MAE40 - Linear Circuits - Winter 20
Final Exam

Instructions

(i) Prior to the exam, you must have completed the Academic Integrity Pledge at `https://academicintegrity.ucsd.edu/forms/form-pledge.html`

(ii) The exam is open book. You may use your class notes and textbook.

(iii) Collaboration is not permitted. The answers you provide should be the result only of your own work

(iv) On the questions for which the answers are given, please provide detailed derivations

(v) The exam has 5 questions for a total of 50 points and 5 bonus points

Good luck!

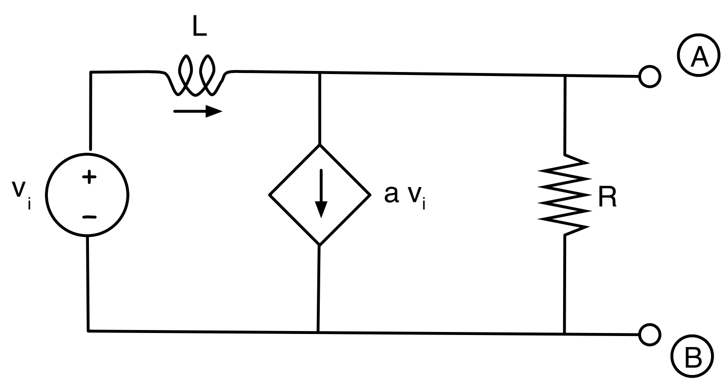

Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

Here, v_i is a constant voltage source and a is a positive constant.

Part I: [2 points] Assuming $i_L(0) = 2A$, transform the circuit in Figure 1 into the s-domain, using a voltage source to account for the initial condition of the inductor.

Part II: [3 points] For the circuit you obtained in Part I, find the open-circuit voltage transform as seen from terminals Ⓐ-Ⓑ. The answer should be given as a ratio of two polynomials.

Part III: [3 points] For the circuit you obtained in Part I, find the short-circuit current transform as seen from terminals Ⓐ-Ⓑ. The answer should be given as a ratio of two polynomials.

Part IV: [2 points] For the circuit you obtained in Part I, find the Thévenin equivalent in the s-domain as seen from terminals Ⓐ-Ⓑ (the impedance should be given as a ratio of two polynomials).

Part V: [Extra 2 points] Break down the open-circuit voltage transform you obtained in Part II as the sum of the zero-state and zero-input response transforms. Do the same as the sum of the forced and natural response transforms.

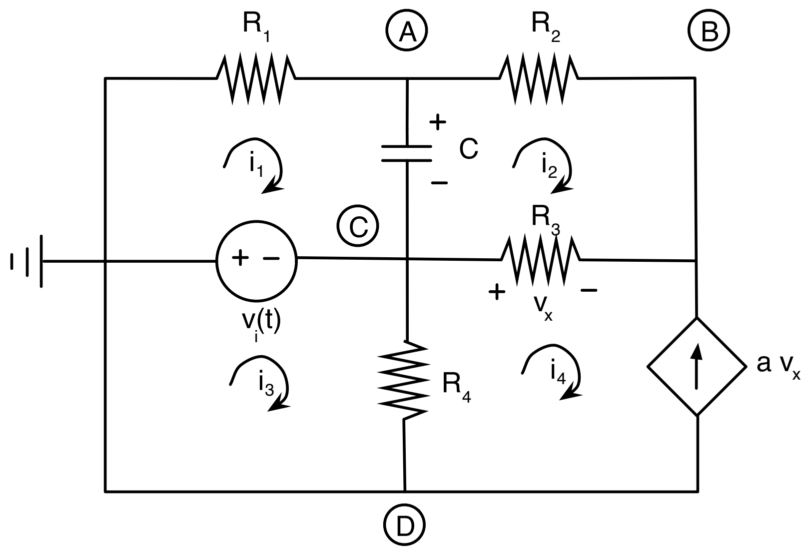

Figure 2: Nodal and Mesh Analysis Circuit for Question 2. a is a positive constant.

2. **Nodal and Mesh Analysis on the s-domain**

Part I: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its node-voltage equations. Use the reference node and other labels as shown in the figure. *Do not assume zero initial conditions.* Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

Part II: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its mesh-current equations. Use the mesh currents shown in the figure. *Do not assume zero initial conditions.* Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

Part III: [Extra 2 points] Express the capacitor's voltage transform $V_C(s)$ in terms of the node voltages. Do the same in terms of the mesh currents.

