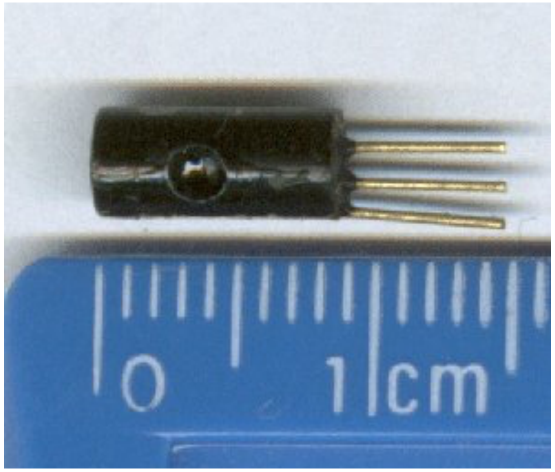
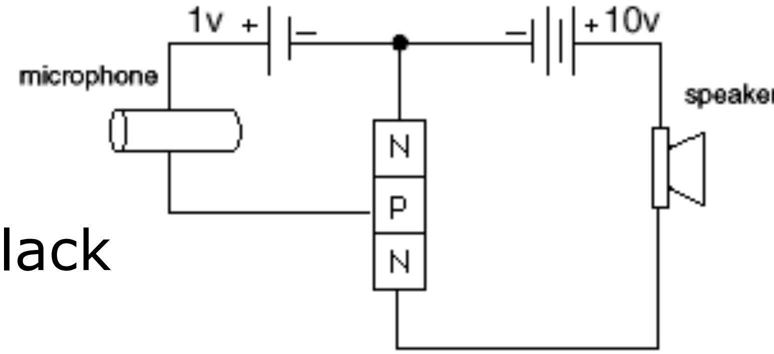
Active Circuits: Life gets interesting

Active cct elements – operational amplifiers (OP-AMPS) and transistors

- Devices which can inject power into the cct
- External power supply – normally comes from connection to the voltage supply "rails"
- Capable of linear operation – amplifiers
  - and nonlinear operation – typically switches
- Triodes, pentodes, transistors



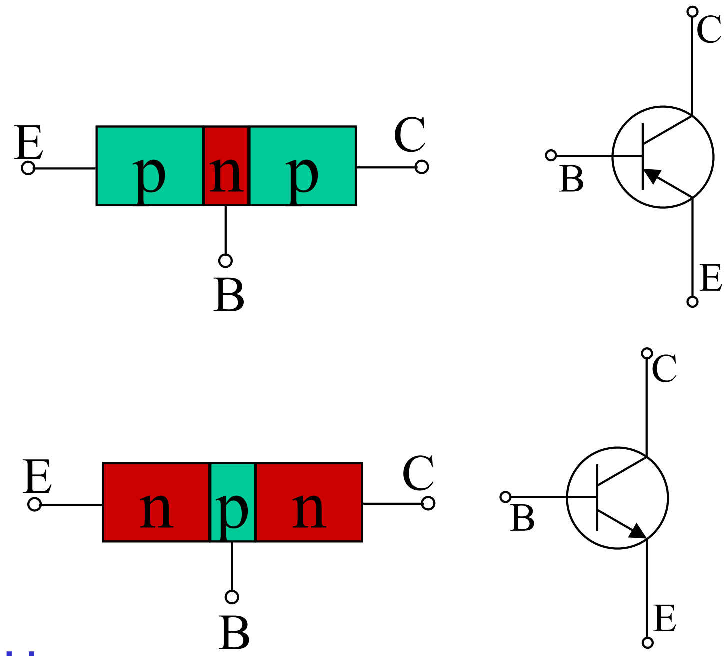
Active Cct Elements



Amplifiers – linear & active

- Signal processors
- Stymied until 1927 and Harold Black
  - Negative Feedback Amplifier
    - Control rescues communications
    - Telephone relay stations manageable against manufacturing variability
- Linearity
  - Output signal is proportional to the input signal
    - Note distinction between signals and systems which transform them
  - Yes! Just like your stereo amplifier
- Idea – controlled current and voltage sources

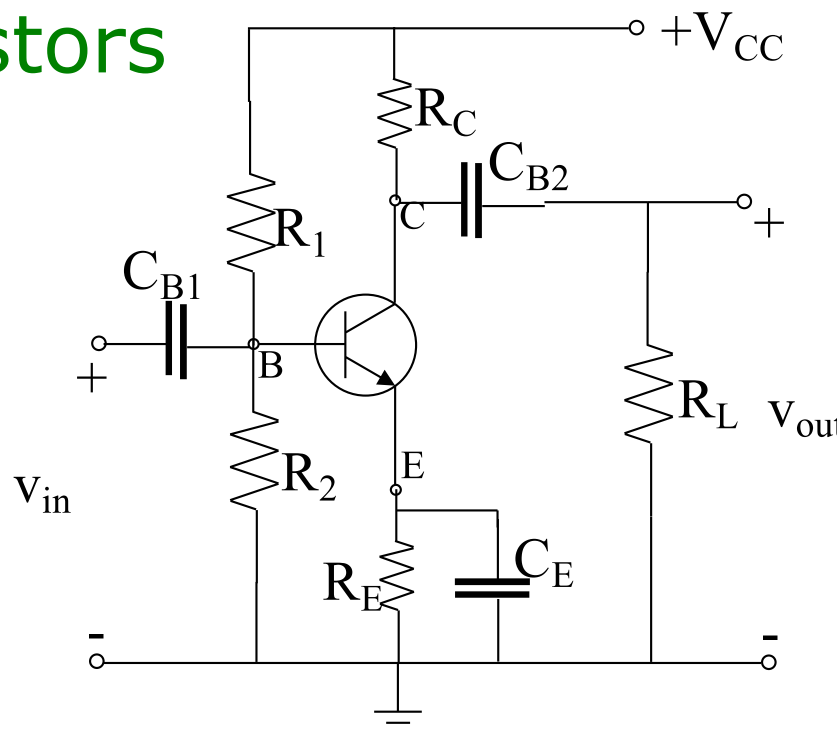
A Brief Aside - Transistors



Bipolar Junction Transistors

- Semiconductors – doped silicon
  - n-doping: mobile electrons
    - Si doped with Sb, P or As
  - p-doping: mobile holes
    - Si doped with B, Ga, In
- Two types npn and pnp
  - Heavily doped Collector and Emitter
  - Lightly doped Base and very thin
  - Collector and Emitter thick and dopey
- Need to bias the two junctions properly
  - Then the base current modulates a strong C→E current
    - Amplification $i_C = \beta i_B$

Transistors



Common Emitter Amplifier Stage

Biasing resistors R_1 and R_2

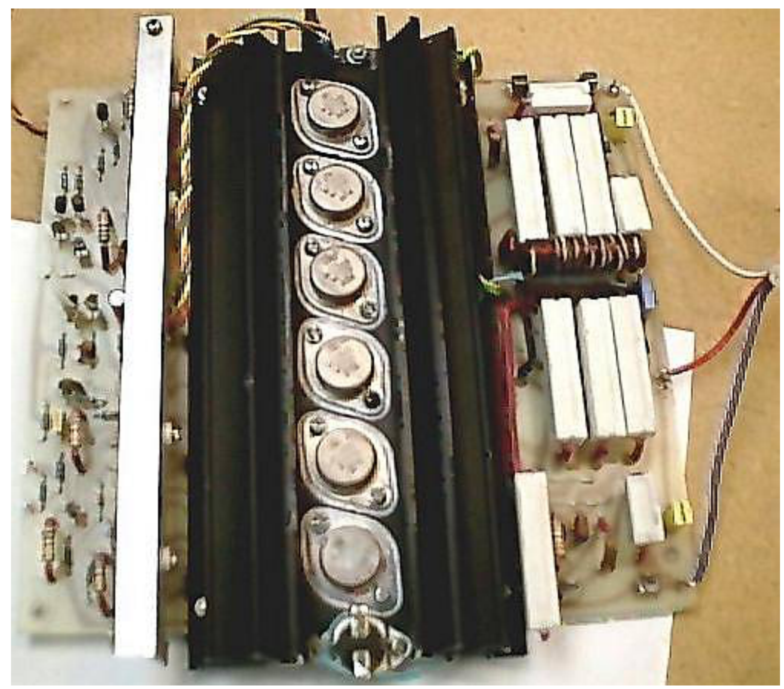
Keep transistor junctions biased in amplifying range

Blocking capacitors C_{B1} and C_{B2}

Keep dc currents out

Feedback capacitor C_E

Grounds emitter at high frequencies



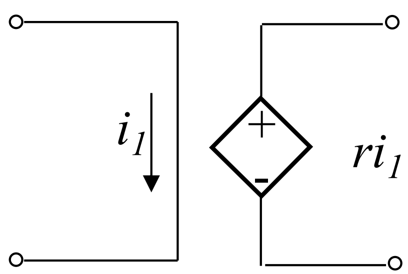
Small changes in v_{in}
Produce large changes in v_{out}

Linear Dependent Sources

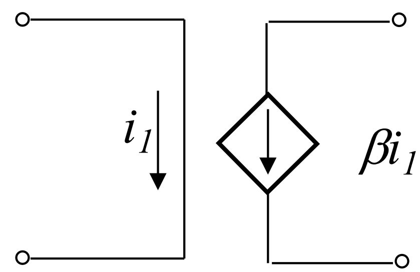
Active device models in linear mode

Transistor takes an input voltage v_i and produces an output current $i_0 = gv_i$ where g is the gain

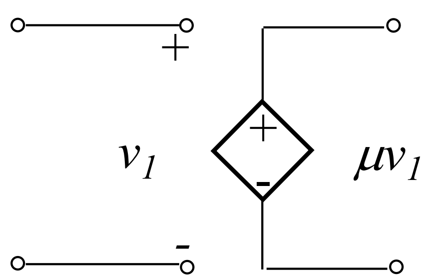
This is a linear voltage-controlled current source VCCS



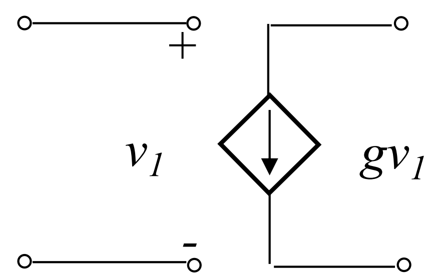
CCVS r transresistance



CCCS β current gain



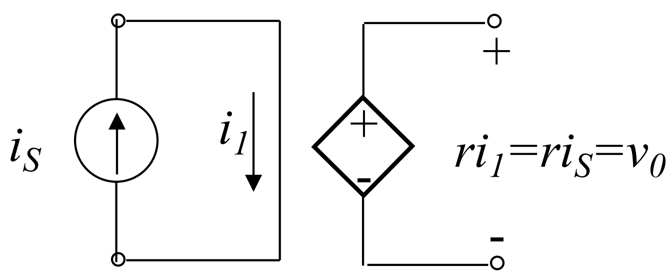
VCVS μ voltage gain



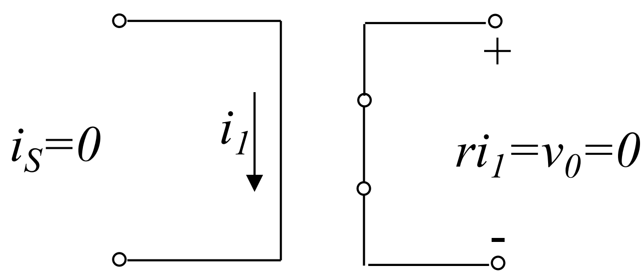
VCCS g transconductance

Linear dependent source (contd)

- Linear dependent sources are parts of active cct models – they are not separate components
  - But they allow us to extend our cct analysis techniques to really useful applications
  - This will become more critical as we get into dynamic ccts
- Dependent elements change properties according to the values of other cct variables



