MAE140 - Linear Circuits - Winter 20
Midterm, February 6

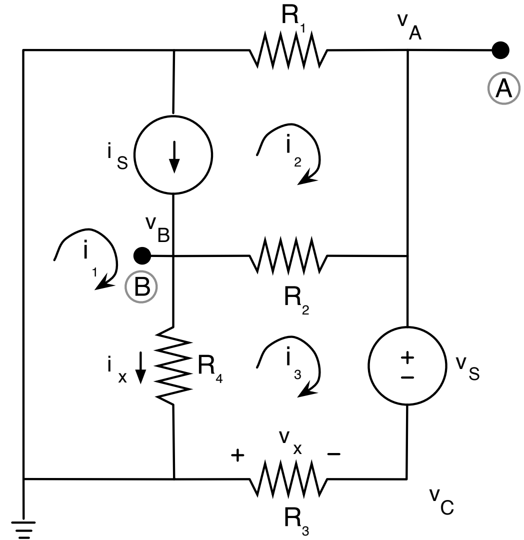


Figure 1: Circuit for all questions.

Instructions

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook

(ii) You may use a hand calculator with no communication capabilities

(iii) You have 75 minutes

(iv) Do not forget to write your **name** and **student number**

(v) The exam has 3 questions, for a total of 30 points

Good luck!

1. **Node voltage analysis**

Part I: [5 points] Formulate node-voltage equations for the circuit in Figure 1. Use the node labels provided in the figure. Clearly indicate how you handle the presence of the voltage source. The final equations in matrix form must depend only on unknown node voltages. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: Part I: There are four nodes in this circuit. The ground node has already been chosen for us. Unfortunately, with this choice, the ground node is not directly connected to the voltage source, so we cannot use method 2 to take care of it. Since they do not let us redraw the circuit, we have to use a supernode. **(+ 1 point)**

The equation defining the supernode is

$$v_A - v_C = v_S \qquad \textbf{(+ 1 point)}$$

KCL for the supernode takes the form

$$G_1 v_A + G_2(v_A - v_B) + G_3 v_C = 0 \qquad \textbf{(+ 1 point)}$$

(where we are using the short-hand notation $G_i = 1/R_i$).
Finally, we write KCL for node B,

$$G_2(v_B - v_A) + G_4 v_B = i_S \qquad \textbf{(+ 1 point)}$$

We can write everything together as a system of 3 equations in 3 unknowns v_A, v_B, v_C,

$$\begin{pmatrix} 1 & 0 & -1 \\ G_1 + G_2 & -G_2 & G_3 \\ -G_2 & G_2 + G_4 & 0 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} v_S \\ 0 \\ i_S \end{pmatrix} \qquad \textbf{(+ 1 point)}$$

Part II: [3 points] Provide expressions for the mesh currents i_1, i_2, and i_3 in terms of the node voltages.

Solution: Part II: The mesh currents can be expressed in terms of the node voltages by using Ohm's law. For instance, i_2 is the current passing through the R_1 resistor. Likewise, i_3 is the current passing through the R_3 resistor. Finally, $i_1 - i_3$ is the current passing through the R_4 resistor. Therefore,

$$i_2 = G_1(-v_A) \qquad \textbf{(+ 1 point)}$$
$$i_3 = G_3 v_C \qquad \textbf{(+ 1 point)}$$
$$i_1 - i_3 = G_4 v_B \Rightarrow i_1 = G_4 v_B + G_3 v_C \qquad \textbf{(+ 1 point)}$$

Part III: [2 points] Provide expressions for the voltage v_x and the current i_x in terms of node voltages.

Solution: Part III: In terms of the node voltages, we have

$$v_x = -v_C \qquad \textbf{(+ 1 point)}$$
$$i_x = G_4 v_B \qquad \textbf{(+ 1 point)}$$

2. **Mesh current analysis**

Part I: [5 points] Formulate mesh-current equations for the circuit in Figure 1. Use the mesh currents shown in the figure and clearly indicate how you handle the presence of the current source. The final equations in matrix form should only depend on the unknown mesh currents. **Do not modify the circuit or the labels** (meaning source transformation or circuit re-drawing are not allowed). No need to solve any equations!

Solution: Part I: There are three meshes in this circuit. The current source belongs to two meshes, instead of one. Without redrawing the circuit, we are forced to use a supermesh.

(+ 1 point)

Consequently, we set

$$i_1 - i_2 = i_S \qquad \textbf{(+ 1 point)}$$

And we write KVL for the supermesh as

$$R_1 i_2 + R_2(i_2 - i_3) + R_4(i_1 - i_3) = 0 \qquad \textbf{(+ 1 point)}$$

Finally, we write KVL for mesh 3 to get

$$R_2(i_3 - i_2) + v_S + R_3 i_3 + R_4(i_3 - i_1) = 0 \qquad \textbf{(+ 1 point)}$$

This gives us a total of 3 equations in the 3 mesh current unknowns i_1, i_2, i_3. In matrix form, we can write this as

$$\begin{pmatrix} 1 & -1 & 0 \\ R_4 & R_1 + R_2 & -R_2 - R_4 \\ -R_4 & -R_2 & R_2 + R_3 + R_4 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} i_S \\ 0 \\ -v_S \end{pmatrix} \qquad \textbf{(+ 1 point)}$$

Part II: [3 points] Provide expressions for the voltage v_x and the current i_x in terms of mesh currents.

