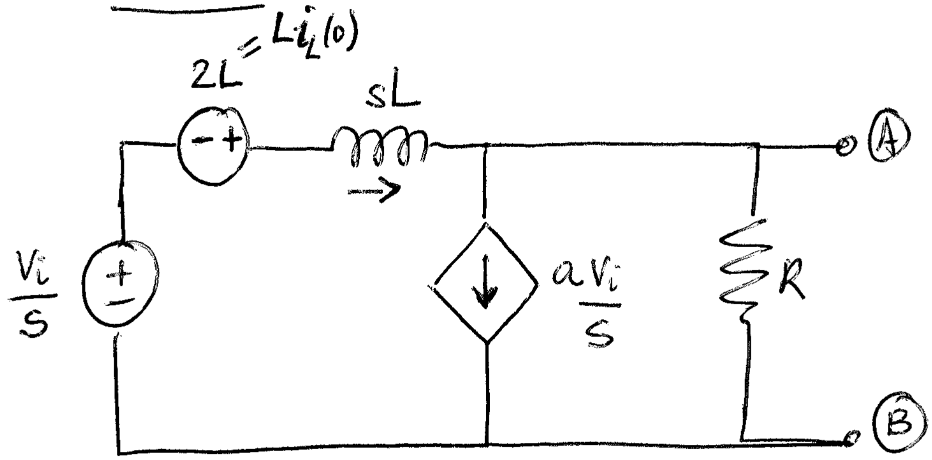
1.- Part I

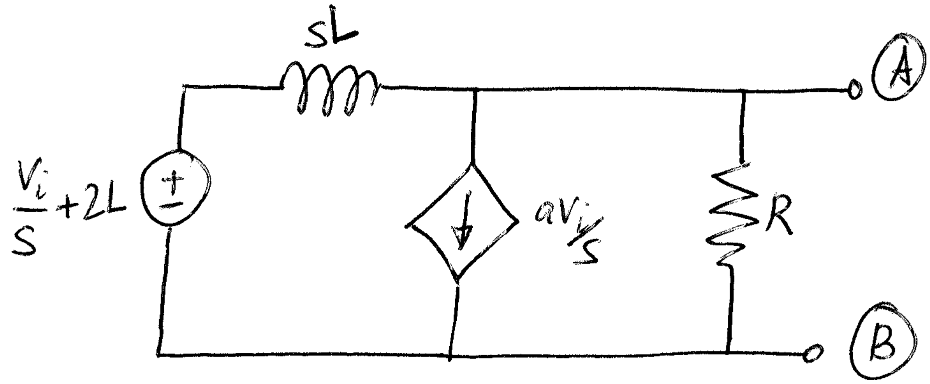


[+1 point for correct initial condition]

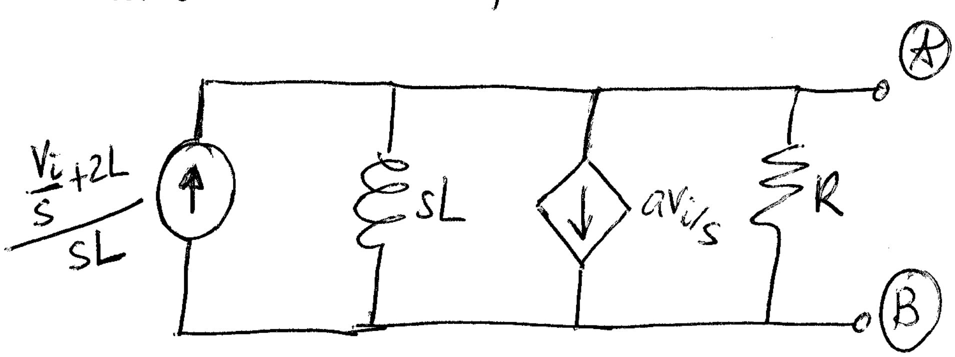
[+1 point for correct overall circuit]

Part II

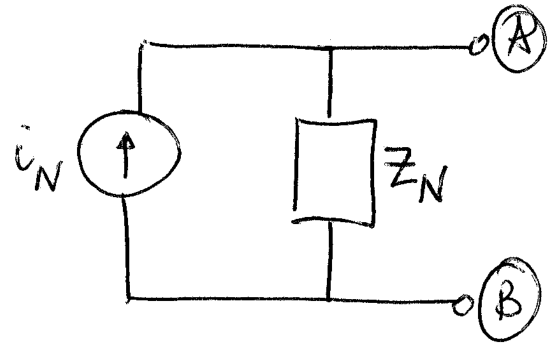
We combine the two voltage sources in series to obtain



We use source transformation with the voltage source in series with the impedance