Source on

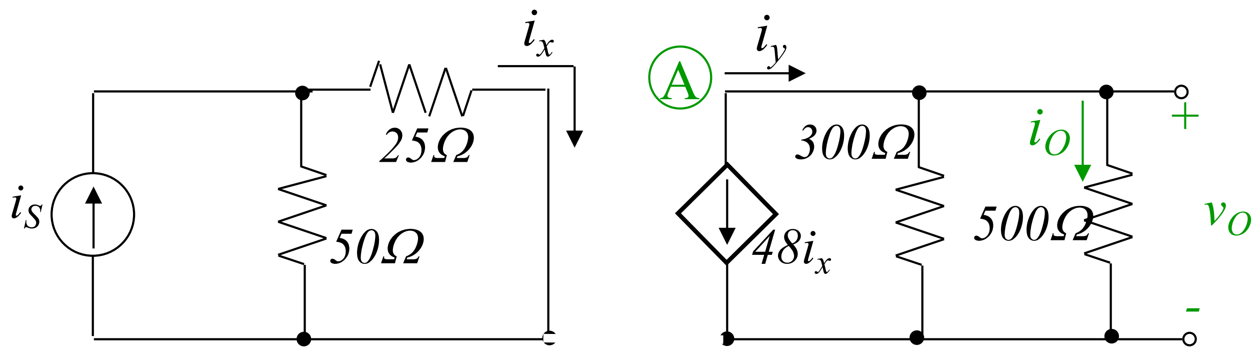


Source off

Cct Analysis with Dependent Sources

Golden rule – do not lose track of control variables

Find i_O, v_O and P_O for the 500Ω load



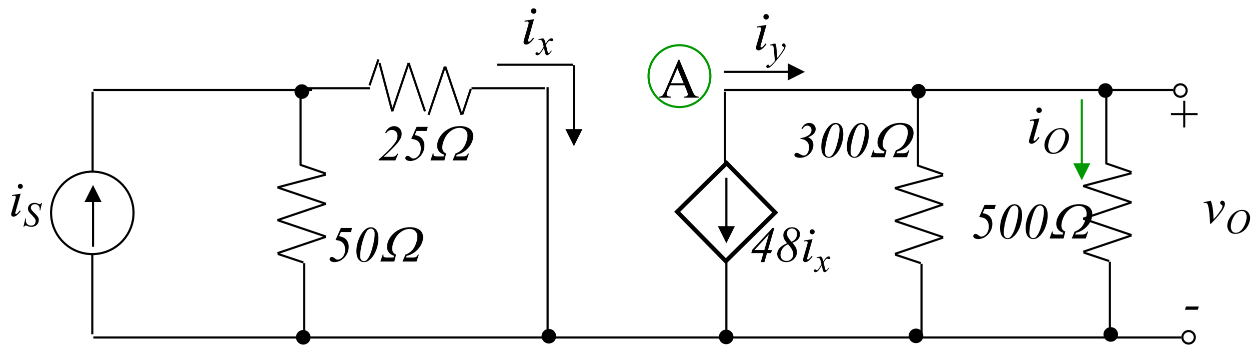
Current divider on LHS $i_x = \frac{2}{3} i_S$

Current divider on RHS $i_O = \frac{3}{8}(-48)i_x = -18i_x = -12i_S$

Ohm's law $v_O = i_O 500 = -6000 i_S$

Power $p_O = i_O v_O = 72{,}000 i_S^2$

Analysis with dependent sources



Power provided by ICS $$p_S = (50\|25)i_S^2 = \frac{50}{3}i_S^2$$

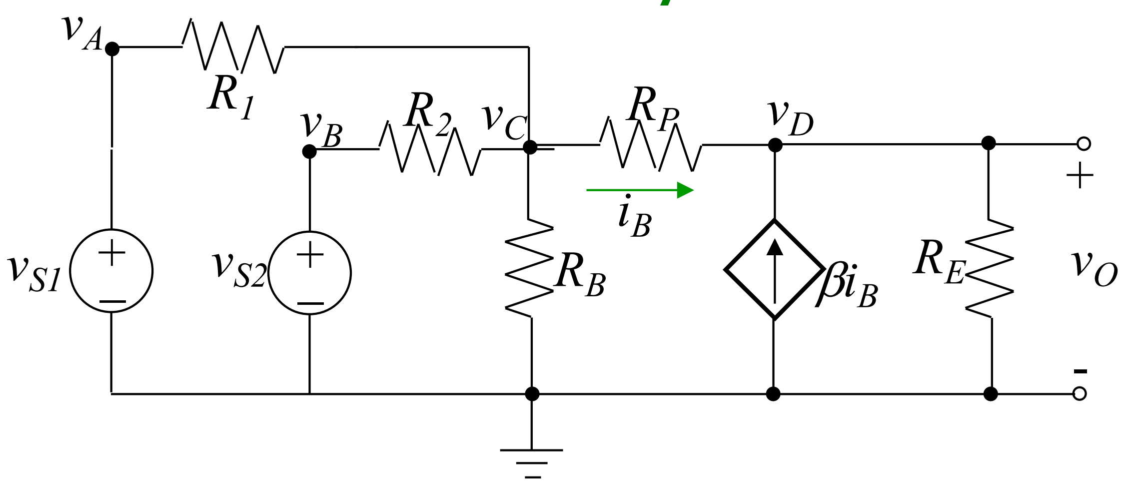
Power delivered to load $$72000 i_S^2$$

Power gain $$G = \frac{p_O}{p_S} = \frac{72000 i_S^2}{\frac{50}{3} i_S^2} = 4320$$

Where did the energy come from?

External power supply

Nodal Analysis with Dependent Source



KCL at node C $G_1(v_C - v_{S1}) + G_2(v_C - v_{S2}) + G_B v_C + G_P(v_C - v_D) = 0$

KCL at node D $G_P(v_D - v_C) + G_E v_D - \beta i_B = 0$

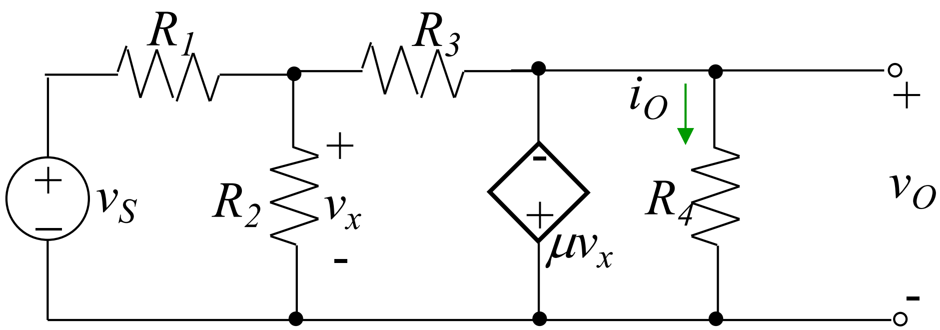
CCCS element description $i_B = G_P(v_C - v_D)$

Substitute and solve

$$(G_1 + G_2 + G_B + G_P)v_C - G_P v_D = G_1 v_{S1} + G_2 v_{S2}$$

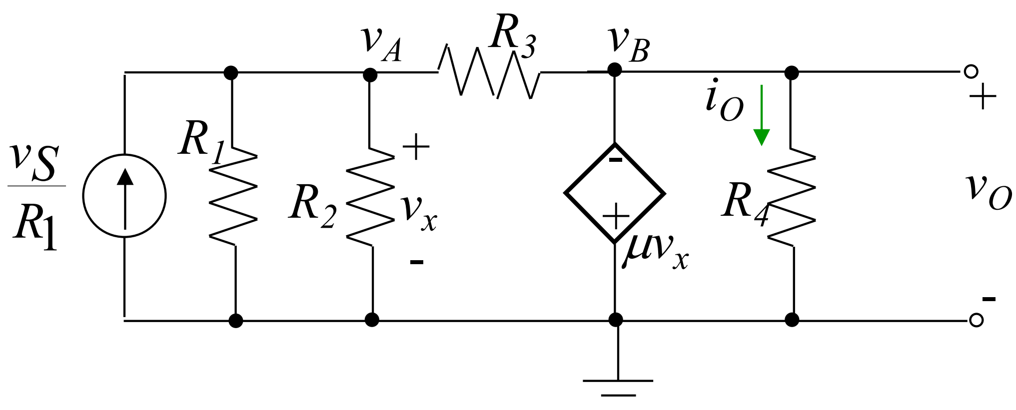
$$-(\beta + 1)G_P v_C + \left[(\beta + 1)G_P + G_E\right]v_D = 0$$

T&R, 5th ed, Example 4-3 p 148



Find v_O in terms of v_S

What happens as $\mu \to \infty$?



Node A:

$$(G_1+G_2+G_3)v_A - G_3 v_B = G_1 v_S$$

Node B:

$$v_B = -\mu v_x = -\mu v_A$$

Solution:

$$v_O = v_B = -\mu v_A = \left(\frac{-\mu G_1}{G_1+G_2+(1+\mu)G_3}\right) v_S$$

For large gains μ: $(1+\mu)G_3 >> G_1+G_2$

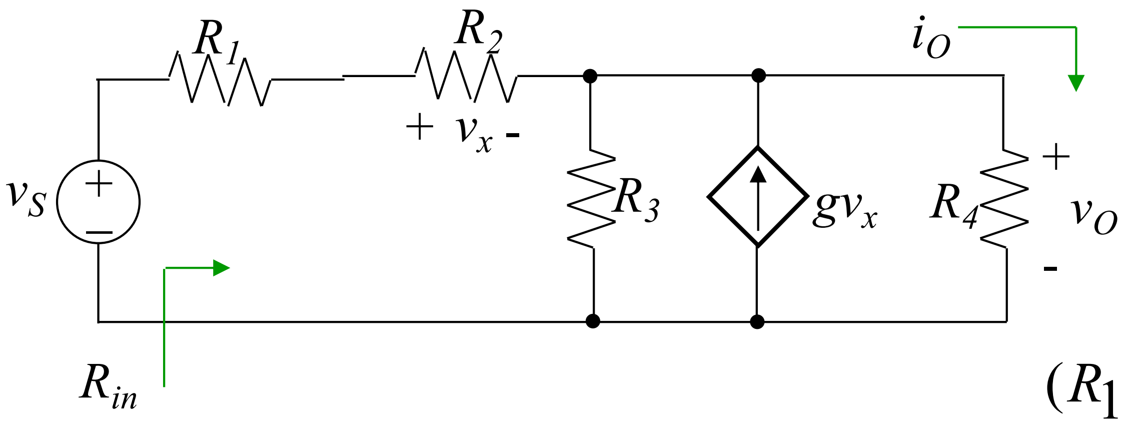
$$v_O \approx \left[\frac{-\mu G_1}{(1+\mu)G_3}\right] v_S \approx -\frac{R_3}{R_1} v_S$$

This is a model of an inverting op-amp

Mesh Current Analysis with Dependent Sources

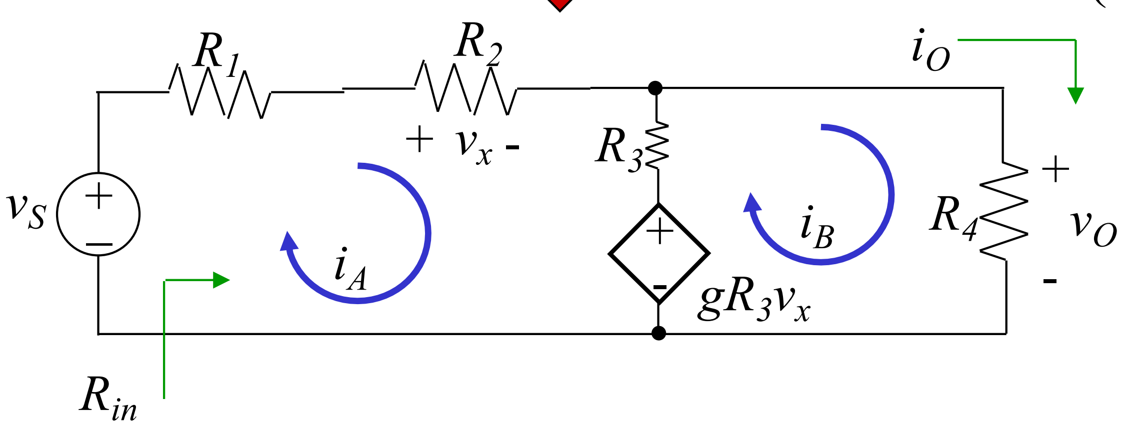
Dual of Nodal Analysis with dependent sources

