

Consider the following transfer function

$$T(s) = \frac{s^2 + 201s + 500}{s^2 + 700s + 10^5}$$

Part I: [3 points] Find real numbers α, β, K_1 and K_2, with $\alpha > \beta$ such that

$$T(s) = \frac{K_1 s}{s + \alpha} + \frac{K_2}{s + \beta}.$$

Part II: [5 points] Design a circuit using OpAmps that implements $T(s)$ as the sum of the two transfer functions from Part I.

Part III: [2 points] What type of filter is $T(s)$?

6. **Loading and the Chain Rule**

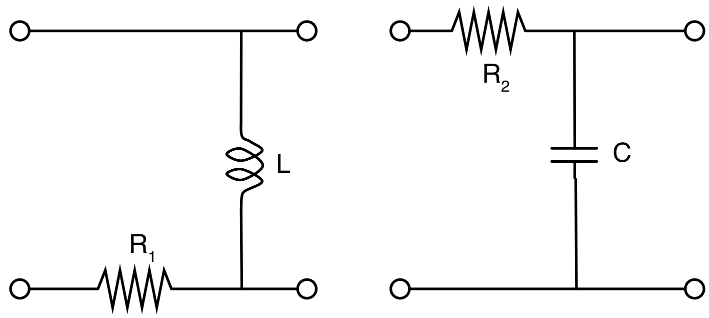

Figure 5: Circuits for Question 6.

A former instructor of MAE140 decided to find the transfer function $T(s) = V_o(s)/V_i(s)$ of the circuit in Question 4 by splitting it in two blocks, as depicted in Figure 5.

Part I: [2 points] Find the transfer functions $T_1(s)$ and $T_2(s)$ for each block in Figure 5.

Part II: [1 point] Once he found the transfer function of each block, the instructor multiplied them and obtained $\tilde{T}(s) = T_1(s)T_2(s)$. Find the expression for $\tilde{T}(s)$.

Part III: [4 points] He was surprised to see that the expression for $\tilde{T}(s)$ was not the same as the expression for $T(s)$ provided in Question 4, Part II. Which one is correct and why? Can you properly explain the instructor's thinking for doing what he did?

Part IV: [3 points] If you were given the task of designing a circuit with transfer function $\tilde{T}(s)$ using the stages in Figure 5, how would you do it? You can employ up to one OpAmp in your design and you should properly justify why it works. Does the order in which you interconnect all the stages matter?