Combining current sources in parallel and impedances in parallel, we obtain



where

$$i_N = \frac{V_i/s + 2L}{sL} - \frac{aV_i}{s} = \frac{V_i/s + 2L - aV_iL}{sL} = \frac{V_i + (2L - aV_iL)s}{s^2L}$$

$$Z_N = Z_T = R \| sL = \frac{RLs}{R + Ls}$$

Therefore

$$V_{oc} = V_T = Z_N \cdot i_N = \frac{R\cancel{L}\cancel{s}}{R+Ls} \cdot \frac{V_i + (2L - aV_iL)s}{s^{\cancel{2}}\cancel{L}} =$$

$$= \frac{RV_i + R(2L - aV_iL)s}{s(R+Ls)} \quad [+3 \text{ points}]$$

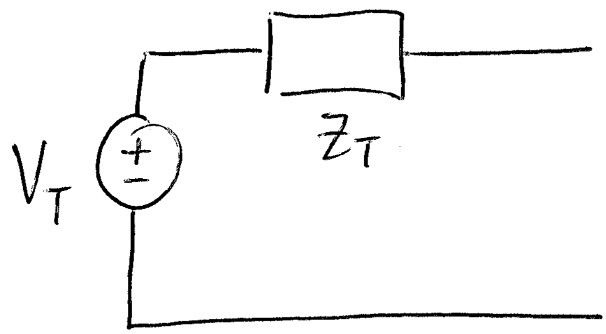
Part III

The short-circuit current is actually the Norton equivalent current, which we obtained in part II. So

$$i_{sc} = i_N = \frac{V_i + (2L - aV_iL)s}{s^2L} \quad [+3 \text{ points}]$$

Part IV

Transforming the Norton eq. circuit we obtained in Part II,



$$V_T = \frac{Rv_i + R(2L - av_i)s}{s(R+Ls)} \qquad \text{[+1 point]}$$

$$Z_T = \frac{RLs}{R+Ls} \qquad \text{[+1 point]}$$

Part V

$$(V_{OC}(s))_{zi} \underset{(v_i = 0)}{=} \frac{2RLs}{s(R+Ls)} \qquad \text{extra [+0.5 point]}$$

$$(V_{OC}(s))_{zs} \underset{\text{(no initial condition)}}{=} \frac{Rv_i(1-as)}{s(R+Ls)} \qquad \text{extra [+0.5 point]}$$

For the forced and natural response, we first break down V_{OC} using partial fractions:

$$V_{OC}(s) = \frac{Rv_i + R(2L - av_i)s}{s(R+Ls)} = \frac{A}{s} + \frac{B}{s + R/L}$$

Using the method of residues,

$$A = \lim_{s \to 0} s \cdot V_{OC}(s) = \lim_{s \to 0} \frac{Rv_i + R(2L - av_iL)s}{R + Ls} = \frac{\cancel{R}v_i}{\cancel{R}} = v_i$$

$$B = \lim_{s \to -R/L} \left(s + R/L\right) \cdot \frac{Rv_i + R(2L - av_iL)s}{s(R + Ls)} =$$

$$= \lim_{s \to -R/L} \frac{Rv_i + R(2L - av_iL)s}{sL} = \frac{Rv_i - R^2(2L - av_iL)\,1/L}{-R} =$$

$$= -v_i + \frac{R}{L}(2L - av_iL) =$$

$$= \frac{-v_iL + R(2L - av_iL)}{L} =$$

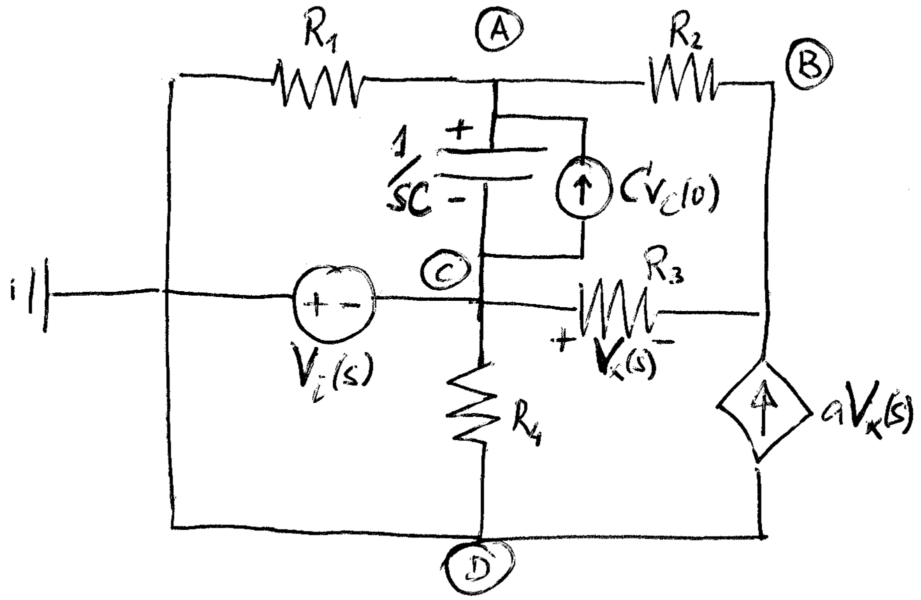
$$= -v_i + R(2 - av_i)$$

Therefore

$$\left(V_{OC}(s)\right)_{fr} = \frac{A}{s}$$ (same poles as input), w/ A as given above [+0.5 extra point]

$$\left(V_{OC}(s)\right)_{nr} = \frac{B}{s + R/L}$$ (poles of the circuit), w/ B as given above. [+0.5 extra point]