Treat the dependent sources as independent and sort out during the solution

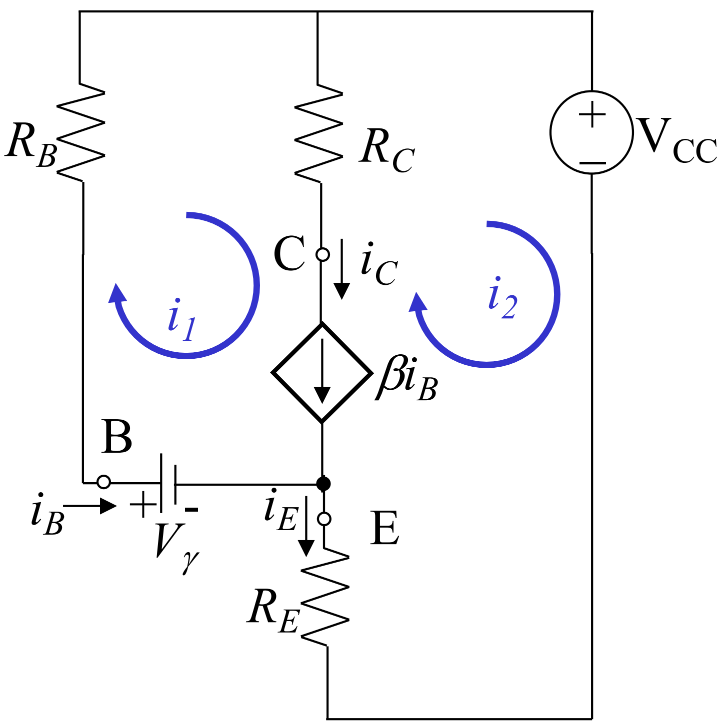


$$(R_1 + R_2 + R_3 + gR_2R_3)i_A - R_3 i_B = v_S$$

$$-(R_3 + gR_2R_3)i_A + (R_3 + R_4)i_B = 0$$



T&R, 5th ed, Example 4-5 BJTransistor



Needs a supermesh

Current source in two loops without *R* in parallel

Supermesh = entire outer loop

Supermesh equation

$$i_2 R_E - V_\gamma + i_1 R_B + V_{CC} = 0$$

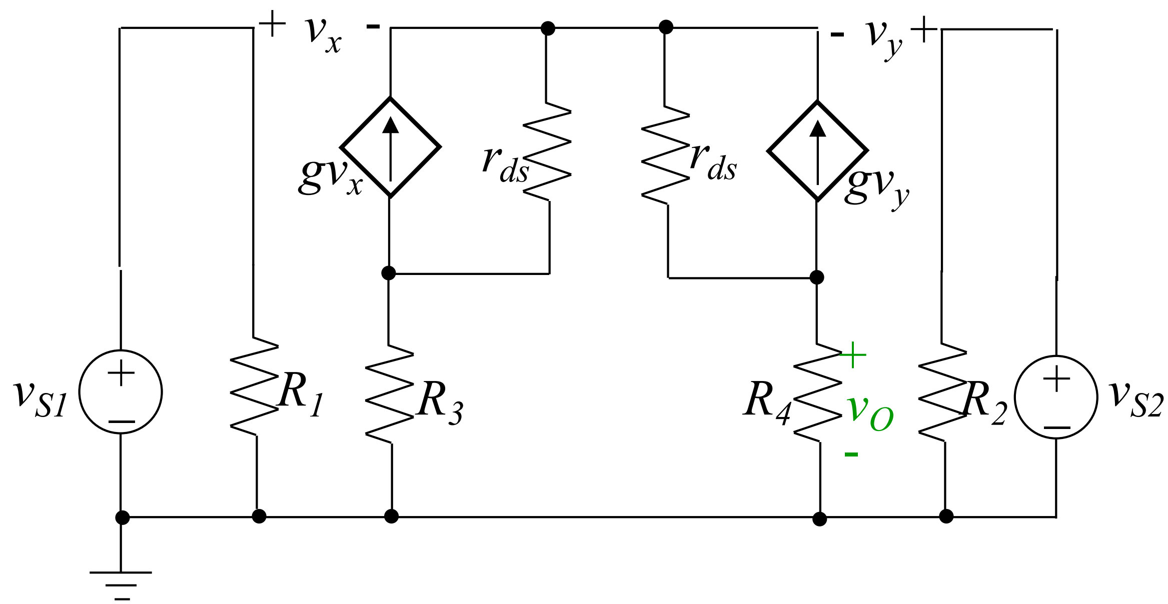
Current source constraint

$$i_1 - i_2 = \beta i_B$$

Solution

$$i_B = -i_1 = \frac{V_{CC} - V_\gamma}{R_B + (\beta + 1) R_E}$$

T&R, 5th ed, Example 4-6 Field Effect Transistor



Since cct is linear $v_O = K_1 v_{S1} + K_2 v_{S2}$

Solve via superposition

First v_{S1} *on* and v_{S2} *off,* then v_{S1} *off* and v_{S2} *on*

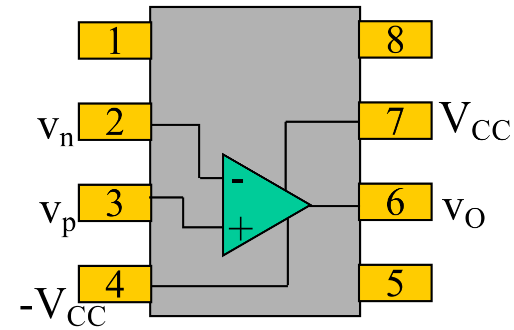
This gives K_1 and K_2

Operational Amplifiers - OpAmps

Basic building block of linear analog circuits

Package of transistors, capacitors, resistors, diodes in a chip

Five terminals



- Positive power supply V_{CC}
- Negative power supply - V_{CC}
- Non-inverting input v_p
- Inverting input v_n
- Output v_O

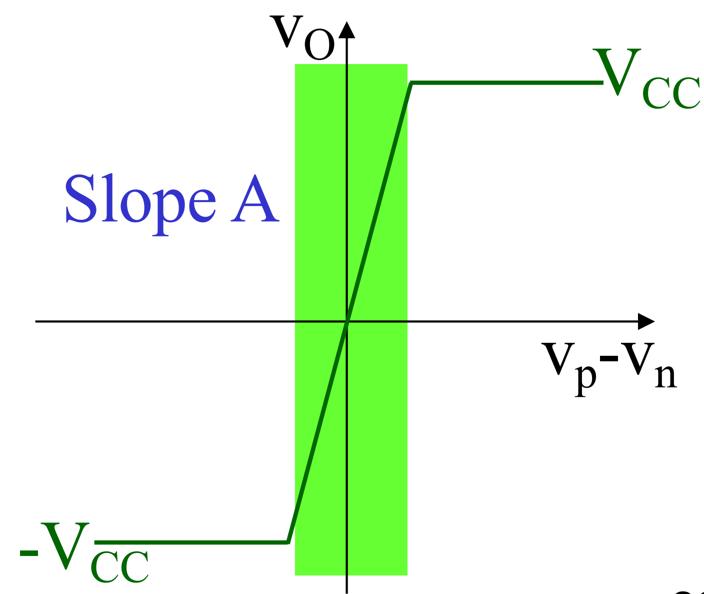
Linear region of operation

$$v_O = A(v_p - v_n)$$

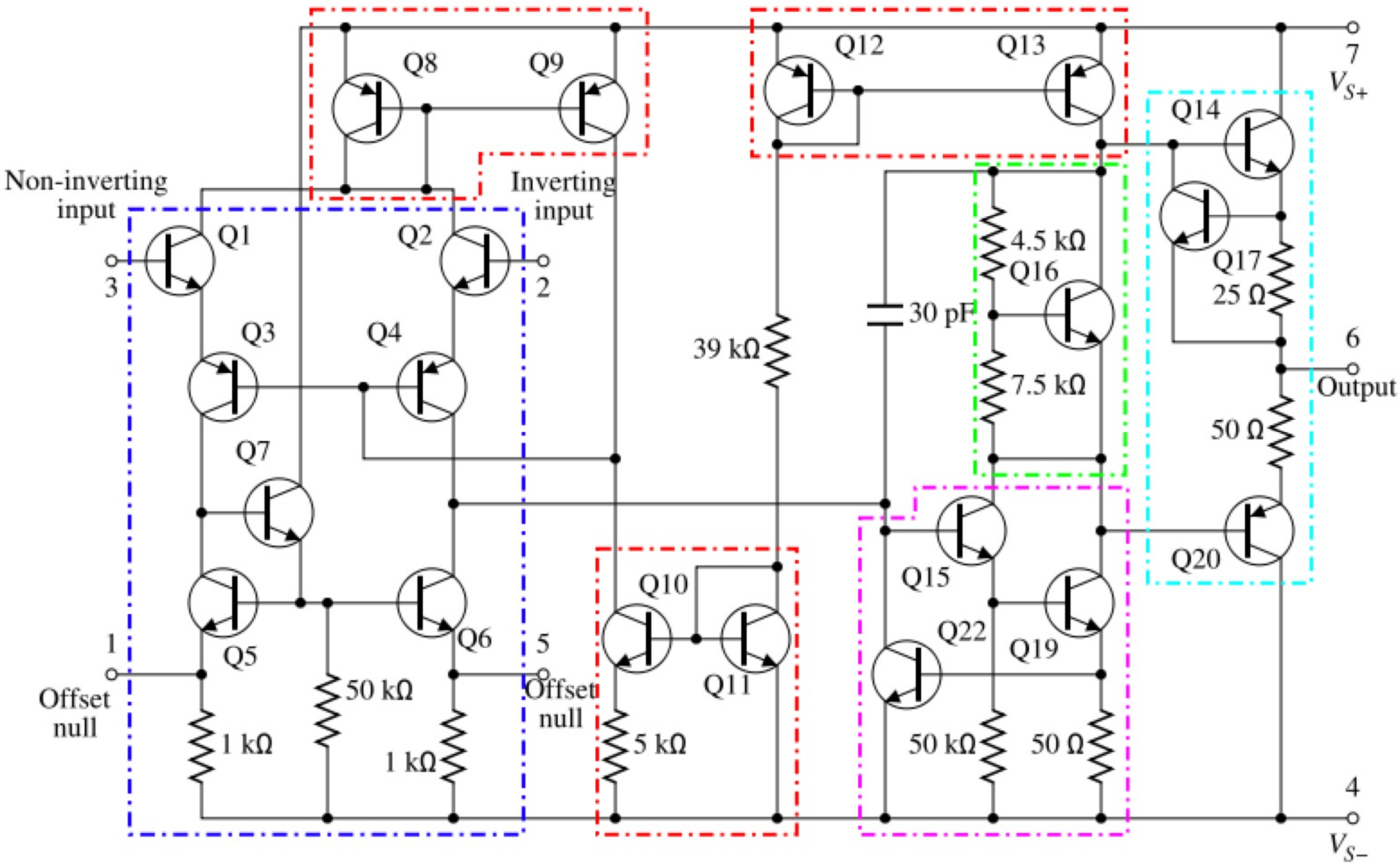
Ideal behavior

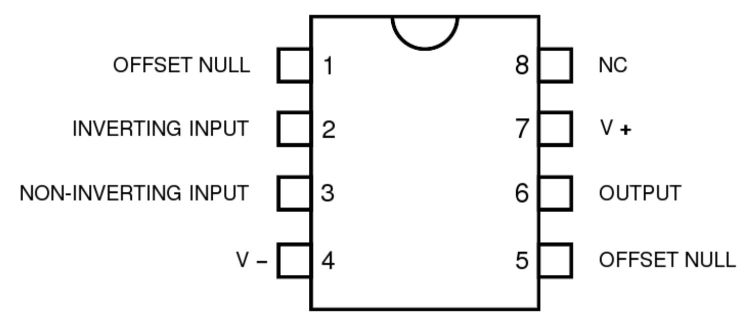
$$10^5 < A < 10^8$$

Saturation at V_{CC}/-V_{CC} limits range

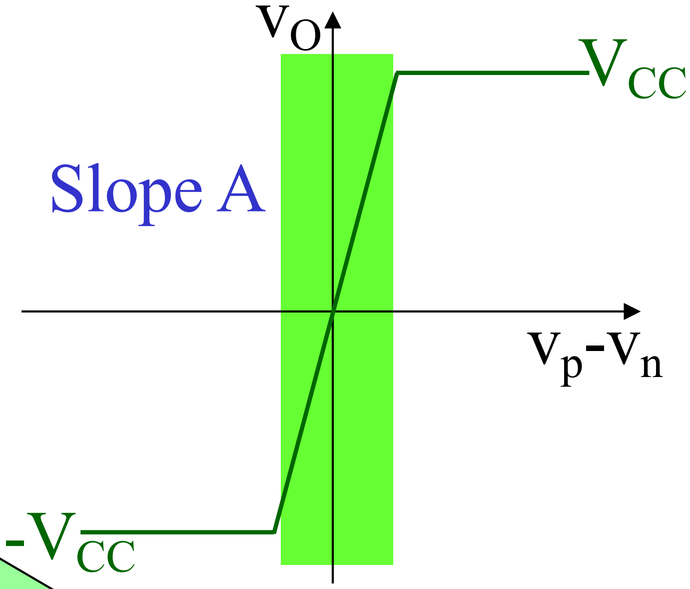


Real OpAmp (u741)