Solution: Part II: In terms of the mesh currents, we have

$$v_x = R_3(-i_3) \qquad \textbf{(+ 1.5 points)}$$
$$i_x = i_1 - i_3 \qquad \textbf{(+ 1.5 points)}$$

Part III: [2 points] If we were allowed to modify the circuit, could you have used source transformation to deal with the current source instead? Justify your answer.

Solution:

Yes, we could have used source transformation (method 1) to make the current source disappear.

(+ 1 point)

This is because the current source is in parallel with the resistor R_4. This would allow us to transform it into a voltage source in series with a resistor R_4, also decreasing the number of meshes by one.

(+ 1 point)

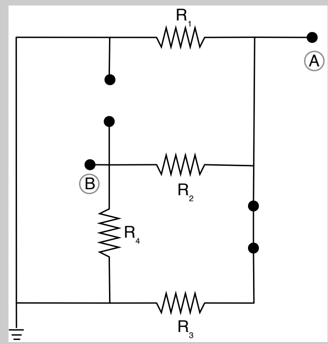
3. **Equivalent circuits**

Part I: [5 points] Turn off all the sources in the circuit of Figure 1 and find the equivalent resistance as seen from terminals Ⓐ and Ⓑ if $R_1 = R_2 = R_3 = 20\Omega$, $R_4 = 10\Omega$ (if you want, after turning off the sources, you can redraw the circuit –respecting its connectivity– to see things more clearly).

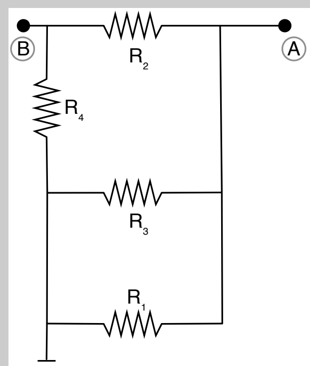
Solution: Part I: We start by switching off the sources.

We substitute the voltage source by a short circuit, and the current source by an open circuit. Then, we get the circuit on the right

(+ 2 points)

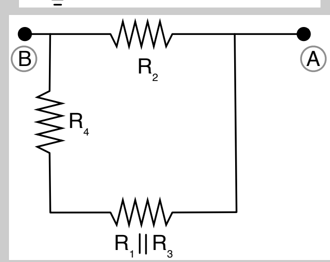


To see things more clearly, we can redraw the circuit respecting its connectivity as follows



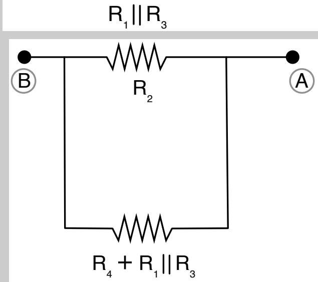
This redrawing makes it more clear that R_1 and R_3 are in parallel, so we get

(+ 1 point)



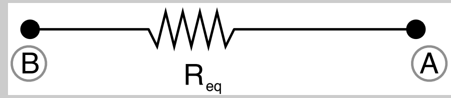
Now R_4 and $R_1 \| R_3$ are in series, which yields

(+ 1 point)



And finally, R_2 is in parallel with $R_4 + R_1 \| R_3$, which yields

$$R_{\text{eq}} = R_2 \| (R_4 + R_1 \| R_3) = 10\Omega$$



(+ 1 point)

Part II: [3 points] Assume that, if you solve for the node voltages with the current source turned off in Problem 1, you get $v_A = 6V$, $v_B = 2V$, and $v_C = -10V$; and if you solve for the node voltages with the voltage source turned off instead, you get $v_A = 3V$, $v_B = 9V$, $v_C = 3V$. Use this information to find the open-circuit voltage as seen from terminals Ⓐ and Ⓑ. [1]

Solution: Part II: With the information provided, by superposition we know that the node voltages in Problem 1 are then

$$v_A = 9V, \quad v_B = 11V, \quad v_C = -7V \qquad \textbf{(+ 1 point)}$$

The open-circuit voltage from Ⓐ to Ⓑ can be expressed in terms of the node voltages as

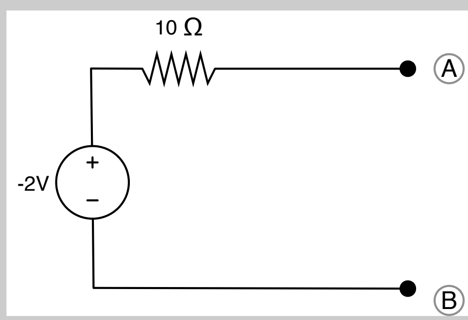
$$v_{AB} = v_A - v_B \qquad \textbf{(+ 1 point)}$$

Therefore, we have that $v_{AB} = -2V$.

(+ 1 point)

Part III: [2 points] Use your answers to Parts I and II to determine the Thévenin equivalent of the circuit as seen from terminals Ⓐ and Ⓑ.

Solution: Part III: We have computed the equivalent resistance from terminals Ⓐ and Ⓑ with all sources turned off in Part I, and the open-circuit voltage in Part II. Therefore, the Thévenin equivalent of the circuit is simply



(+ 2 points)

[1]The original statement had the typo $v_C = 8V$ with the current source off and $v_B = -1V$ with the voltage source off. If you relied on this values to use superposition, then you get full credit here and in Part III. If you instead relied on the equations obtained in Problem 1, Part I to get consistent values for the node voltages, then you get full credit too.