MAE40 Linear Circuits (for non-electrical engs)

Topics covered

Circuit analysis techniques

- Kirchoff's Laws – KVL, KCL
- Nodal and Mesh Analysis
- Thévenin and Norton Equivalent Circuits
- Resistive circuits, RLC circuits
- Steady-state and dynamic responses
  - Impulse and step responses   Laplace transforms
- Sinusoidal steady-state response

Circuit design techniques

- Active circuit elements – dependent sources and operational amplifiers
- Feedback basics
- Signal filtering

What do I expect you to know?

Prerequisites

Mathematics 21D or 20D (some are doing this concurrently)

- Solution of sets of linear equations
  - $Ax=b$ equations for vector x given matrix A and vector b
- Solution of constant coefficient linear ordinary differential equations
  - Laplace transform
  - Initial conditions and forced response
- Complex analysis
  - Numbers, arithmetic, magnitudes and phases, poles, zeros

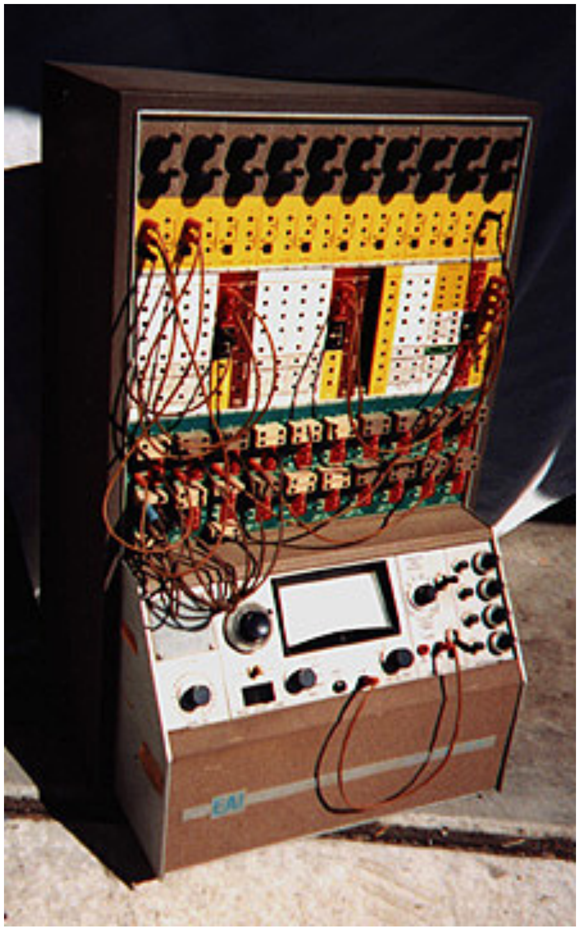
Physics 2B

- Behavior of circuit elements: *R, L* and *C*
  - Underlying physical principles

Why should you be excited about MAE40?

Linear circuits are exceptionally well described by simple and powerful mathematics

The quality of concordance between measured physical behavior and mathematical description is amazing



This is an analog computer from around 1963

It was used to solve nonlinear ODEs via analog circuitry

It contained: about 20 integrators based on valves or transistors, some nonlinear function blocks using diodes, comparators

This was the state of the art for much physical system simulation

It relies on ideas underpinning MAE40

The tools of MAE40 are immediately useful in design

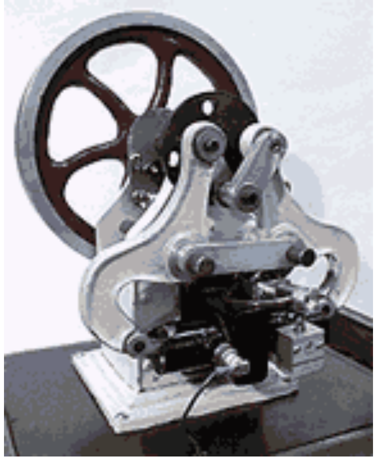
Why should you still be excited about MAE40?

The *front and back ends* of your digital cell phone are comprised of analog circuits

- This is pretty much true of all *digital* technology

Why does the stagecoach wheel appear to rotate backwards?

- Aliasing
  - A high frequency masquerading as a low frequency
- Anti-aliasing filters MUST be used in all sampled data systems



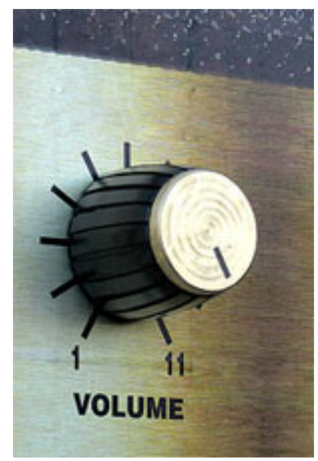
After MAE40 you will be able to start designing such anti-aliasing filters

- The real thing!

Why should you still be excited about MAE40?