Ideal OpAmp



Equivalent linear circuit

Dependent source model

$10^6 < R_I < 10^{12}\Omega$ $\infty\Omega$

$10 < R_O < 100\Omega$ 0Ω

$10^5 < A < 10^8$ ∞

Need to stay in linear range

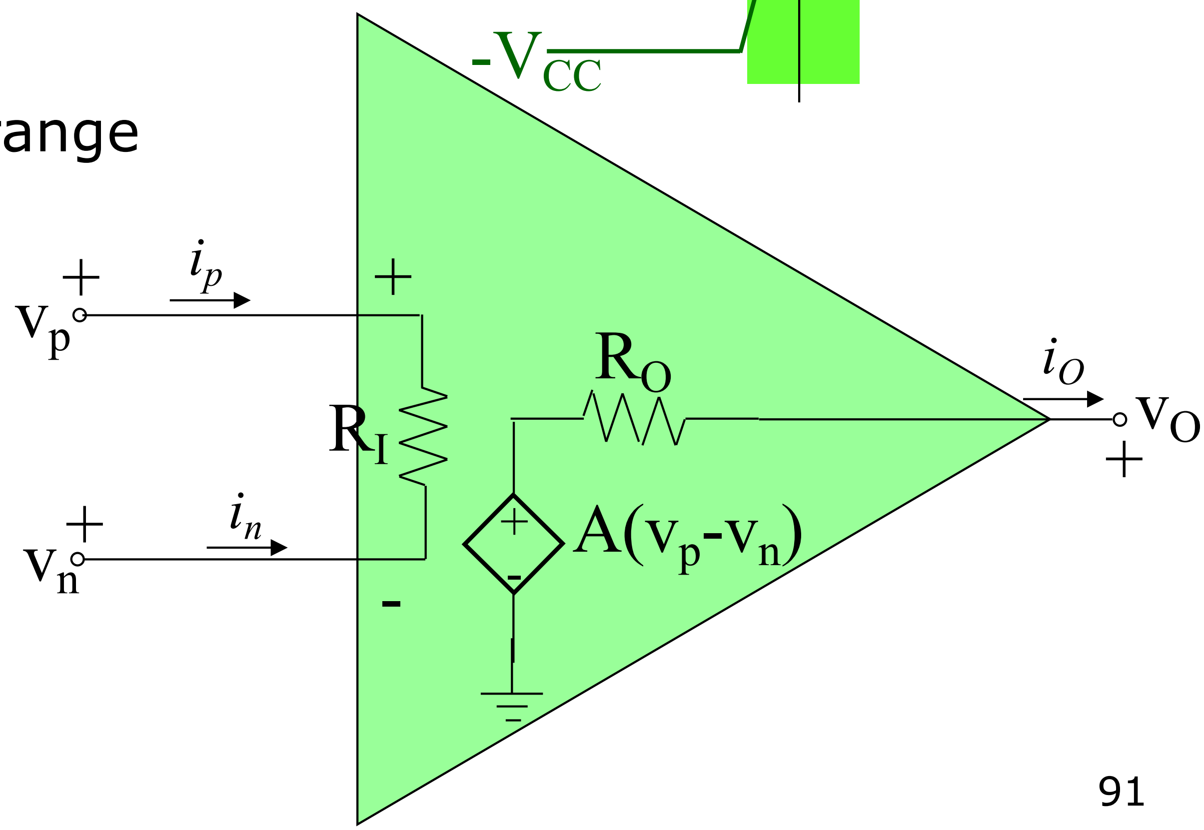
$$-V_{CC} \leq v_O \leq V_{CC}$$

$$-\frac{V_{CC}}{A} \leq v_p - v_n \leq \frac{V_{CC}}{A}$$

Ideal conditions

$$v_p = v_n$$

$$i_p = i_n = 0$$

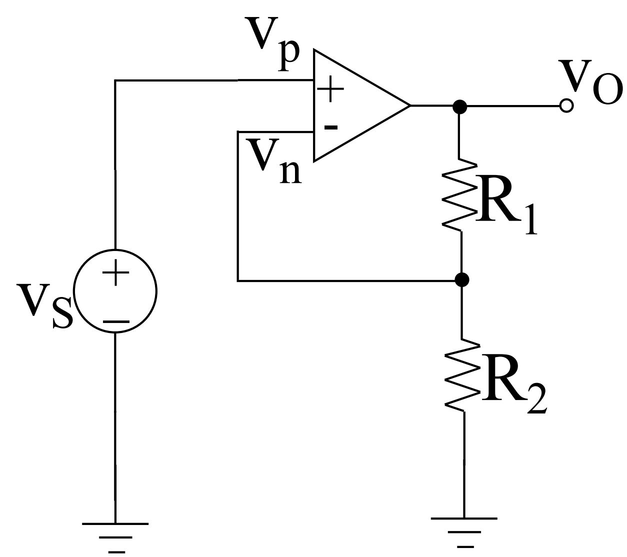


Non-inverting OpAmp - Feedback

What happens now?

Voltage divider feedback

$$v_n = \frac{R_2}{R_1 + R_2} v_O$$



Operating condition $v_p = v_S$

$$v_O = \frac{R_1 + R_2}{R_2} v_S$$

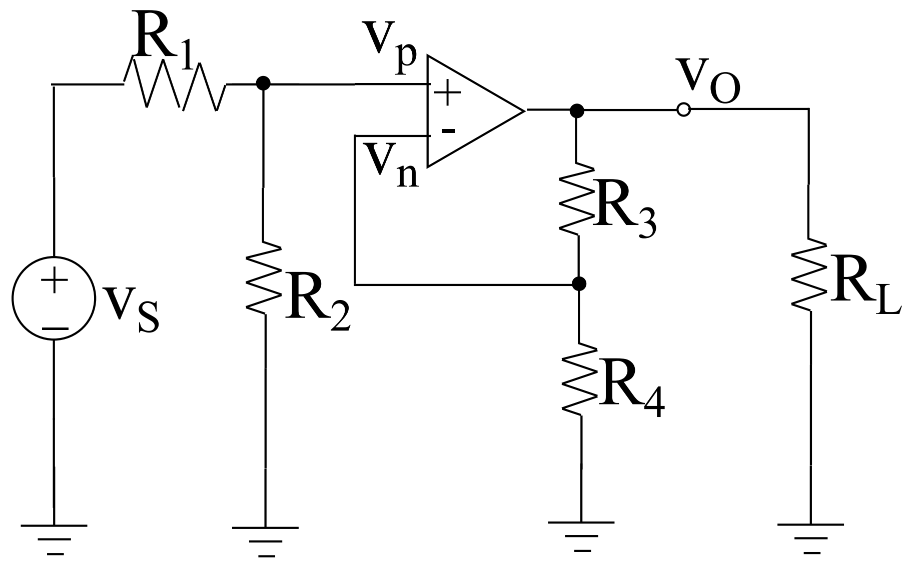
Linear non-inverting amplifier

Gain K= $\frac{R_1 + R_2}{R_2}$

With dependent source model

$$v_O = \frac{R_I A(R_1 + R_2) + R_2 R_O}{R_I(AR_2 + R_O + R_1 + R_2) + R_2(R_1 + R_O)} v_S$$

T&R, 5th ed, Example 4-13



Analyze this

$$i_p = 0$$

$$K_S = \frac{v_p}{v_S} = \frac{R_2}{R_1 + R_2}$$

Ideal OpAmp has zero output resistance

R_L does not affect v_O

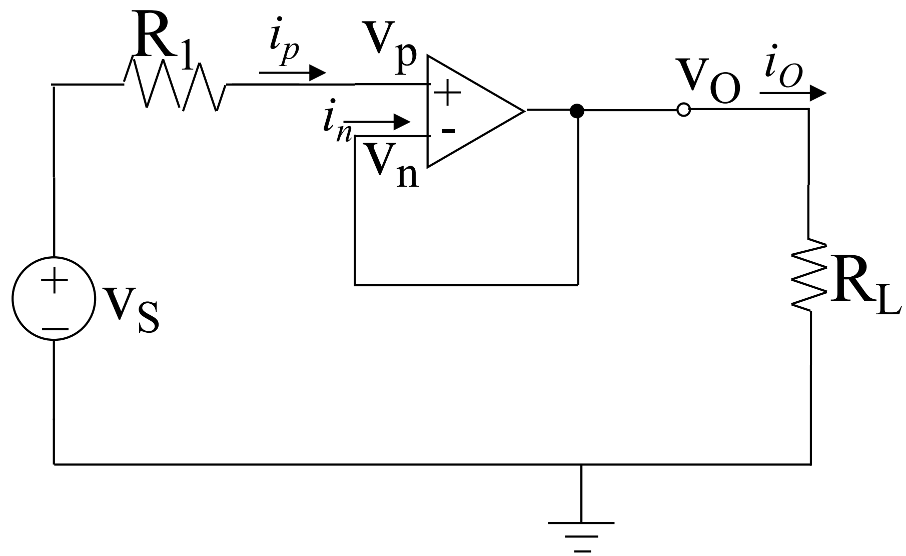
$$K_{\text{AMP}} = \frac{v_O}{v_p} = \frac{R_3 + R_4}{R_4}$$

$$K_{\text{Total}} = K_S K_{\text{AMP}} = \frac{v_O}{v_S} = \left[\frac{R_2}{R_1 + R_2}\right]\left[\frac{R_3 + R_4}{R_4}\right]$$

Voltage Follower - Buffer

Feedback path

$$v_n = v_O$$



Infinite input resistance

$$i_p = 0, \quad v_p = v_S$$

Ideal OpAmp

$$v_p = v_n$$

$$v_O = v_S \qquad i_O = \frac{v_O}{R_L}$$

Loop gain is 1

Power is supplied from the Vcc/-Vcc rails

OpAmp Ccts – inverting amplifier

Input and feedback applied at same terminal of OpAmp

R_2 is the feedback resistor

So how does it work?

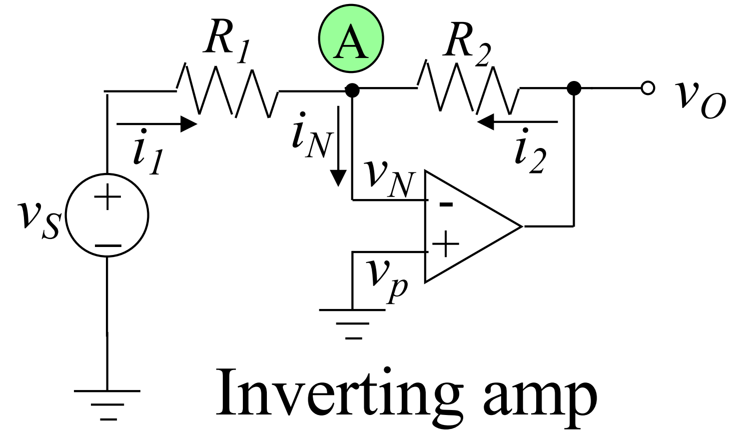
KCL at node A

$$\frac{v_N - v_S}{R_1} + \frac{v_N - v_O}{R_2} + i_N = 0$$

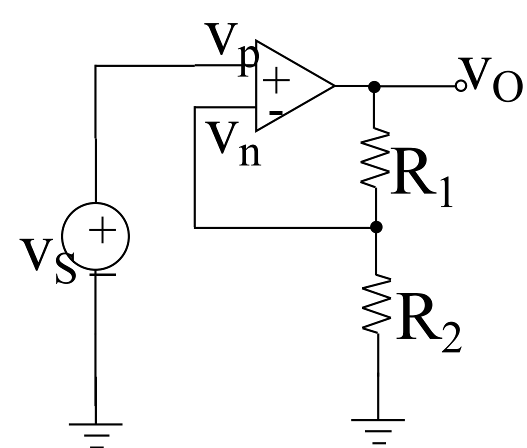
$$i_N = 0,\ v_N = v_p = 0$$

$$\boxed{v_O = -\frac{R_2}{R_1} v_S}$$

v_O=-Kv_S hence the name



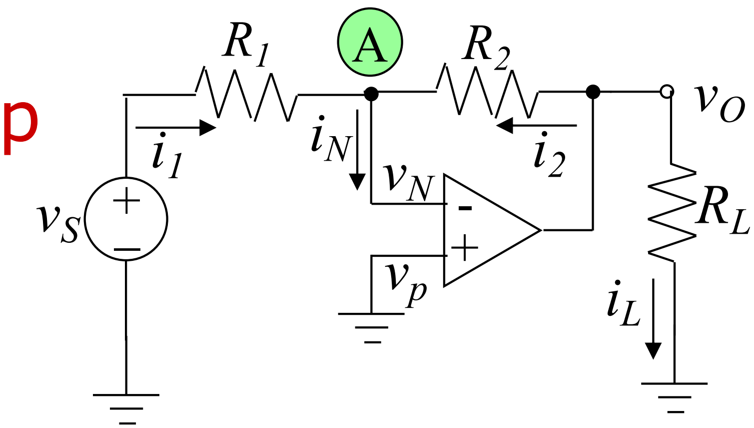
Inverting amp



Non-inverting amp

Inverting Amplifier (contd)