2.– <u>Part I</u>



Note we have used a current source to accommodate the initial condition of the capacitor [because we're using nodal analysis] [+1 point]

For nodal analysis, the voltage source represents a problem. However, we can easily take care of it using method 2 (the choice of ground). By looking at the circuit, we see that

$$V_C(s) = -V_i(s)$$ [+1 point]

Therefore, we only need to write KCL eqs for nodes A & B. (Note that D is ground)

KCL @ A,

$$G_1 V_A(s) + G_2\left(V_A(s) - V_B(s)\right) + sC\left(V_A(s) - V_C(s)\right) = Cv_C(0)$$ [+1 point]

KCL @ B,

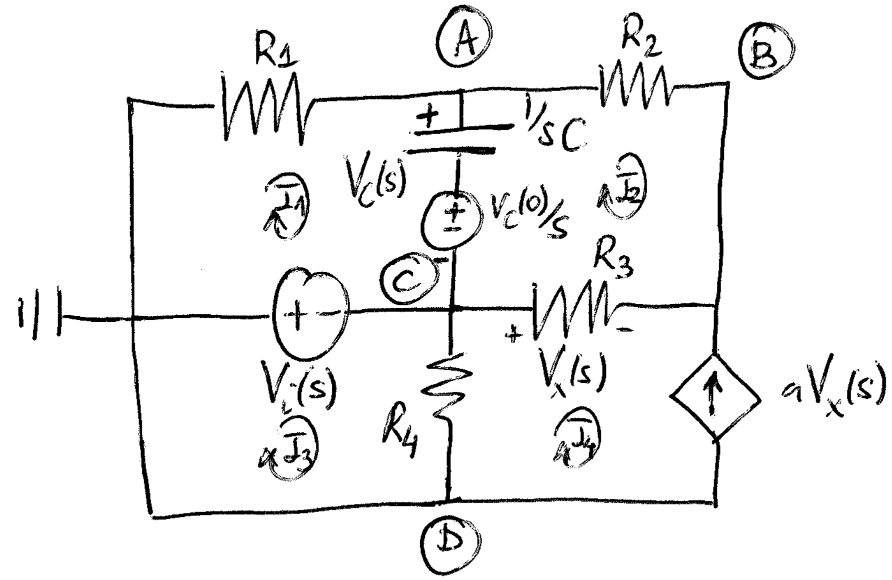
$$G_2\left(V_B(s) - V_A(s)\right) + G_3\left(V_B(s) - V_C(s)\right) = aV_x(s)$$ [+1 point]

Finally, we take care of the dependent source by setting

$$V_x(s) = V_C(s) - V_B(s)$$ [+1 point]

This gives us a total of 4 eqs in 4 unknowns ($V_A(s)$, $V_B(s)$, $V_C(s)$, and $V_x(s)$).

Part II



Note we have used a voltage source to accommodate the initial condition of the capacitor [because we're using mesh current analysis] [+0.5 point]

For mesh analysis, the current source is a problem. Luckily, it only belongs to one mesh, so we can use method 2 and write

$$I_4(s) = -a V_x(s)$$ [+0.5 point]

We now have to write KVL eqs for meshes 1, 2, 3.

KVL @ mesh 1:

$$R_1 I_1(s) + \frac{1}{sC}\left(I_1(s) - I_2(s)\right) + \frac{V_C(0)}{s} - V_i(s) = 0$$ [+1 point]

KVL @ mesh 2:

$$R_2 I_2(s) + R_3\left(I_2(s) - I_4(s)\right) - \frac{V_C(0)}{s} + \frac{1}{sC}\left(I_2(s) - I_1(s)\right) = 0$$ [+1 point]

KVL @ mesh 3:

$$V_i(s) + R_4\left(I_3(s) - I_4(s)\right) = 0 \qquad \text{[+1 point]}$$

Finally, we take care of the dependent source by setting

$$V_x(s) = R_3\left(I_4(s) - I_2(s)\right) \qquad \text{[+1 point]}$$

This gives us a total of 5 eqs in 5 unknowns ($I_1(s)$, $I_2(s)$, $I_3(s)$, $I_4(s)$, $V_x(s)$).

Part III

$$V_C(s) = V_A(s) - V_C(s) \qquad \text{[+1 extra point]}$$

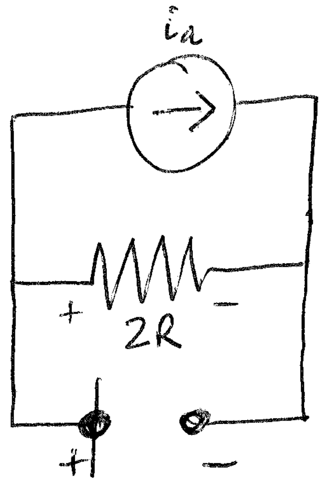
$$V_C(s) = \frac{1}{sC}\left(I_1(s) - I_2(s)\right) + \frac{V_C(0)}{s} \qquad \text{[+1 extra point]}$$

3.- Part I

The switch has been in position A for a very long time. This means that the capacitor has been subject to DC excitation, and therefore behaves as an open circuit.