You will learn how these knobs and plugs work:

We will not have time to cover this one:



Why should you still be excited about MAE40?

theguardian

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US nearly detonated atomic bomb over North Carolina – secret document

Exclusive: Journalist uses Freedom of Information Act to disclose 1961 accident in which one switch averted catastrophe

Ed Pilkington in New York
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The Guardian, Friday 20 September 2013 12.03 EDT
Jump to comments (1115)

The bomb that nearly exploded over North Carolina was 260 times more powerful than the device which devasted Hiroshima in 1945. Photo: Three Lions/Getty Images

A secret document, published in declassified form for the first time by the Guardian today, reveals that the US Air Force came dramatically close to detonating an atom bomb over North Carolina that would have been 260 times more powerful than the device that devastated Hiroshima.

The document, obtained by the investigative journalist Eric Schlosser under the Freedom of Information Act, gives the first conclusive evidence that the US was narrowly spared a disaster of monumental proportions when two Mark 39 hydrogen bombs were accidentally dropped over

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Eric Schlosser: 'The people who are most anti-nuclear are the ones who know most about it'
The US author talks about his six-year all-out immersion in the terrifying and surreal world of nuclear weapons for his latest book, Command and Control

"The bombs fell to earth after a B-52 bomber broke up in mid-air, and one of the devices behaved precisely as a nuclear weapon was designed to behave in warfare: its parachute opened, its trigger mechanisms engaged, and only **one low-voltage switch** prevented untold carnage."

"… a senior engineer in the Sandia national laboratories responsible for the mechanical safety of nuclear weapons concludes that 'one simple, dynamo-technology, **low voltage switch** stood between the United States and a major catastrophe.'"

Circuit variables (T&R Chap 1)

Charge and Energy

-1 coulomb (C) = 6.25×10^{18} electrons' charge

q

1 ampere (A) = 1 coulomb/second

$$i = \frac{dq}{dt}$$

- Current is a measure of charge passing
- Direction of flow is of positive charge
  - This is important for understanding some devices

Voltage/potential difference/electromotive force/tension

- Voltage measures the energy gained by a charge

1 joule(J)/coulomb = 1 volt (V)

$$v = \frac{dw}{dq}$$

- Voltage is measured between two points (potential difference)
  - This is akin to gravitational potential and fluid flow or temperature and heat flow

Circuit variables (contd)

Power is the rate of energy change per unit time $p = \dfrac{dw}{dt}$ watts (W)

Note the chain rule $$p = \left(\frac{dw}{dq}\right)\left(\frac{dq}{dt}\right) = vi$$

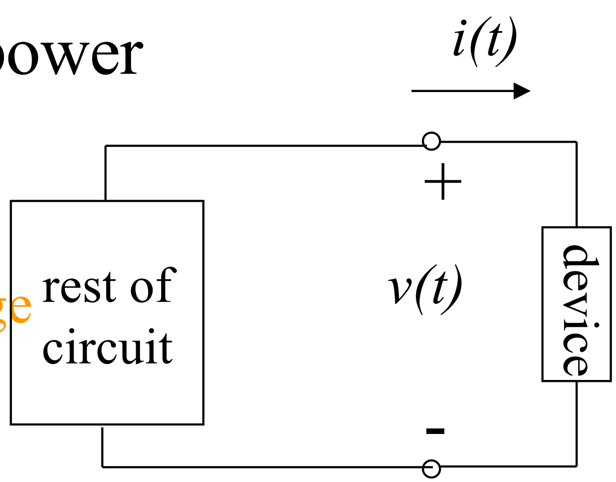
The instantaneous electrical power associated with a voltage and a current flow is given by the product $v \times i$

Passive sign convention

Power is positive when a device absorbs power

Signs are measured as illustrated

"The current variable must be defined so positive current enters the positive voltage terminal of the device"



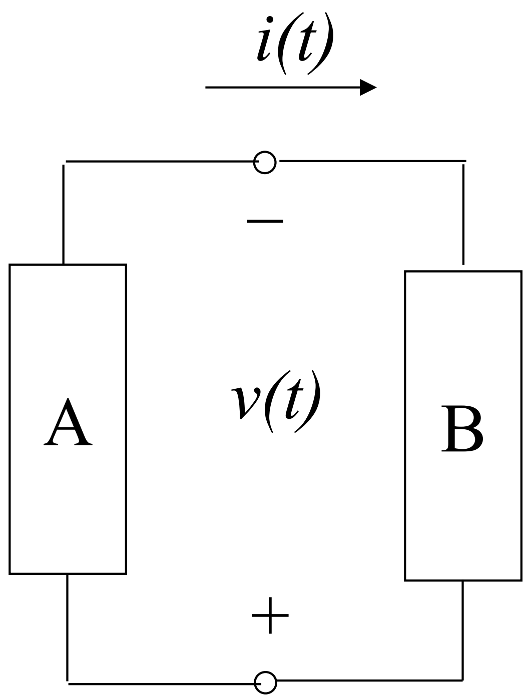
Note that $v(t)$ or $i(t)$ can be negative

In the illustration the current into the device is the negative of that into the rest of the circuit

An example

The figure at right depicts the circuit inside my flashlight

- The devices are the battery and the lamp
- A voltmeter reads $v(t)$ is 12V
- An ammeter reads $i(t)$ is –1.5A



Which device is the battery and which is the lamp? How much power is being consumed by the lamp?