Current flows in the inverting amp

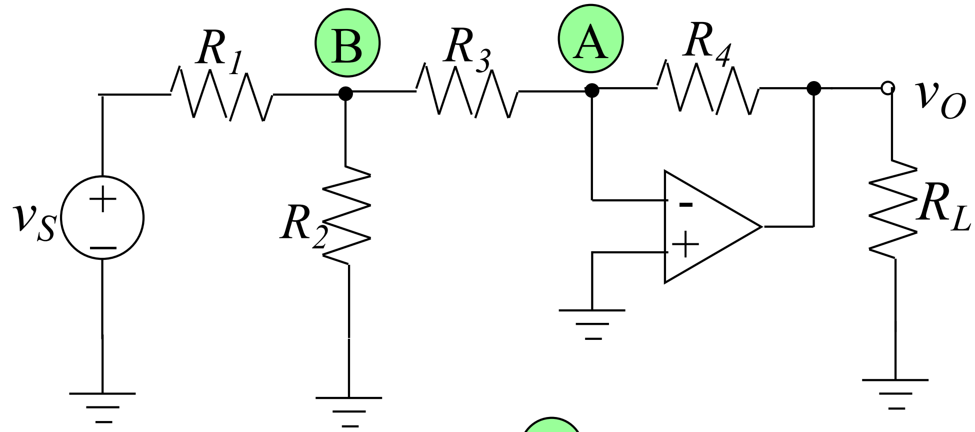


$$i_1 = \frac{v_S}{R_1}, \ R_{in} = R_1$$

$$i_2 = \frac{v_O}{R_2} = \frac{-v_S}{R_1} = -i_1$$

$$i_L = \frac{v_O}{R_L} = -\frac{R_2}{R_1} \times \frac{1}{R_L} \times v_S$$

OpAmp Analysis – T&R, 5th ed, Example 4-14

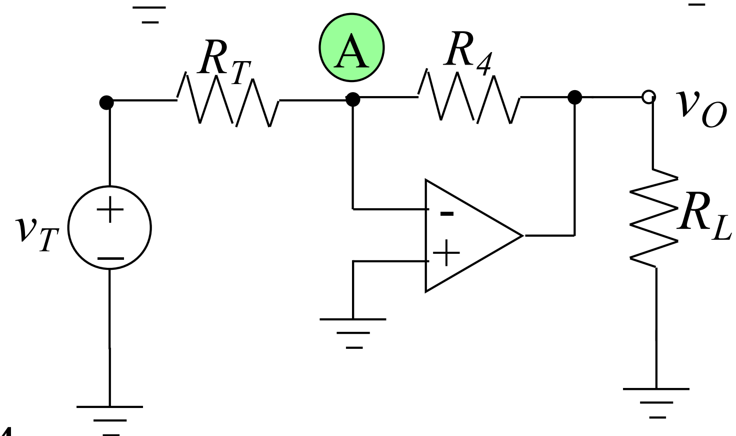


Compute the input-output relationship of this cct

- Convert the cct left of the node A to its Thévenin equivalent

$$v_T = v_{OC} = \frac{R_2}{R_1+R_2} v_S$$

$$R_T = R_{in} = R_3 + \frac{R_1 R_2}{R_1+R_2} = \frac{R_1R_2+R_1R_3+R_2R_3}{R_1+R_2}$$



Note that this is not the inverting amp gain times the voltage divider gain

- There is interaction between the two parts of the cct (R_3)

This is a feature of the inverting amplifier configuration

$$v_O = -\frac{R_4}{R_T} v_T$$

$$= -\left[\frac{R_4(R_1+R_2)}{R_1R_2+R_1R_3+R_2R_3}\right]\left[\frac{R_2}{R_1+R_2}\right] v_S$$

$$= -\frac{R_2R_4}{R_1R_2+R_1R_3+R_2R_3} v_S$$

Summing Amplifier - Adder

So what happens?

Node A is effectively grounded

$v_n = v_p = 0$

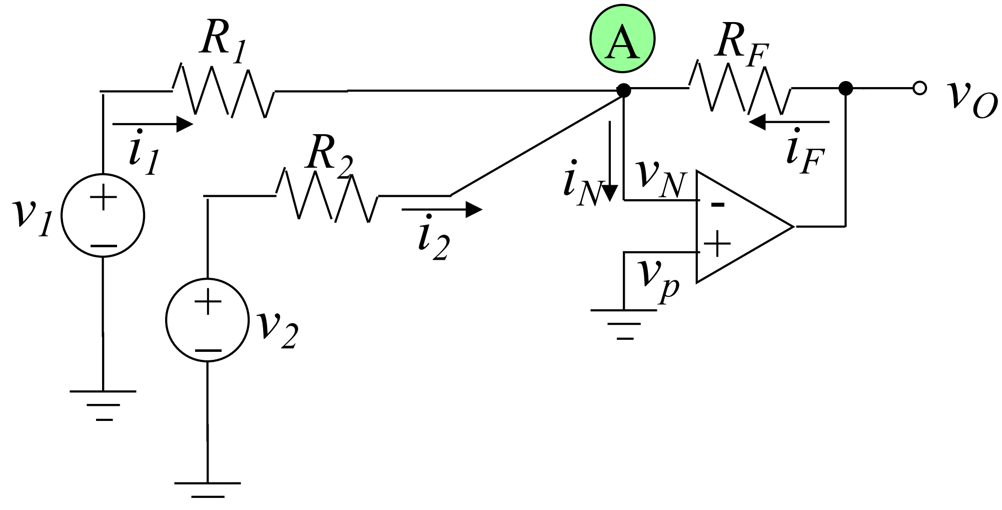
Also $i_N = 0$ because of R_{in}

So

$$i_1 + i_2 + i_F = 0$$

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_O}{R_F} = 0$$

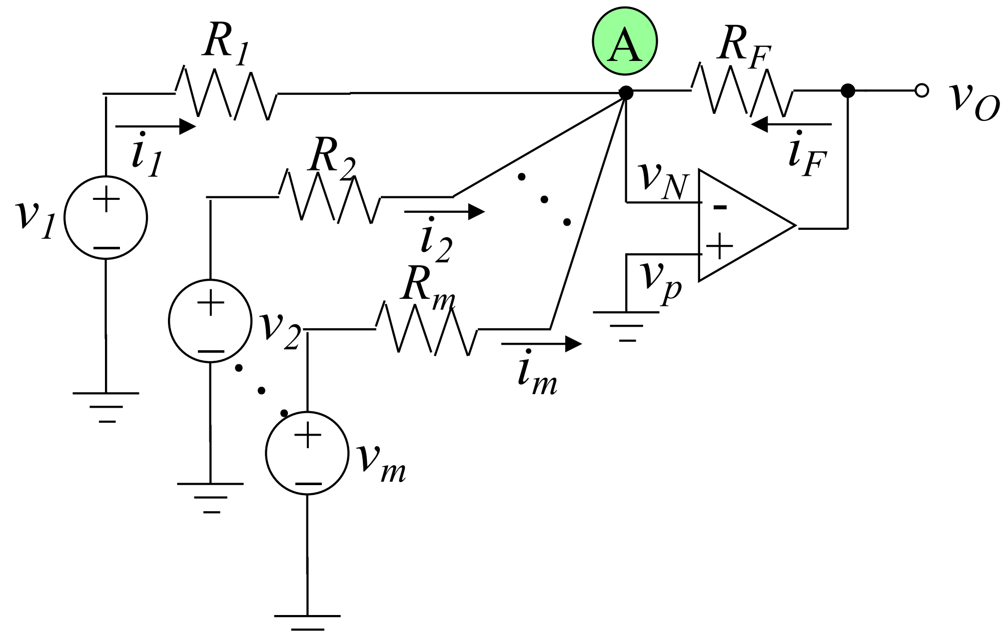
This is an inverting summing amplifier



$$v_O = \left(-\frac{R_F}{R_1}\right)v_1 + \left(-\frac{R_F}{R_2}\right)v_2$$
$$= K_1 v_1 + K_2 v_2$$

Ever wondered about audio mixers? How do they work?

Mixing desk – Linear ccts



$$v_O = \left(-\frac{R_F}{R_1}\right)v_1 + \left(-\frac{R_F}{R_2}\right)v_2 + \ldots + \left(-\frac{R_F}{R_m}\right)v_m$$

$$= K_1 v_1 + K_2 v_2 + \cdots + K_m v_m$$

Virtual ground at v_n

Currents add

Summing junction

Permits adding signals to create a composite

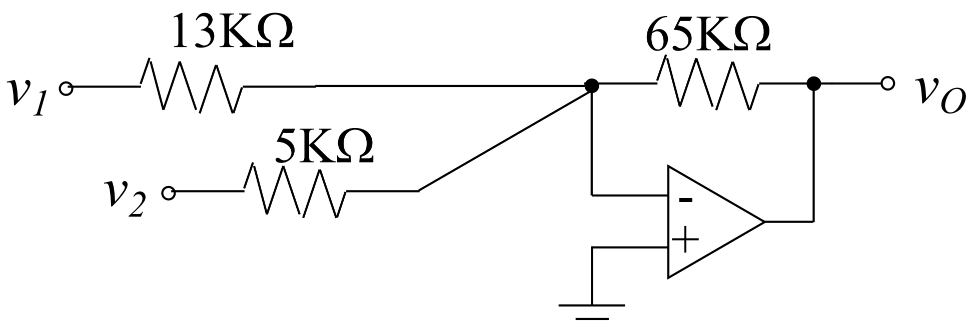
Strings+brass+woodwind+percussion

Guitars+bass+drums+vocal+keyboards

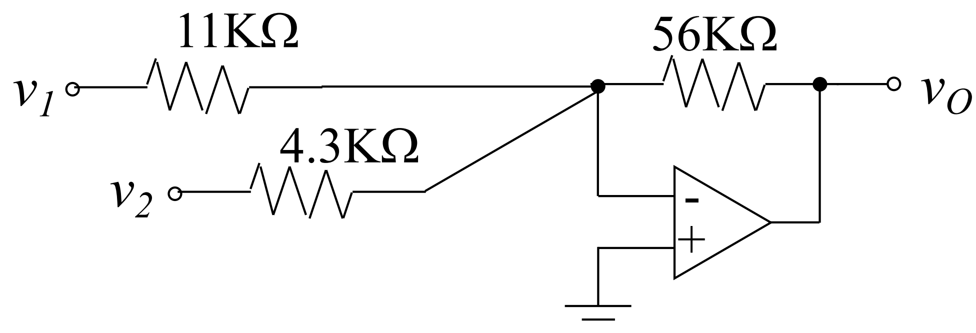
T&R, 5th ed, Design Example 4-15

Design an inverting summer to realize $v_O = -\left(5v_1 + 13v_2\right)$

Inverting summer with $\dfrac{R_F}{R_1} = 5,\ \dfrac{R_F}{R_2} = 13$



Nominal values



Standard values

If v_1=400mV and V_{CC}=±15V what is max of v_2 for linear op^n?