[+1 point]

We then have

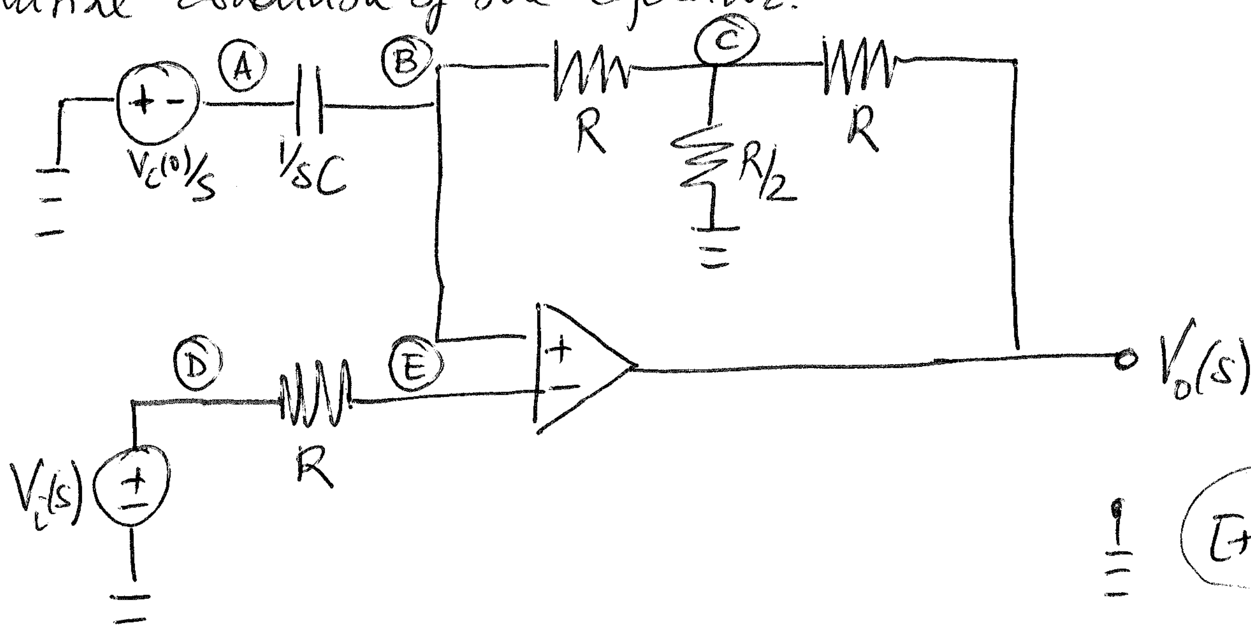


From the plot, we see $V_C(0) = V_{2R} = 2R(-i_a)$, as stated.

[+1 point]

Part II

At time 0, it is moved to position B. We then transform the circuit to the s-domain, taking special care of the initial condition of the capacitor.



[+1 point]

Looking at the circuit, we don't recognize any of the basic building blocks, so we use nodal analysis. This means we do not write KCL for the output node of the op-amp, and that we employ ideal op-amp conditions.

[+~~0.5~~ point]

Just by looking at the circuit, we can set

$$V_A(s) = -\frac{V_C(0)}{s} = -\left(-\frac{2Ri_a}{s}\right) = \frac{2Ri_a}{s}$$ [+0.5 point]

$$V_D(s) = V_i(s)$$ [+0.5 point]

We write KCL at nodes B, C and E [+0.5 point]

KCL @ B: $sC(V_B(s) - V_A(s)) + G(V_B(s) - V_C(s)) = 0$ $\left(G = \frac{1}{R}\right)$

KCL @ C: $G(V_C(s) - V_B(s)) + 2G\,V_C(s) + G(V_C(s) - V_0(s)) = 0$ [+0.5 point]

KCL @ E: $G(V_E(s) - V_D(s)) = 0$ [+0.5 point]

Additionally, ideal-op-amp gives us $V_B(s) = V_E(s)$ [+0.5 point]

Therefore,

$$V_E(s) = V_D(s) = V_i(s) = V_B(s)$$

From KCL @ C, we get

$$V_0(s) = 4V_C(s) - V_B(s) = 4V_C(s) - V_i(s)$$

From KCL @ B, we get

$$V_C(s) = \frac{G+sC}{G} V_B(s) - \frac{sC}{G} V_A(s) = (1+RCs)V_B(s) - RC\cancel{s} \cdot \frac{2Ri_a}{\cancel{s}}$$

$$V_C(s) = (1+RCs)V_B(s) - 2R^2 C i_a = (1+RCs)V_i(s) - 2R^2 C i_a$$

Finally,

$$V_0(s) = 4(1+RCs)V_i(s) - 8R^2 C i_a - V_i(s) =$$

$$= (3+4RCs)V_i(s) - 8R^2 C i_a$$

[+0.5 point]

Part III

$v_i(t) = te^{-t}u(t)\ V$ $\qquad V_i(s) = \dfrac{1}{(s+1)^2}$

$i_a = 1A$ $\qquad R = 10^3\ \Omega$

$C = 10^{-3}\ F$

Therefore,

$$V_0(s) = (3+4RCs)\frac{1}{(s+1)^2} - 8R^2 C i_a$$

[+1 point]