Using the passive sign convention

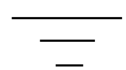
- The power absorbed by device A is 12×-1.5 W = -18 W
- A is the battery and B is the lamp, which consumes 18W

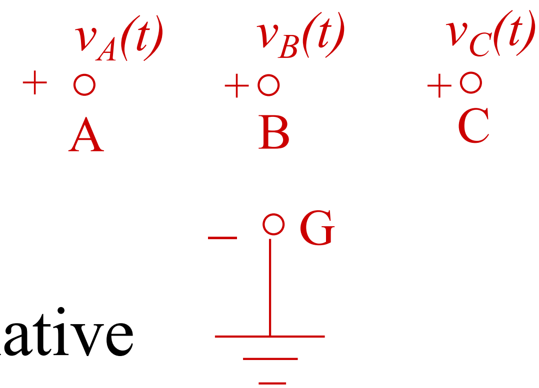
Circuit variables (contd)

Ground

Because all voltages are measured between two points, there is no absolute voltage - it is a potential

In circuits we often refer all voltages of points as being relative to a fixed point or *ground* voltage

This is depicted via the  symbol



The terms v_A etc mean the voltage at A relative to G

This is like referring heights relative to sea level

Gravity is also a potential

Check out the elevator buttons in the Math & App Phys Building!

Circuit variables (contd)

A *circuit* is a collection of interconnected electrical devices

For us:

- All electromagnetic interactions in the circuit take place within the devices (This is a *lumped-parameter* circuit)
- The circuit devices are connected together by wires which are ideal
  - They have the same voltage at both ends instantaneously
  - They propagate current without loss instantaneously
  - They may be stretched arbitrarily without changing properties
- All circuit devices have at least two terminals and are assumed not to accumulate charge – current in equals current out

A *node* is the junction of terminals of two or more devices

A *loop* is a closed path formed by tracing through an ordered sequence of nodes without passing through any node more than once

Circuit variables

The important physical circuit variables are currents into and voltages across circuit elements and their associated power consumption or production.

Circuit analysis consists of solving for the circuit variables in a given circuit

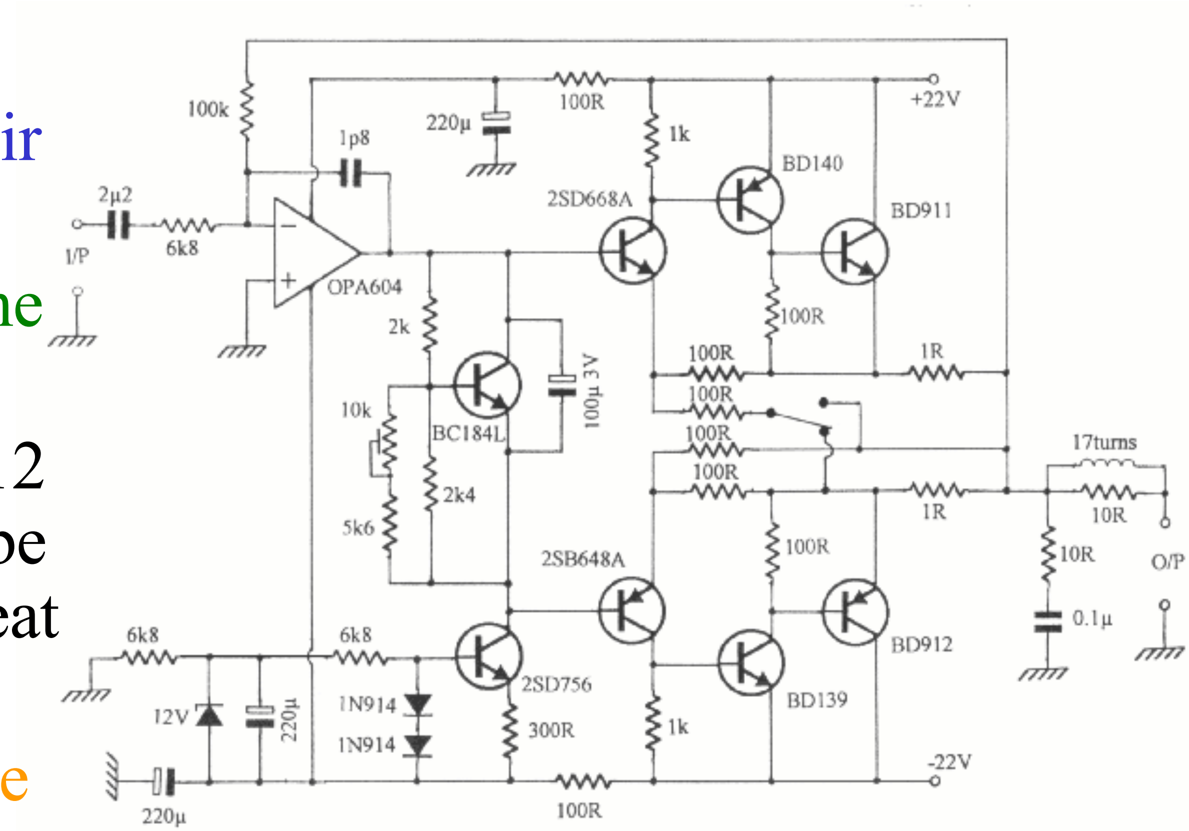
Circuit design or *synthesis* consists of constructing a circuit whose circuit variables exhibit a specified behavior