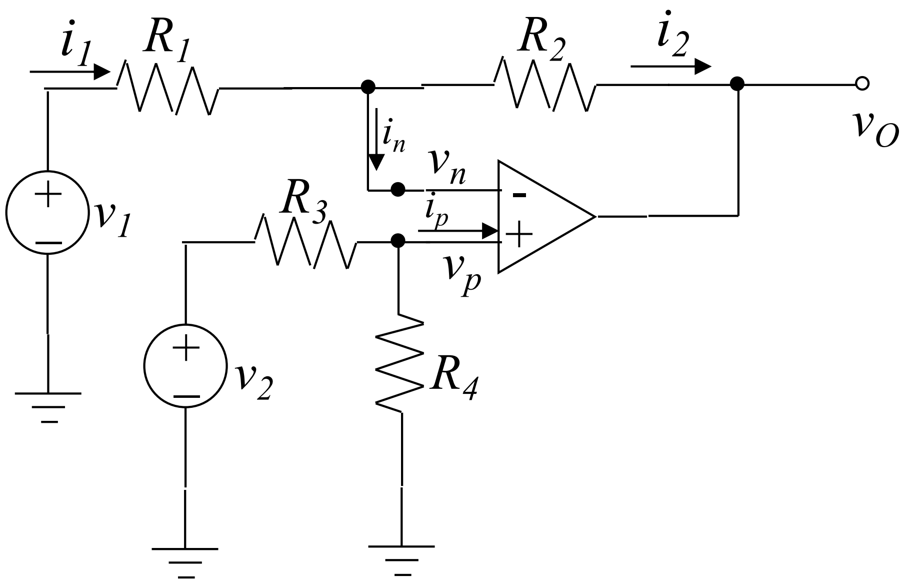
Need to keep v_O>-15V

$$-15 < -\left(5v_1 + 13v_2\right)$$

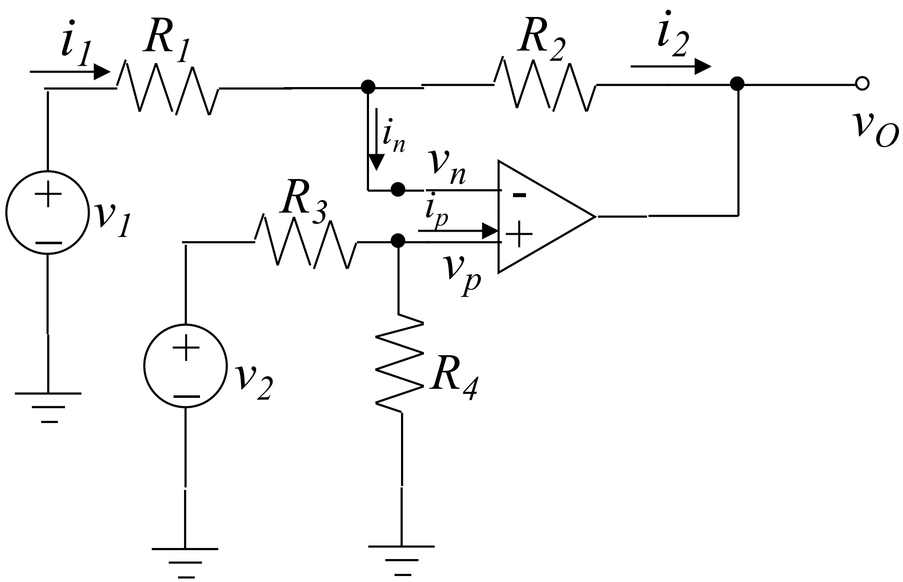
$$15 > 5v_1 + 13v_2$$

$$v_2 < \frac{15 - 5 \times 0.4}{13} = 1\text{V}$$

OpAmp Circuits – Differential Amplifier



OpAmp Circuits – Differential Amplifier



Use superposition to analyze

v_2=0: inverting amplifier

$$v_{O1} = -\frac{R_2}{R_1} v_1$$

v_1=0: non-inverting amplifier plus voltage divider

$$v_{O2} = \left[\frac{R_4}{R_3 + R_4}\right]\left[\frac{R_1 + R_2}{R_1}\right] v_2$$

$$\begin{aligned} v_O &= v_{O1} + v_{O2} \\ &= -\left[\frac{R_2}{R_1}\right] v_1 + \left[\frac{R_4}{R_3 + R_4}\right]\left[\frac{R_1 + R_2}{R_1}\right] v_2 \\ &= -K_1 v_1 + K_2 v_2 \end{aligned}$$

K_1 inverting gain

K_2 non-inverting gain

T&R, 5th ed, Exercise 4-13

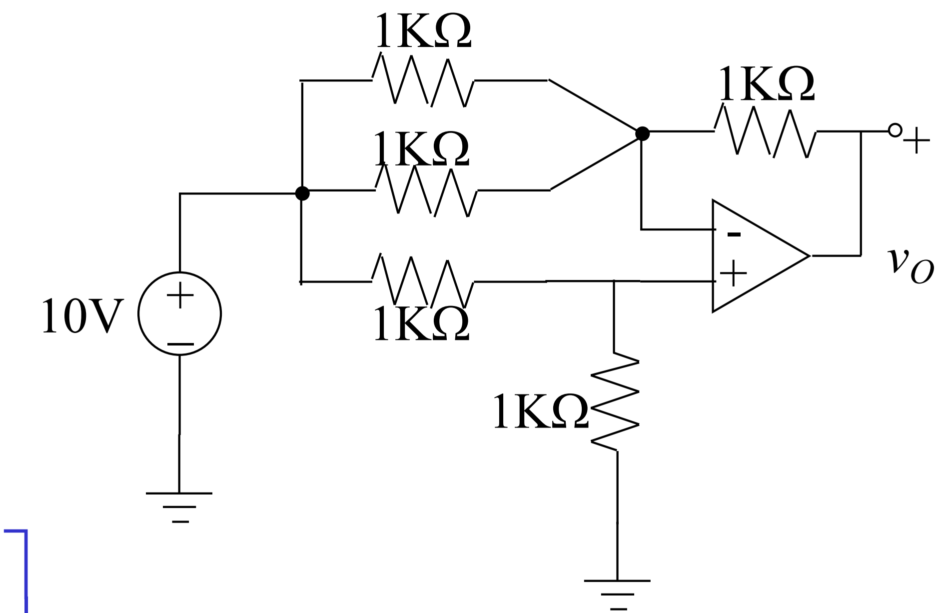
What is v_O?

This is a differential amp

v_1 is 10V, v_2 is 10V

R_1=1KΩ||1KΩ=500Ω

R_2=R_3=R_4=1KΩ

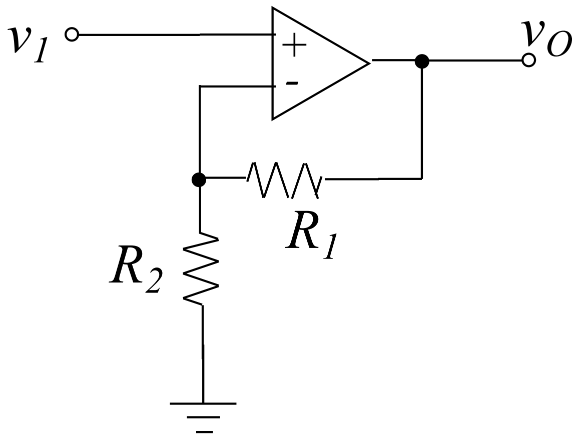


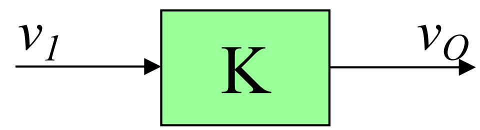
$$v_O = K_1 v_1 + K_2 v_2$$

$$= -\frac{R_2}{R_1} v_1 + \left[\frac{R_1 + R_2}{R_1}\right]\left[\frac{R_4}{R_3 + R_4}\right] v_2$$

$$= -20 + 3 \times \frac{1}{2} \times 10 = -5V$$

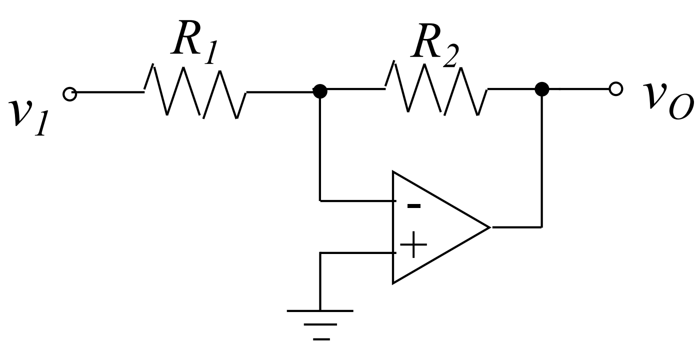
Lego Circuits

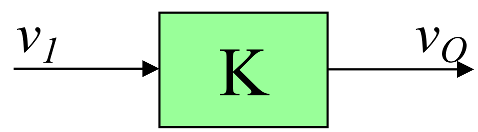




$$K = \frac{R_1 + R_2}{R_2}$$

Non-inverting amplifier

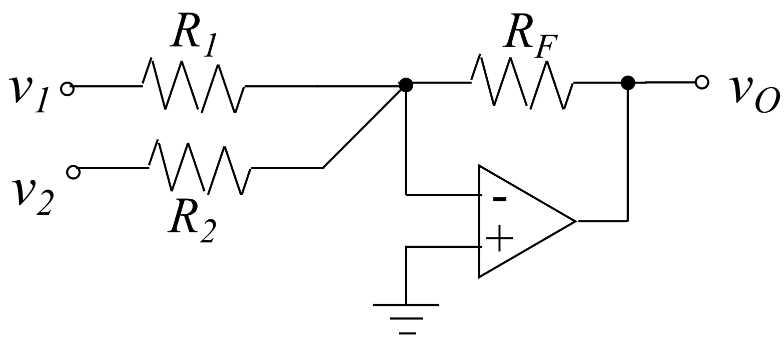


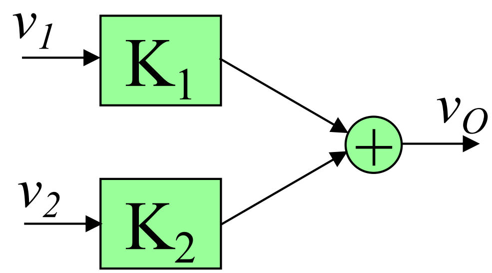


$$K = -\frac{R_2}{R_1}$$

Inverting amplifier

Lego Circuits (contd)