We use partial fractions to break down the first summand

$$\frac{3+4RCs}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{(s+1)^2} = \frac{A(s+1)+B}{(s+1)^2}$$

Hence $\quad 3 = A+B$ $\quad \Rightarrow \quad B = 3-4RC$

$4RC = A$

[+1 point]

So,

$$V_0(s) = \frac{4RC}{s+1} + \frac{3-4RC}{(s+1)^2} \cancel{+} - 8R^2 C i_a$$

[+1 point]

Taking inverse Laplace transforms,

$$V_o(t) = \left[4RC\, e^{-t} + (3-4RC)\, e^{-t}\cdot t - 8R^2C\, i_a\, \delta(t)\right] u(t)$$

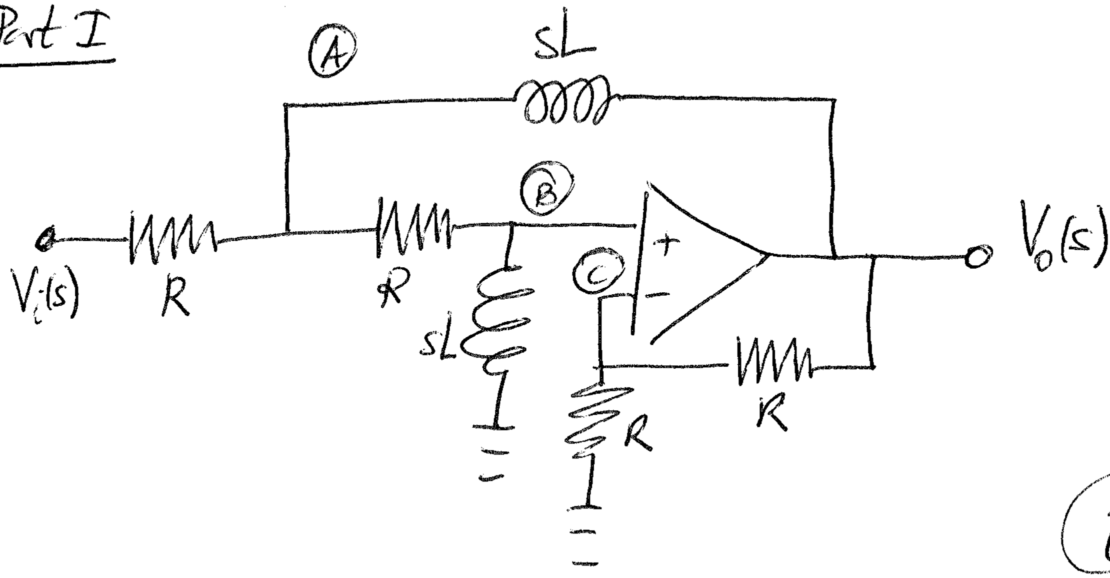
$$= \left(4e^{-t} \cancel{+} - te^{-t} - 8000\,\delta(t)\right) u(t)$$

Part IV

No, using nodal analysis, we will normally employ a current source. However, in this case, given that the capacitor is connected to ground, we know we can easily take care of the voltage source the statement asks us to employ.

[+1 extra point]

4.- Part I



[+1 point]

Part II

We use nodal analysis.

KCL @ A

$$\left(G=\frac{1}{R}\right) \qquad G\,(V_A(s)-V_i(s)) + \frac{1}{sL}\,(V_A(s)-V_o(s)) + G\,(V_A(s)-V_B(s)) = 0$$

[+0.5 point]

KCL @ B

$$G\,(V_B(s)-V_A(s)) + \frac{1}{sL}\,V_B(s) = 0 \quad \Rightarrow \quad V_A(s) = \frac{G+1/sL}{G}\,V_B(s)$$

[+0.5 point]

KCL @ C

$$G\,(V_C(s)) + G\,(V_C(s)-V_o(s)) = 0 \quad \Rightarrow \quad V_o(s) = 2\,V_C(s)$$

[+0.5 point]

Plus ideal op-amp conditions,

$$V_B(s) = V_C(s)$$

[+0.5 point]

From KCL @ A, we get

$$G\cdot V_i(s) = \left(2G+\frac{1}{sL}\right) V_A(s) - \frac{1}{sL}\,V_o(s) - G\,V_B(s)$$

$$= \left(2G+\frac{1}{sL}\right) \frac{G+1/sL}{G}\,V_B(s) - \frac{1}{sL}\,2V_B(s) - G\,V_B(s)$$

$$= \left(\left(2G + \frac{1}{sL} \right) \frac{GLs+1}{GsL} - \frac{2}{sL} - G \right) V_B(s)$$

$$= \left(\frac{2GLs+1}{sL} \cdot \frac{GLs+1}{GsL} - \frac{2}{sL} - G \right) V_B(s)$$

$$= \left(\frac{2G^2L^2s^2 + 3GLs + 1}{G s^2 L^2} - \frac{2 + GLs}{sL} \right) V_B(s)$$