The *topology* of a circuit is the key structure

- It can be stretched and wrapped around without changing circuit variables
  - Nodes can be stretched and shrunk to cover many terminals

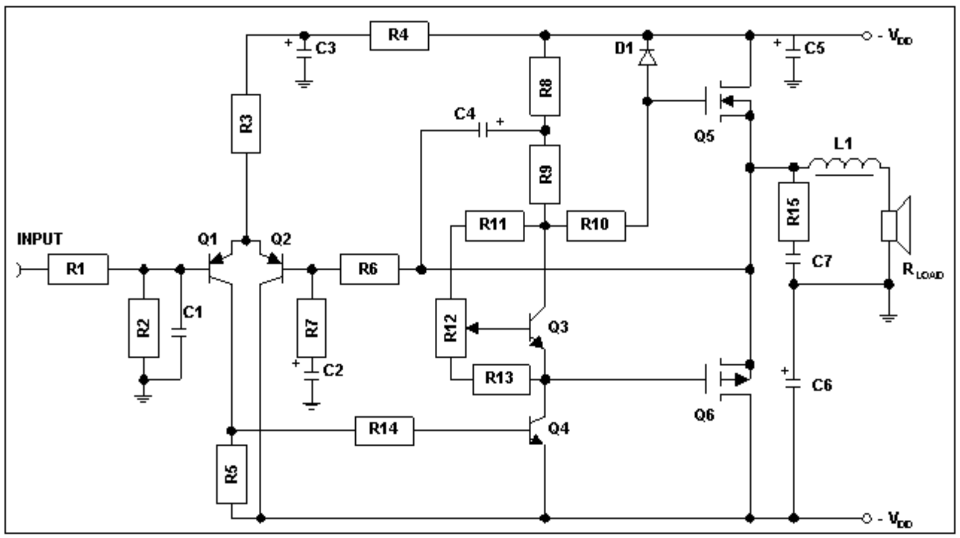
Circuit Topology

- This is the circuit diagram of an audio amplifier
- It shows the relationship between parts but not their actual physical location
- The ground/earth is likely the chassis or case
- The output transistors BD912 and BD911 will need to be attached to an external heat sink
- The op-amp OPA604 will be board mounted
- The volume control needs to be accessible



Circuit Topology – high power amplifier with complementary HEXFETS

Circuit Diagram



Circuit Board Layout

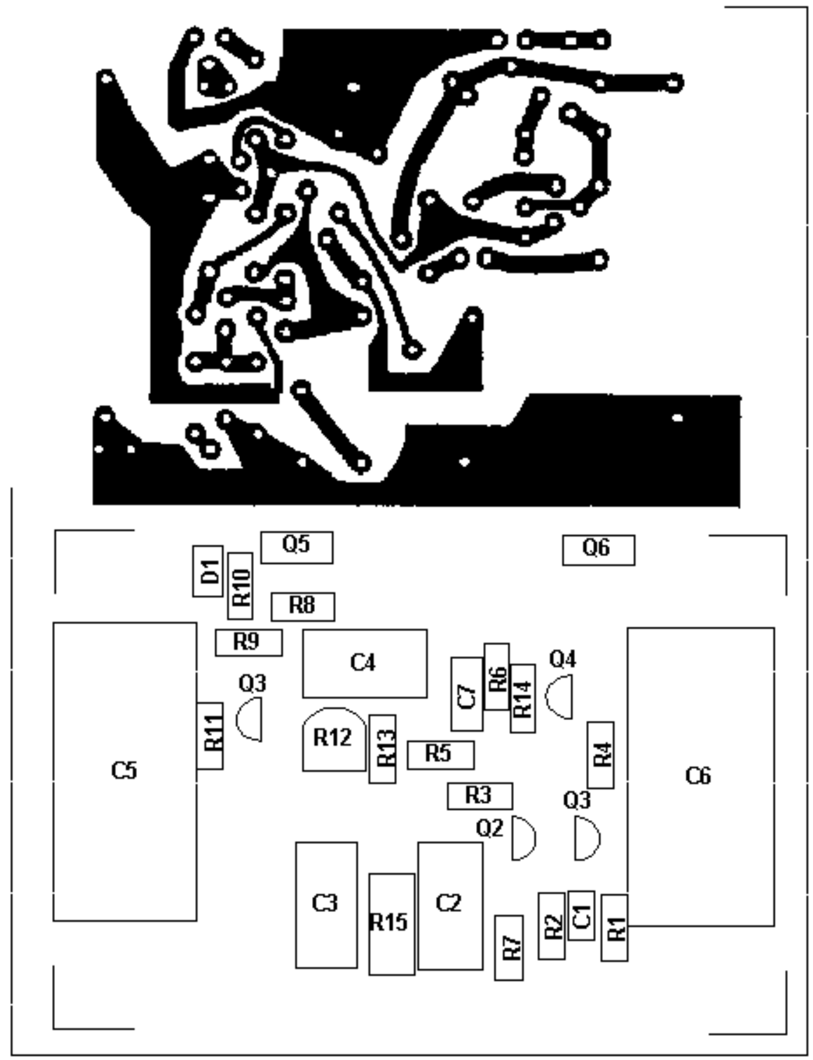


Figure 2. Amplifier Printed Circuit Board Layout

HEXFET – power MOSFET from International Rectifier

Basic Circuit Analysis (T&R Chap 2)