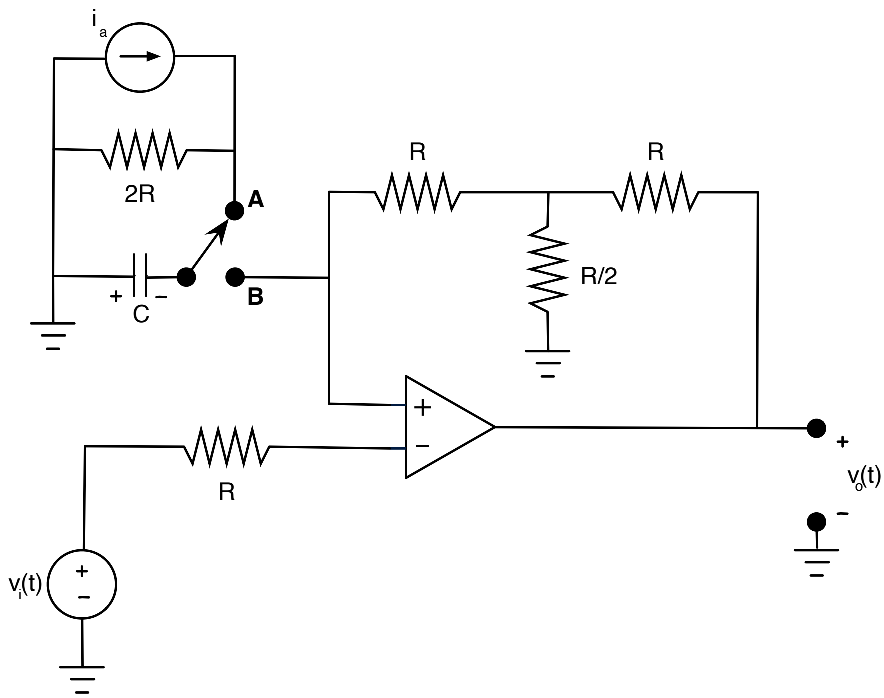

Figure 3: RCL circuit for Laplace Analysis for Question 3.

3. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 3. The value i_a of the current source is constant. The switch is kept in position **A** for a very long time. At $t = 0$, it is moved to position **B**. Show that the initial condition for the capacitor is given by

$$v_C(0^-) = -2Ri_a.$$

[Show your work]

Part II: [5 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the capacitor in which the initial condition appears as a voltage source. Use nodal analysis to express the output response transform $V_o(s)$ as a function of $V_i(s)$ and i_a.

Part III: [3 points] Use partial fractions and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $i_a = 1\,\text{A}$, $v_i(t) = te^{-t}u(t)\,V$, $C = 1\,m\text{F}$, and $R = 1\,K\Omega$ is

$$v_o(t) = \Big(4e^{-t} - te^{-t} - 8000\,\delta(t)\Big)u(t).$$

Part IV: [Extra 1 point] In Part II we have used nodal analysis, yet the statement asks us to use a voltage source to represent the initial condition. Generally, is this what we would do? Why? Can you explain why the statement asked us to do it?

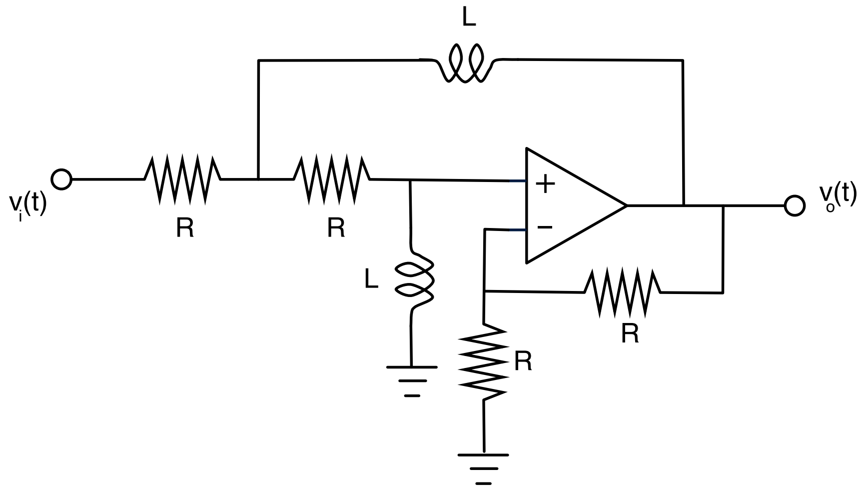

Figure 4: Frequency Response Analysis for Question 4.

4. **Frequency Response Analysis**

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 4 into the s-domain.

Part II: [3 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ is given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{2L^2s^2}{L^2s^2 + LRs + R^2}.$$

[Show your work]

Part III [5 points] Let $R = 1\,\Omega$ and $L = 100\,\text{mH}$. Show that with these values, the transfer function takes the form

$$T(s) = \frac{2s^2}{s^2 + 10s + 100}$$

Compute the gain and phase functions of $T(s)$. What are the DC gain and the ∞-freq gain? What are the corresponding values of the phase function? What is the cut-off frequency? Sketch plots for the gain and phase functions. What type of filter is this one?
[Explain your answer]

Part IV [1 point] Using what you know about frequency response, compute the steady-state response $v_o^{SS}(t)$ of this circuit when $v_i(t) = \cos(5t + \frac{\pi}{2})$ using the same values of R and L as in Part III.

5. **OpAmp Design**

Consider the following transfer function

$$T(s) = \frac{1 + 10s}{1000s^2 + 10s}$$

Part I: [4 points] Design a circuit as the series connection of two stages, each with 1 OpAmp, that implements $T(s)$. You can also use 10Ω and 100Ω-resistors, and $1F$-capacitors, but not inductors. Be sure to properly justify why the overall transfer function of your design is the product of the individual transfer function of each stage.

Part II: [4 points] Design a circuit as the series connection of two stages, one with 1 OpAmp and the other without, that implements $T(s)$. You can also use 10Ω and 100Ω-resistors, and $1F$-capacitors, but not inductors. Be sure to properly justify why the overall transfer function of your design is the product of the individual transfer function of each stage.

Part III: [2 points] Would the order in which you interconnect the two stages in Part I affect the resulting transfer function? How about in Part II?