$$= \frac{2G^2L^2s^2 + 3GLs + 1 - 2GLs - G^2L^2s^2}{Gs^2L^2} V_B(s)$$

$$= \frac{G^2L^2s^2 + GLs + 1}{Gs^2L^2} V_B(s) = G\, V_i(s)$$

Hence,

$$V_B(s) = \frac{G^2L^2s^2}{G^2L^2s^2 + GLs + 1} V_i(s) =$$

$$= \frac{L^2s^2}{L^2s^2 + RLs + R^2} V_i(s)$$

E

Finally, $V_0(s) = \dfrac{2L^2s^2}{L^2s^2 + RLs + R^2} V_i(s)$, and hence

$$T(s) = \frac{V_0(s)}{V_i(s)} = \frac{2L^2s^2}{L^2s^2 + RLs + R^2}$$

[+1 point]

Part III

With the values provided,

$$T(s) = \frac{2\cdot 10^{-2} s^2}{10^{-2}s^2 + 10^{-1}s + 1} = \frac{2s^2}{s^2 + 10s + 100}$$

And we have

$$T(j\omega) = \frac{-2\omega^2}{-\omega^2 + 10j\omega + 100} = \frac{-2\omega^2}{(100-\omega^2) + 10\omega j}$$

$$|T(j\omega)| = \frac{2\omega^2}{\sqrt{(100-\omega^2)^2 + 100\omega^2}} = \frac{2\omega^2}{\sqrt{10^4 + \omega^4 - 200\omega^2 + 100\omega^2}} =$$

$$= \frac{2\omega^2}{\sqrt{10^4 + \omega^4 - 100\omega^2}}$$

[+0.5 point]

$$\angle T(j\omega) = \pi - \arctan \frac{10\omega}{100-\omega^2}$$

[+0.5 point]

$|T(j0)| = 0$ [+0.5 point] $\angle T(j0) = \pi$ rad [+0.5 point]

$|T(j\infty)| = 2$ [+0.5 point] $\angle T(j\infty) = \pi - \pi = 0$ rad

[+0.5 point] for large ω, $(100-\omega^2, 10\omega)$ is in this quadrant

For $\omega \gg 1$, $|T(j\omega)| \approx \frac{2\omega^2}{\omega^2} = 2$

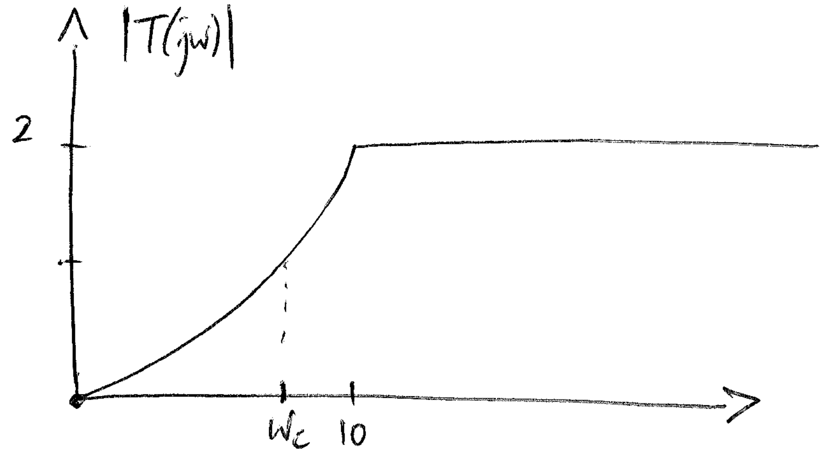
For $\omega \ll 1$, $|T(j\omega)| \approx \frac{2\omega^2}{10^2} = \frac{2\omega^2}{100}$

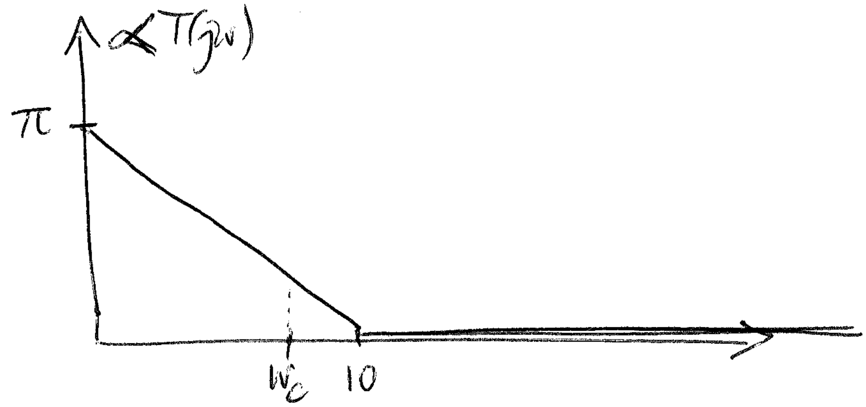
$T_{max} = 2$

$$\frac{2}{\sqrt{2}} = |T(j\omega_c)| = \frac{2\omega_c^2}{\sqrt{10^4 + \omega_c^4 - 100\omega_c^2}} \iff \omega_c = 5\sqrt{2(\sqrt{5}-1)} \approx 7.86151 \text{ rad}$$