Kirchoff's Current Law (KCL)

The algebraic sum of currents entering a node is zero at every instant

- This is a restatement of the principle of conservation of charge
- Alternatively, but not so nice,

The sum of currents entering a node is equal to the sum of currents leaving a node at every instant

- KCL provides linear constraints between the currents in a circuit

In a circuit containing a total of N nodes there are only N-1 independent KCL connection equations

T&R, 5th ed, Exercise 2-1 p 22

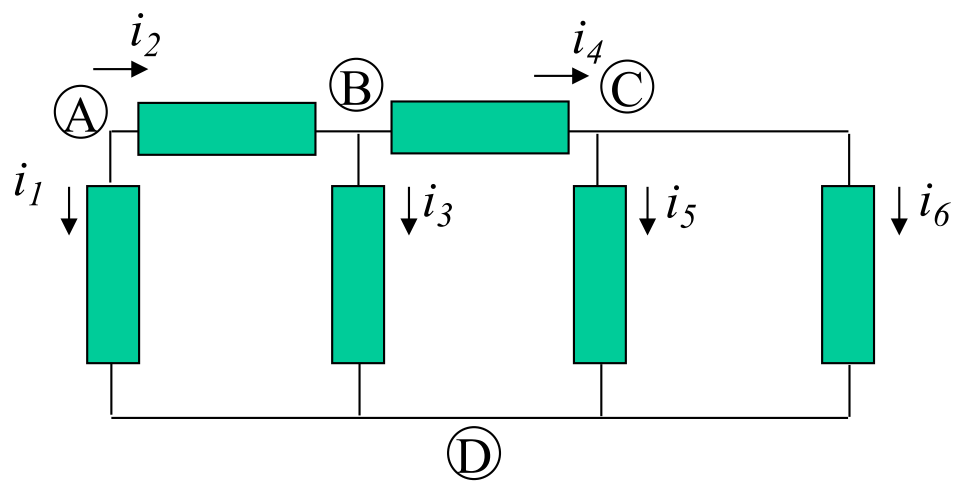
Write KCL at nodes *A, B, C, D*

Node A: $-i_1 - i_2 = 0$

Node B: $i_2 - i_3 - i_4 = 0$

Node C: $i_4 - i_5 - i_6 = 0$

Node D: $i_1 + i_3 + i_5 + i_6 = 0$



If i_1=–1mA, i_3=0.5mA, i_6=0.2mA, find the rest

i_2=1mA

i_4=0.5mA

i_5=0.3mA

$$\begin{pmatrix} -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Note that we have 4 nodes

But only 3 independent equations

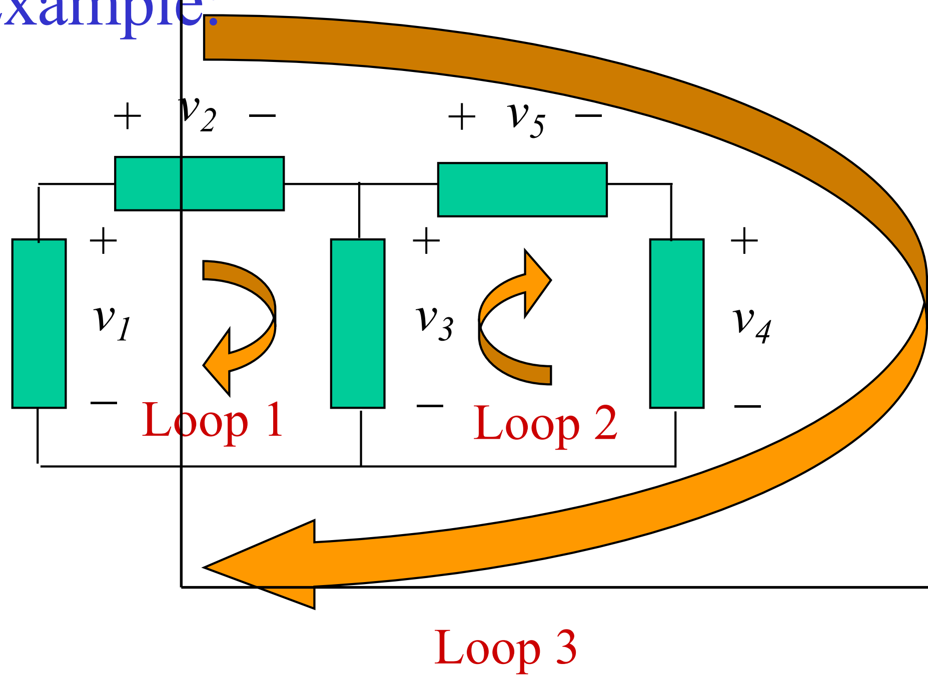
Kirchoff's Voltage Law (KVL)

The algebraic sum of voltages around a loop is zero at every instant

The picture can't be displayed.