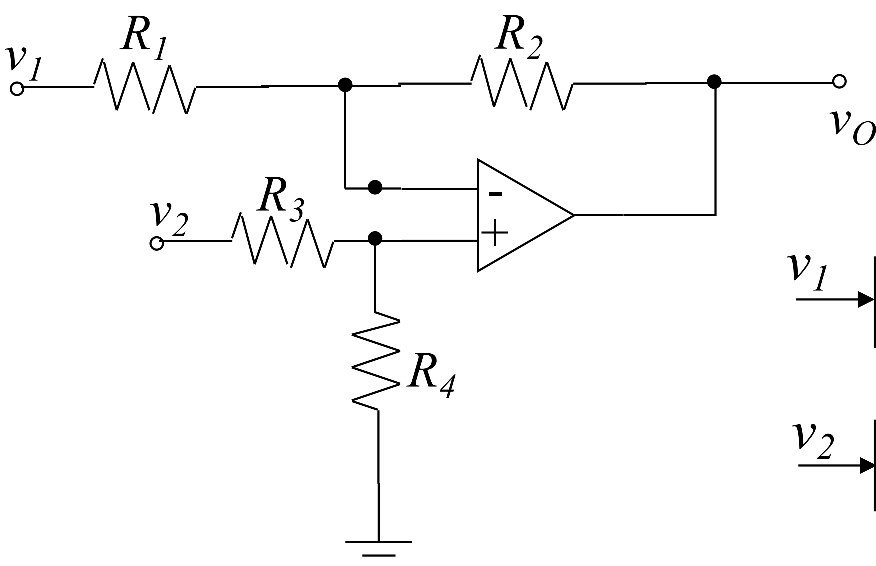


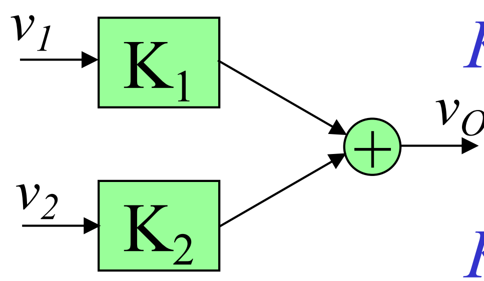


$$K_1 = -\frac{R_F}{R_1}$$

$$K_2 = -\frac{R_F}{R_2}$$

Inverting summer



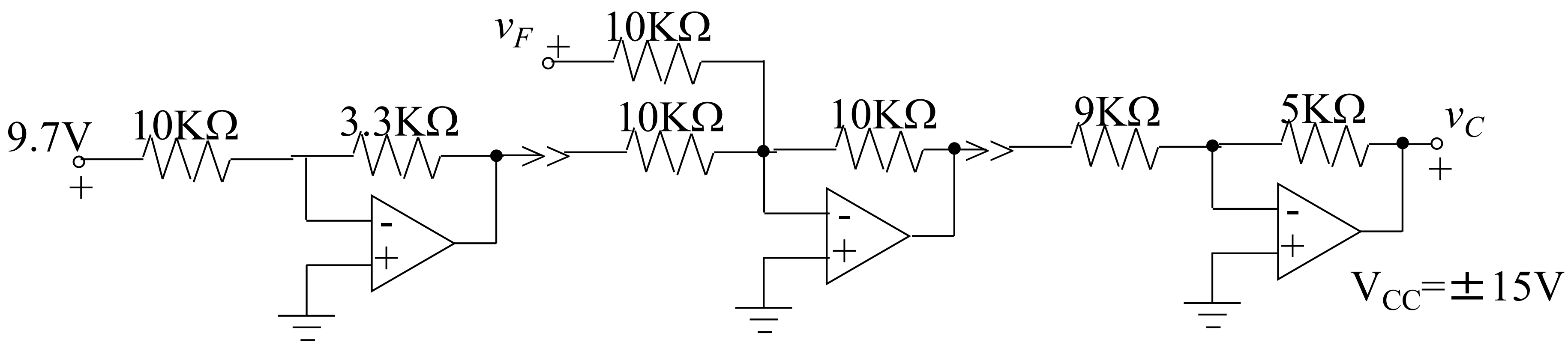


$$K_1 = -\frac{R_2}{R_1}$$

$$K_2 = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)$$

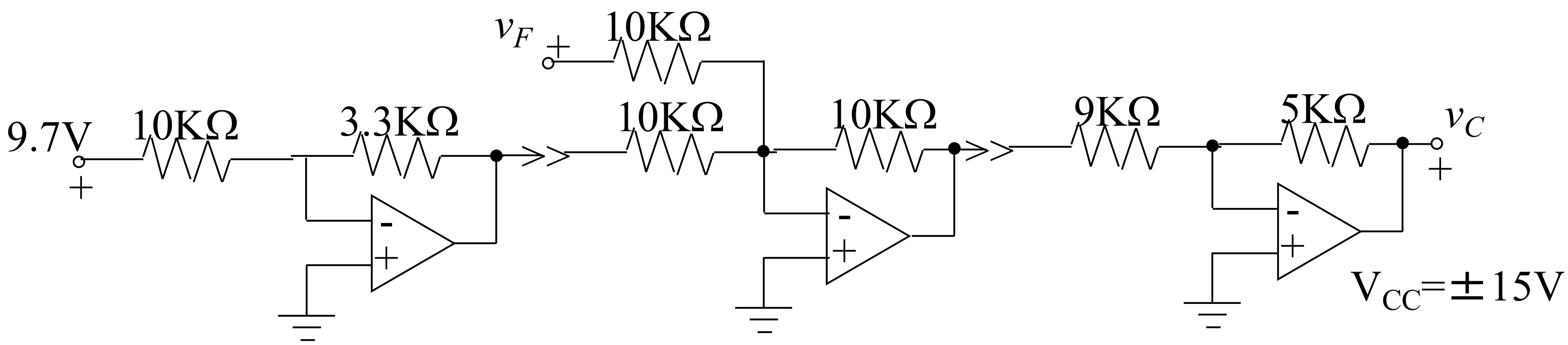
Differential amplifier

T&R, 5th ed, Example 4-16: OpAmp Lego

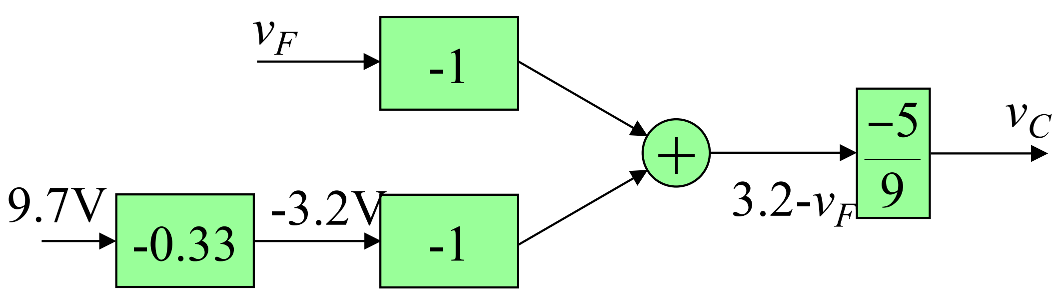


So what does this circuit do?

Example 4-16: OpAmp Lego



So what does this circuit do?



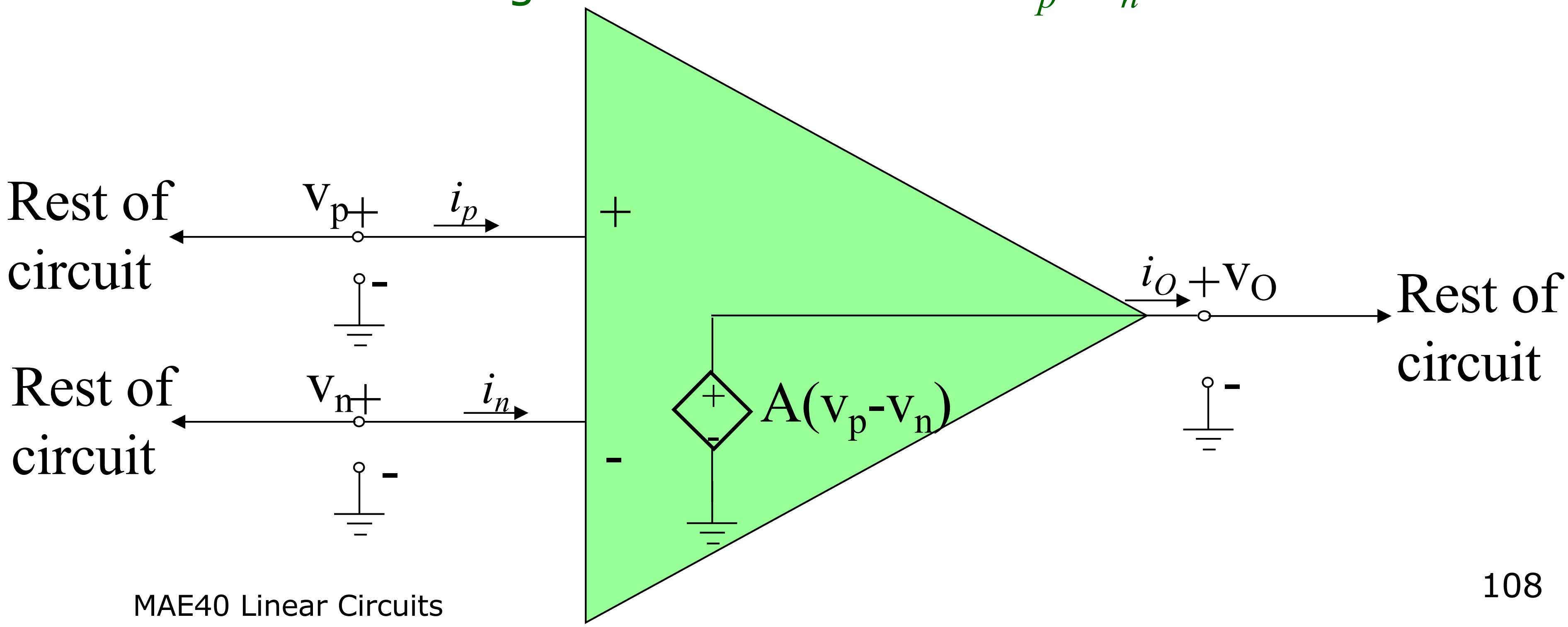
It converts tens of ºF to tens of ºC

Max current drawn by each stage is 1.5mA

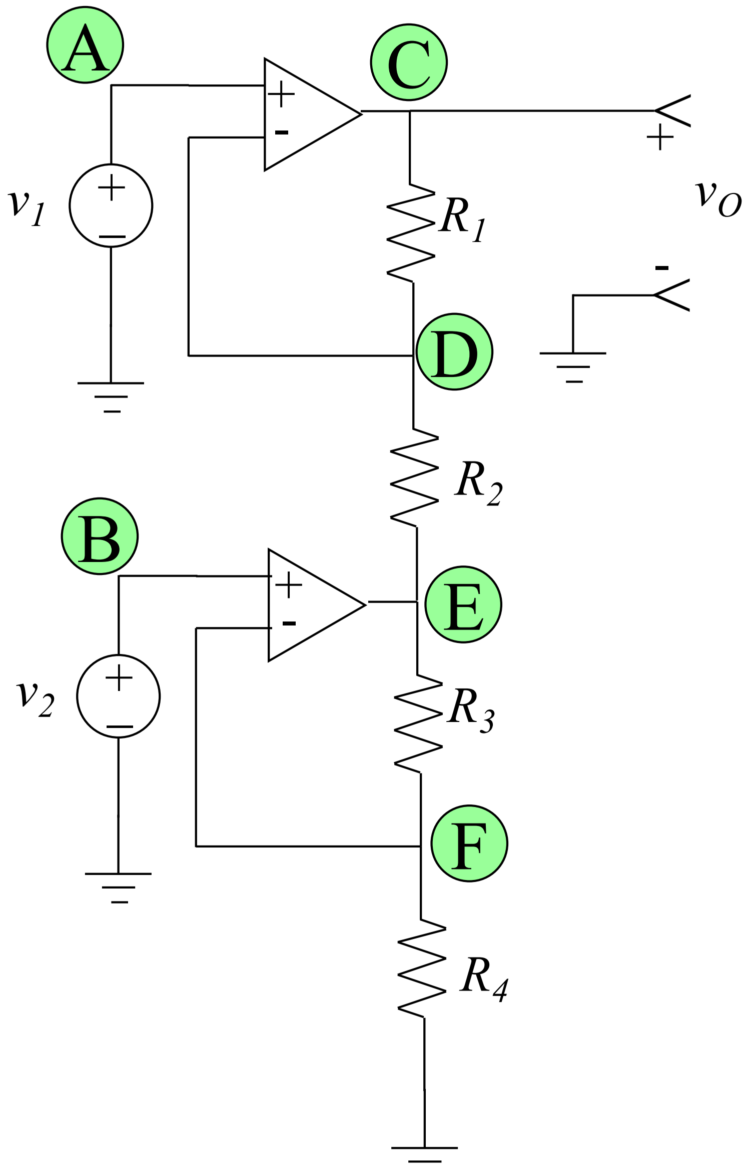
OpAmp Cct Analysis

What if circuit is not simple interconnection of basic building blocks? OpAmp Nodal Analysis

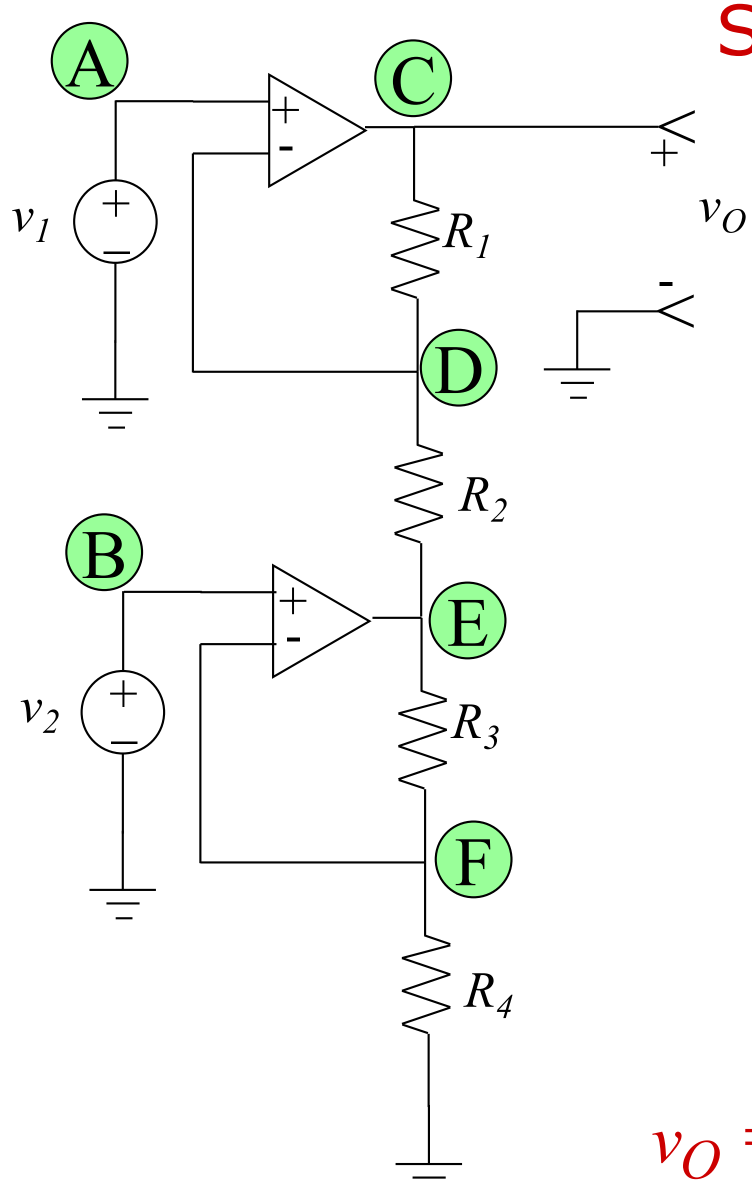
- Use dependent voltage source model
    - Identify node voltages
    - Formulate input node equations
        - Solve using ideal characteristic $v_p = v_n$