[+1 point]

Sketch





[+0.5 point]

This is a high-pass filter. [+0.5 point]

Part IV

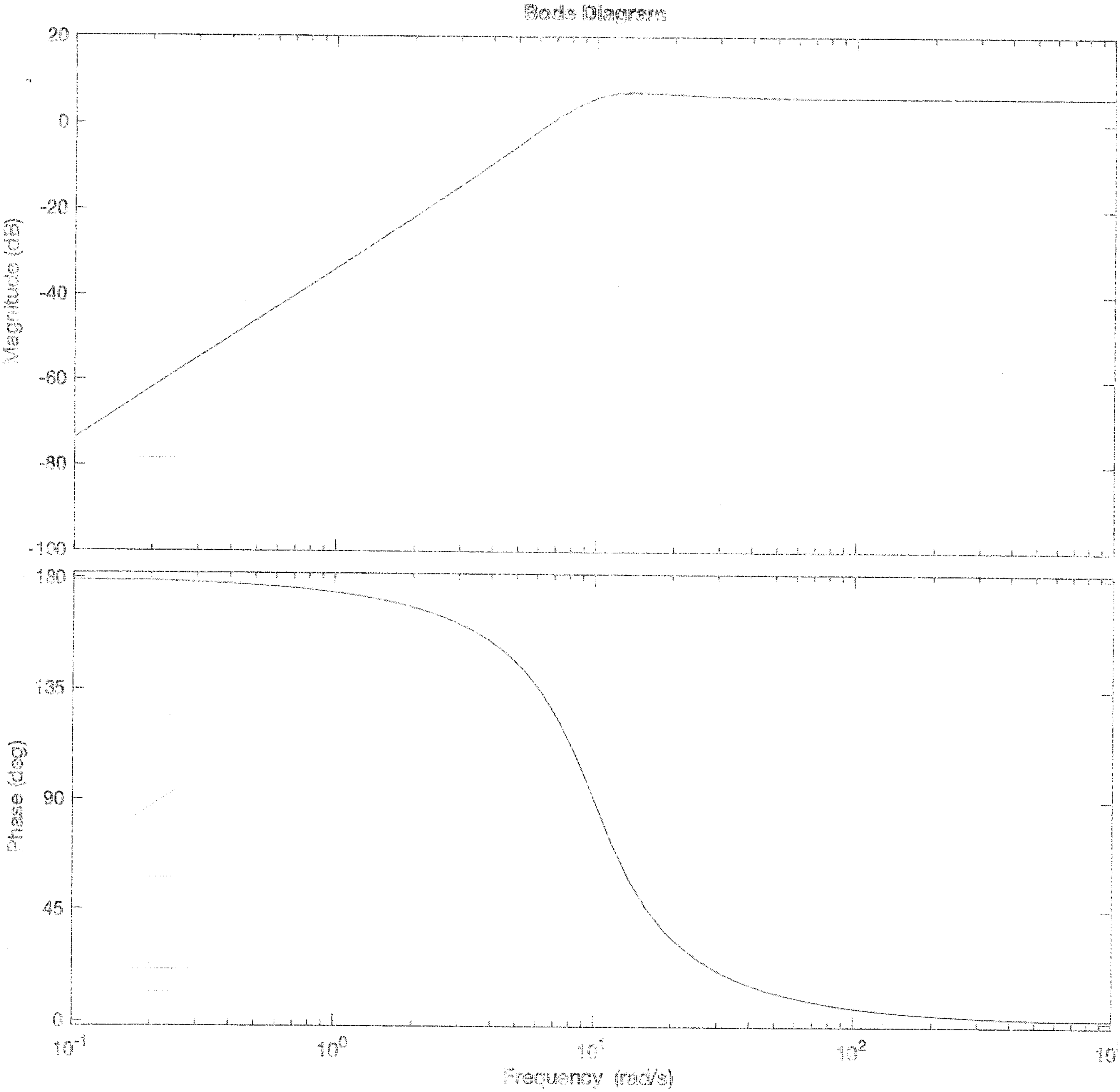
$$v_i(t) = \cos(5t + \frac{\pi}{2})$$

$$v_o^{ss}(t) = |T(j5)| \cdot \cos(5t + \frac{\pi}{2} + \measuredangle T(j5))$$

$$|T(j5)| = \frac{50}{\sqrt{10^4 + 5^4 - 2500}} = 0.5547$$ [+0.5 point]

$$\measuredangle T(j5) = \pi - \arctan\frac{2}{3} = 2.55359 \text{ rad}$$ [+0.5 point]

So $v_o^{ss}(t) = 0.5547 \cdot \cos(5t + \frac{\pi}{2} + 2.55359)$

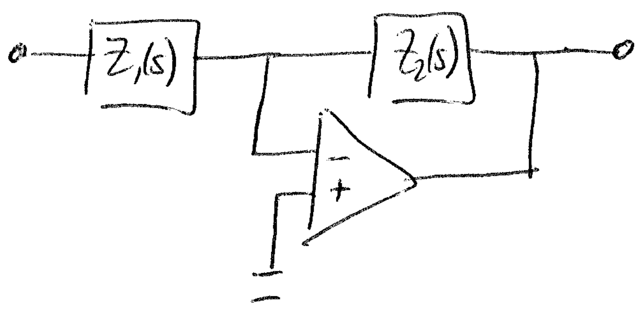

Bode Diagram
Magnitude (dB)
20
0
-20
-40
-60
-80
-100
Phase (deg)
180
135
90
45
0
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