This is really a restatement of voltage as a potential, effectively a statement of the principle of conservation of energy

Example:



$$\text{Loop 1}: -v_1 + v_2 + v_3 = 0$$

$$\text{Loop 2}: -v_3 + v_4 + v_5 = 0$$

$$\text{Loop 3}: -v_1 + v_2 + v_4 + v_5 = 0$$

$$\begin{pmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ -1 & 1 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Circuit Analysis

One way to consider the analysis of a circuit (i.e. the computation of the currents in and voltage across the circuit elements) is through the specification of constraints on these variables

- KCL and KVL provide one set of constraints on circuit variables reflecting connection topology
- The *i-v* properties of the devices themselves provide further constraints
  - *N* nodes and *E* circuit elements
    - *N-1* independent KCL relations
    - *E-N+1* independent KVL relations (number of independent loops)
    - *E* independent *i-v* characteristics
  - *2E* circuit variables

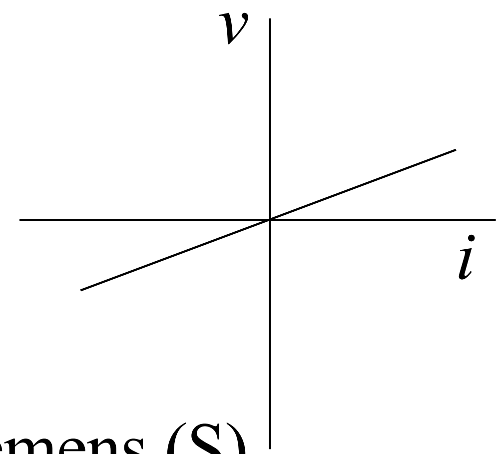
If we keep to linear devices then we have linear circuit analysis

- *2E* Linear algebraic equations for 2*E* variables

Linear Circuit Elements

Linearity in v-i relations

- Doubling v doubles i and vice versa $i=\alpha v$
  - Straight lines in i-v plane
- Note that $i=\alpha v+\beta$ describes an *affine* relation



Resistor

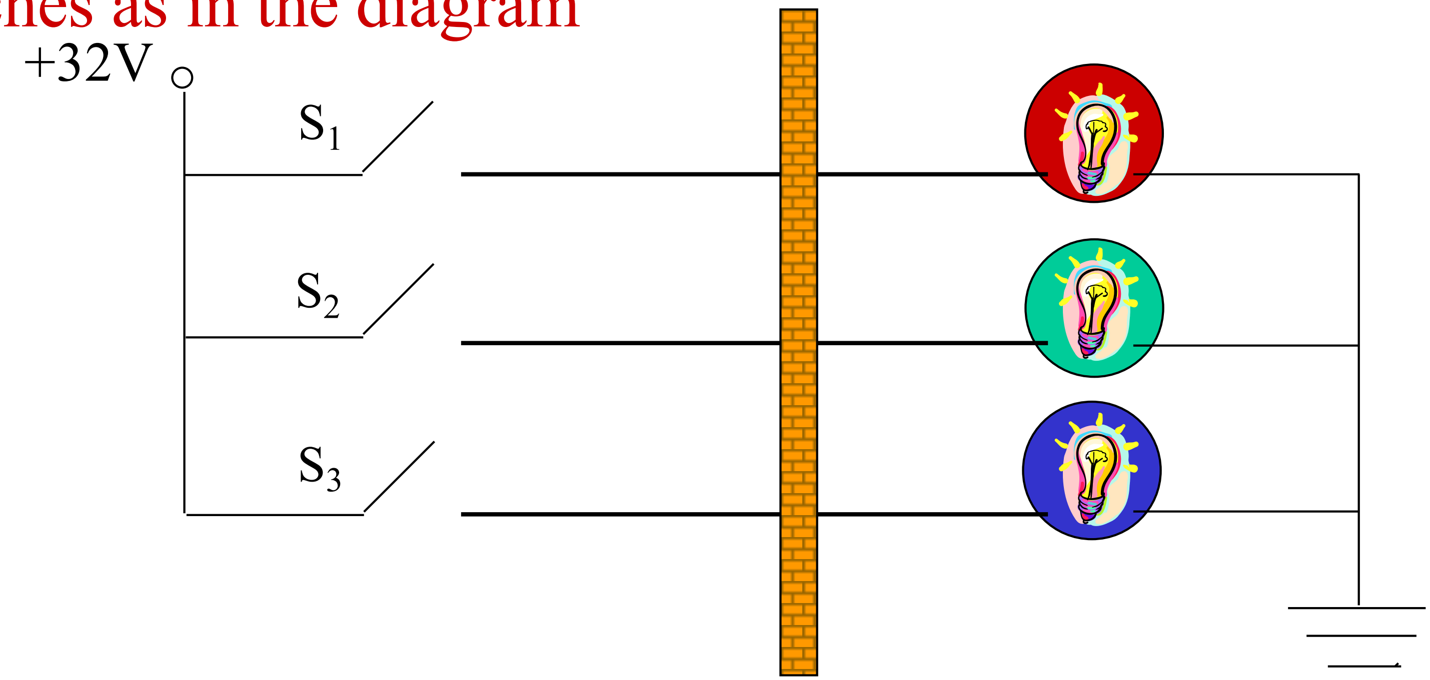
- $v=Ri$ or $i=Gv$ Ohm's Law
- R resistance in ohms (Ω), G conductance in siemens (S)
- Power $P=i^2R= v^2/R=v^2G$
- A resistor always absorbs power – i-v line has positive slope

This is an instantaneous linear element

- We will look at circuit elements with memory like C and L later in the course

Time out puzzle

We have three lamps – red, green and blue – connected to three switches as in the diagram



The lamps are outside the switch room and we are alone

What single test can we perform by playing with the switches to determine which switch is connected to which light?