5._ Part I

$$T(s) = \frac{1+10s}{10s\,(100s+1)} = \frac{1+10s}{10s} \cdot \frac{1}{100s+1}$$

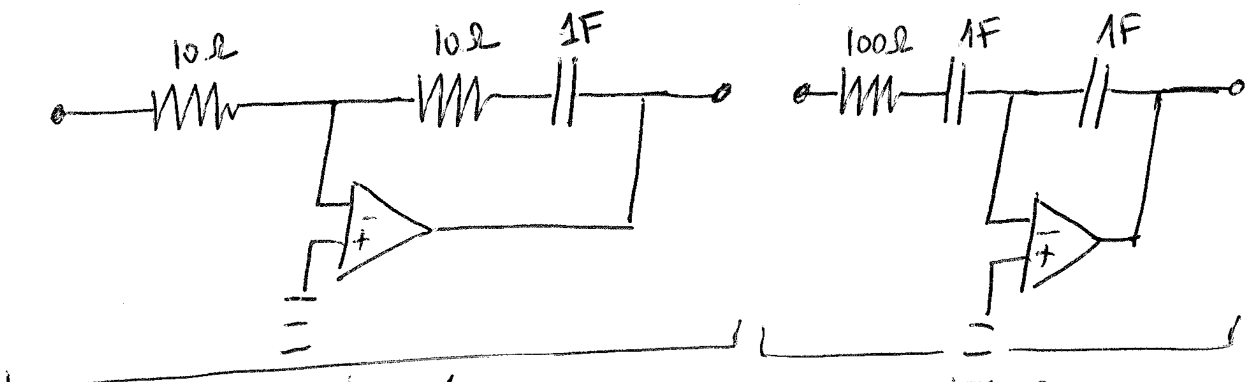
$$= \underbrace{\left(-\frac{\frac{1}{s}+10}{10}\right)}_{\text{inverting op amp}} \underbrace{\left(-\frac{1/s}{100+1/s}\right)}_{\text{inverting op-amp}} \qquad [+1 \text{ point}]$$

[we have divided by s the numerator and denominator to be able to generate them without inductors]



$$T(s) = -\frac{Z_2(s)}{Z_1(s)}$$

So we design



stage 1 stage 2

$$T_1(s) = -\frac{1/s + 10}{10} \qquad T_2(s) = \frac{-1/s}{1/s + 100}$$

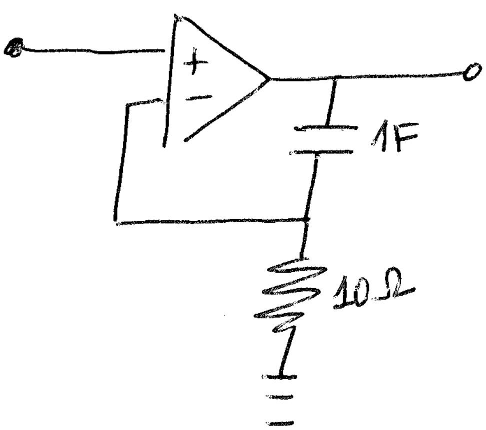
[+1 point] [+1 point]

Note that there is no loading because of the zero output impedance of stage 1. [+1 point]

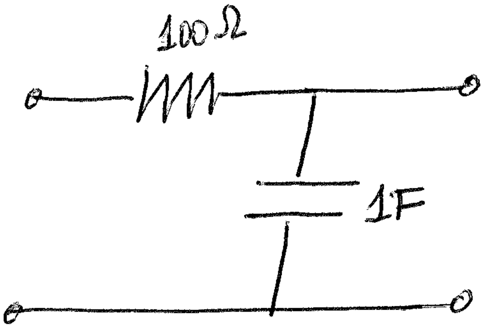
Part II

$$T(s) = \frac{1+10s}{10s} \cdot \frac{1}{100s+1} = \underbrace{\frac{\frac{1}{s}+10}{10}}_{\text{non-inverting op-amp}} \cdot \underbrace{\frac{\frac{1}{s}}{100+\frac{1}{s}}}_{\text{voltage divider}}$$

[as before, we have divided numerator and denominator by s to be able to generate it without inductors] [+1 point]



stage 1



stage 2

$$T_1(s) = \frac{\frac{1}{s}+10}{10}$$ [+1 point]

$$T_2(s) = \frac{\frac{1}{s}}{\frac{1}{s}+100}$$ [+1 point]

Note that there is no loading because of the zero output impedance of the non-inverting op-amp. [+1 point]

Part III

The order will not affect the resulting transfer function in either design, because there is no loading. In Part I, because of the 0-output impedance of each stage. In Part II, because of the ∞-input and 0-output impedance of the non-inverting op-amp. [+2 points]