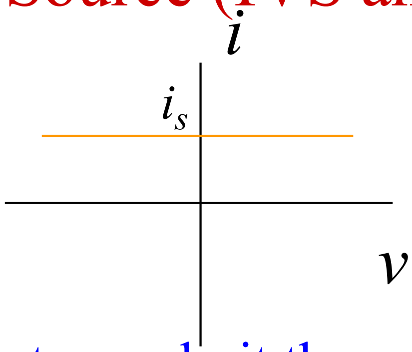
I know of two solutions, one of physicists, one of engineers

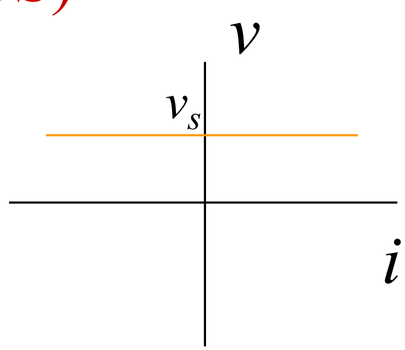
Linear Circuit Elements (contd)

Independent Voltage and Current Source (IVS and ICS)

IVS $v=v_s$ for any i

ICS $i=i_s$ for any v





Affine elements rather than linear, but we admit them

Most sources of electrical energy (batteries, generators, power systems) are modeled as voltage sources. Examples of current sources include diode and transistor current sources

Open circuit and short circuit – variants of ICS and IVS resp.

Open circuit $i=0$ for any v Same as $R=\infty$

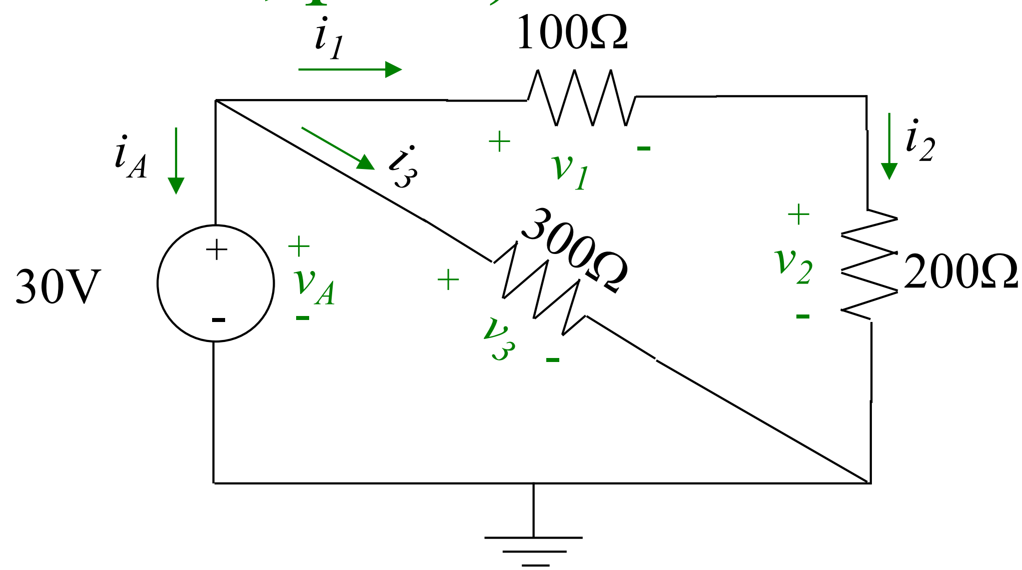
Short circuit $v=0$ for any i Same as $R=0$

These are truly linear since the curves pass through (0,0)

Switches – either o.c. or s.c.

Example 2-10 (T&R, 5th ed, p. 31)

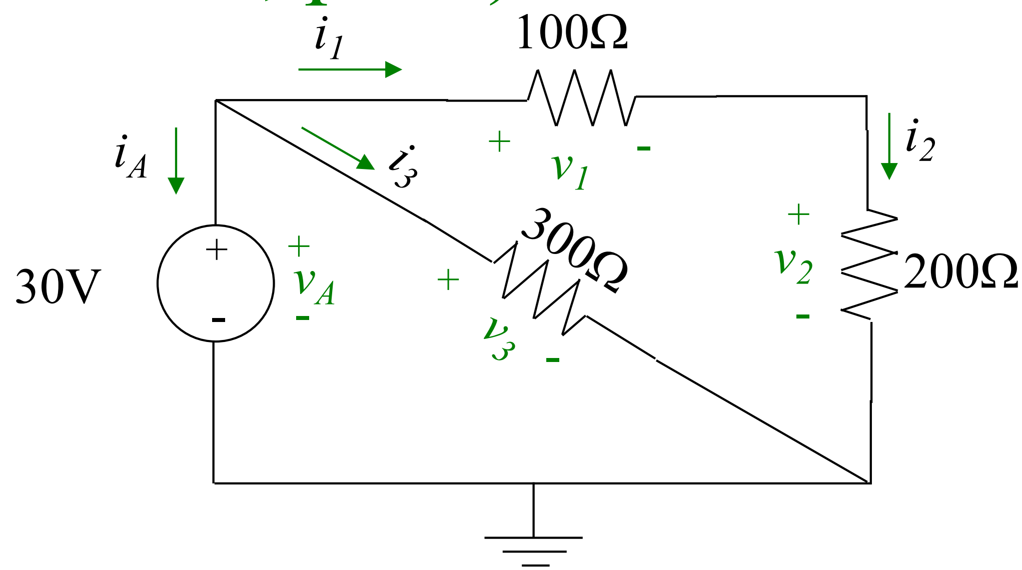
Write a complete set of equations for each element



Write a complete set of connection equations

Solve these equations for all currents and voltages

Example 2-10 (T&R, 5th ed, p. 31)



Write a complete set of equations

for each element

$v_A=30V;$

$v_1=100i_1;$

$v_2=200i_2;$

$v_3=300i_3;$

Write a complete set of connection equations

$-i_A-i_1-i_3=0;\quad i_1-i_2=0;\quad -30+v_3=0;\quad v_1+v_2-v_3=0;$

Solve these equations for all currents and voltages

$v_A=30V;\quad v_1=10V;\quad v_2=20V;\quad v_3=30V;$

$i_A=-200mA;\quad i_1=i_2=i_3=100mA;$

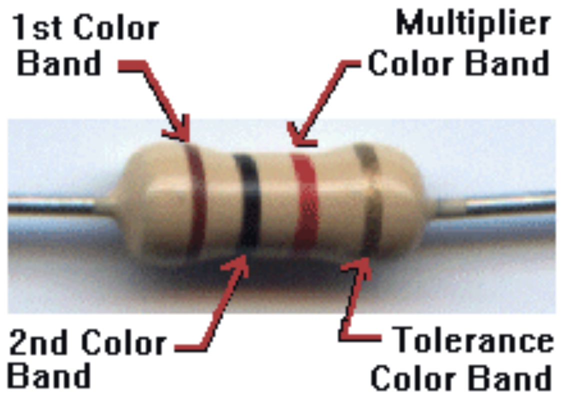
All you ever wanted to know about resistors…

Resistors are color coded to indicate their value

Black	Brown	Red	Orange	Yellow	Green	Blue	Violet	Grey	White
0	1	2	3	4	5	6	7	8	9

and tolerance

5%	10%	20%
Gold	Silver	Empty

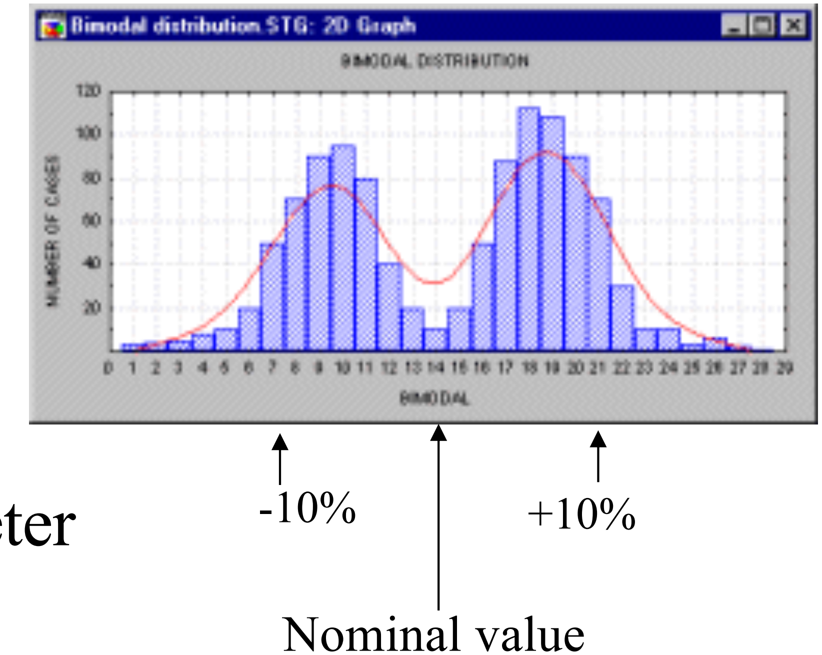


This is a 1KΩ resistor with 5% tolerance

Resistors come at standard values (e.g., at 10% tolerance)

1	1.2	1.5	1.8	2.2	2.7	3.3	3.9	4.7	5.6	6.8	8.2	Nominal
.9	1.08	1.35	1.62	1.98	2.43	2.97	3.51	4.23	5.04	6.12	7.38	Lower
1.1	1.32	1.65	1.98	2.42	2.97	3.63	4.29	5.17	6.16	7.48	9.02	Upper

Even more than you ever wanted to know about resistors …



The distribution of resistances

Why should this be so?

How can we get a 258.3Ω resistor?

Keep testing the ones in the box

A really finely tunable potentiometer (variable resistor) – a *trimpot*

A coarsely tunable potentiometer in parallel with a lower *R*

Make one from high precision components

Another important characteristic of *R* is the power rating

Why?