OpAmp Analysis – T&R, 5th ed, Example 4-18



OpAmp Analysis – T&R, 5th ed, Example 4-18



Seemingly six non-reference nodes: A-E

- Nodes A, B: connect to reference voltages v_1 and v_2
- Node C, E: connected to OpAmp outputs (forget for the moment)
- Node D: $(G_1 + G_2)v_D - G_1 v_C - G_2 v_E = 0$
- Node F: $(G_3 + G_4)v_F - G_3 v_E = 0$
- OpAmp constraints

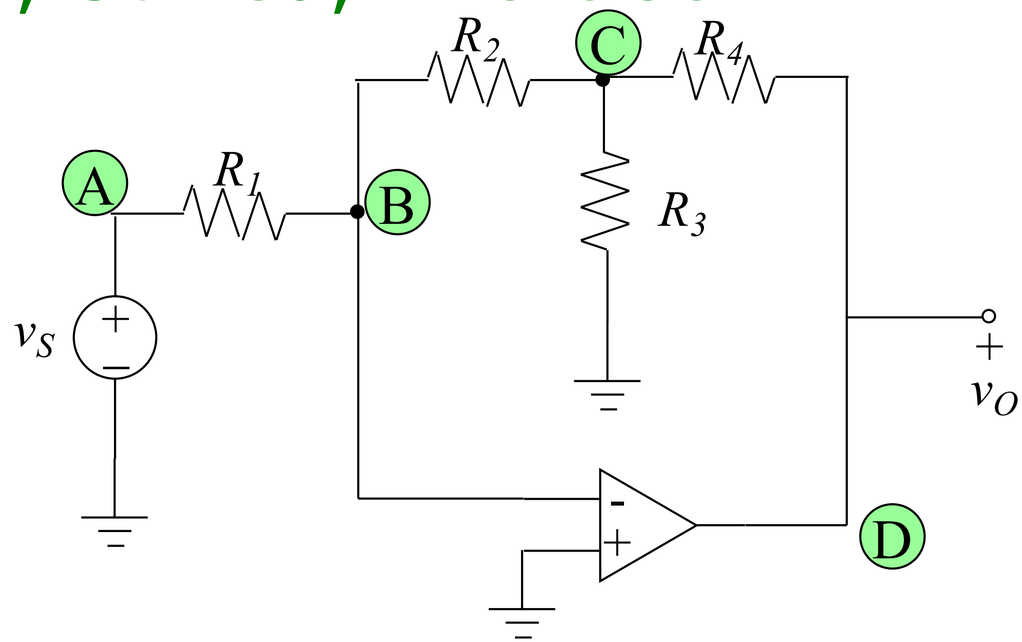
$$v_A = v_1 = v_D, \; v_B = v_2 = v_F$$

$$G_1 v_C + G_2 v_E = (G_1 + G_2)v_1$$

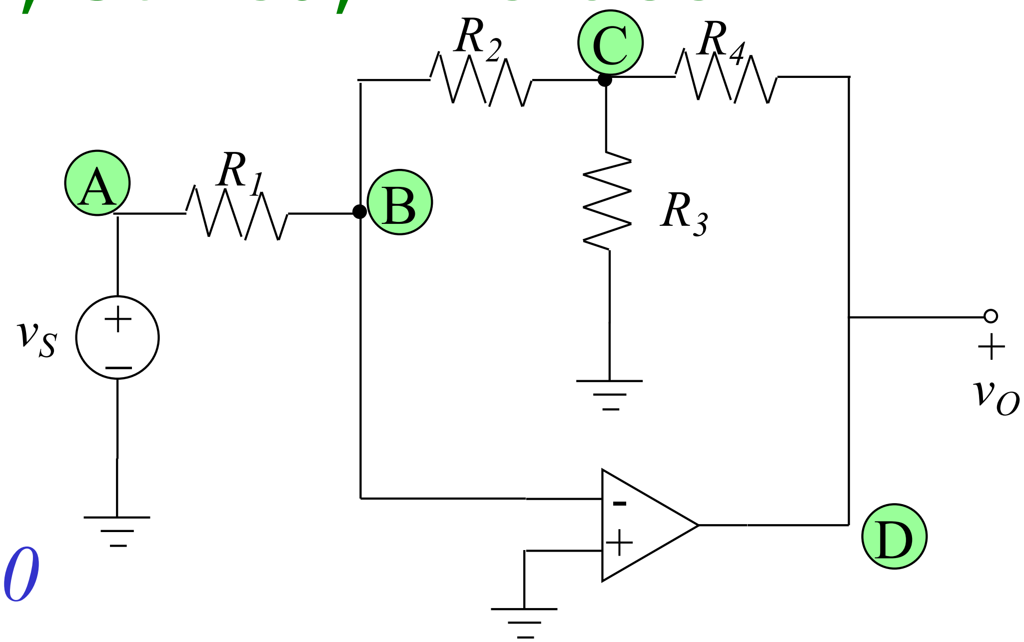
$$G_3 v_E = (G_3 + G_4)v_2$$

$$v_O = v_C = \left[\frac{G_1 + G_2}{G_1}\right]v_1 - \frac{G_2}{G_1}\left[\frac{G_3 + G_4}{G_3}\right]v_2$$

OpAmp Analysis – T&R, 5th ed, Exercise 4-14



OpAmp Analysis – T&R, 5th ed, Exercise 4-14



Node A: $v_A = v_S$

Node B:

$(G_1+G_2)v_B - G_1 v_A - G_2 v_C = 0$

Node C:

$(G_2+G_3+G_4)v_C - G_2 v_B - G_4 v_D = 0$

Constraints

$v_B = v_p = v_n = 0$

Solve

$$v_C = -\frac{G_1}{G_2} v_S$$

$$v_O = v_D$$

$$v_O = \frac{(G_2+G_3+G_4)}{G_4} \times \frac{-G_1}{G_2} v_S$$

$$= -\frac{(R_2R_3 + R_2R_4 + R_3R_4)}{R_1R_3} v_S$$

OpAmp Circuit Design – the whole point

Given an input-output relationship design a cct to implement it

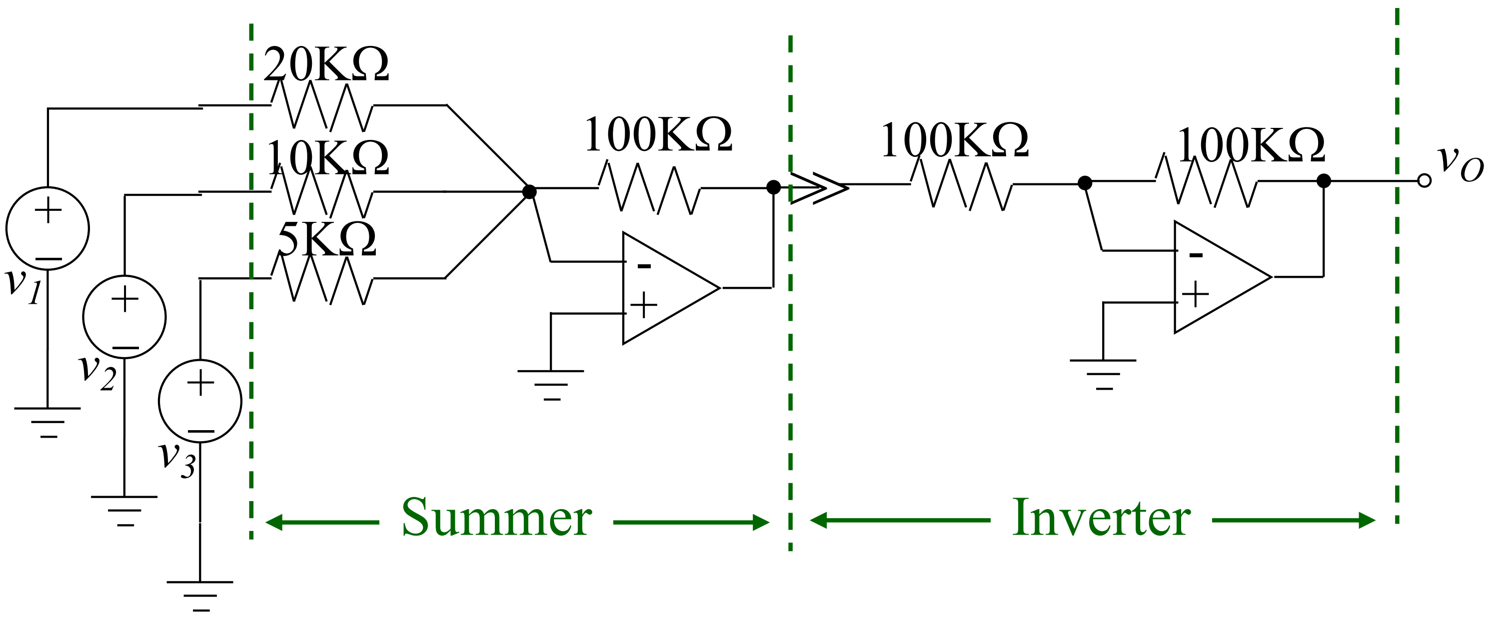
Build a cct to implement $v_O=5v_1+10v_2+20v_3$

OpAmp Circuit Design – the whole point

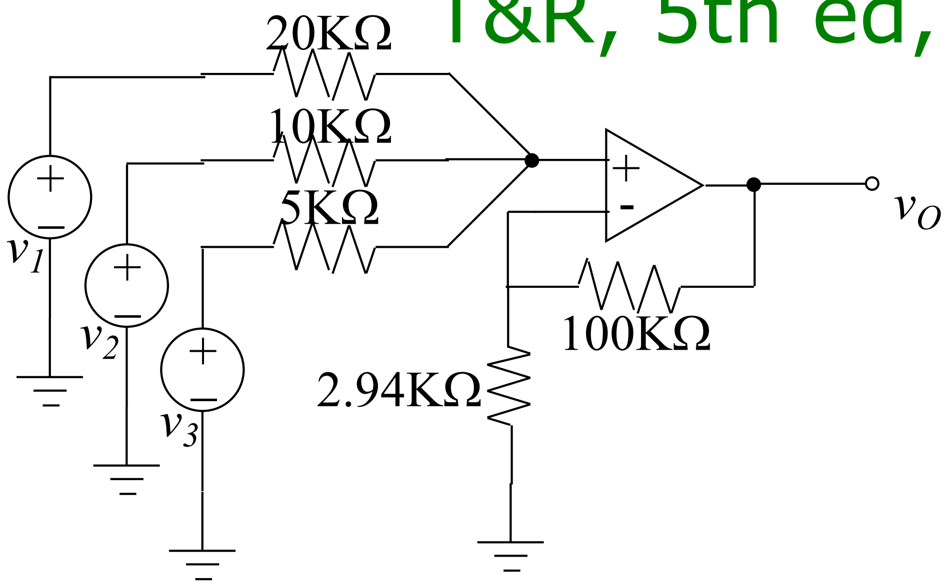
Given an input-output relationship design a cct to implement it

Build a cct to implement $v_O = 5v_1 + 10v_2 + 20v_3$

Inverting summer followed by an inverter

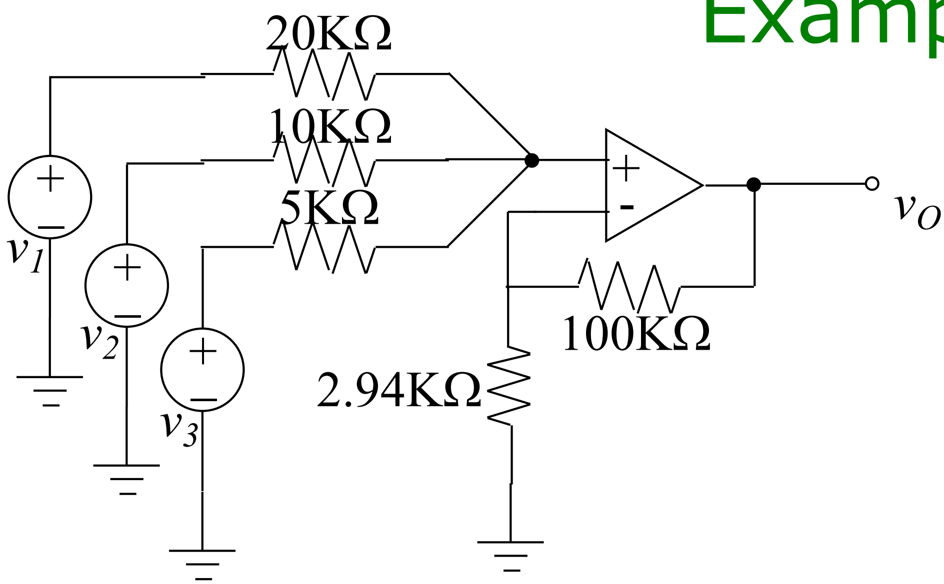


T&R, 5th ed, Example 4-21



How about this one?

Example 4-21



How about this one?

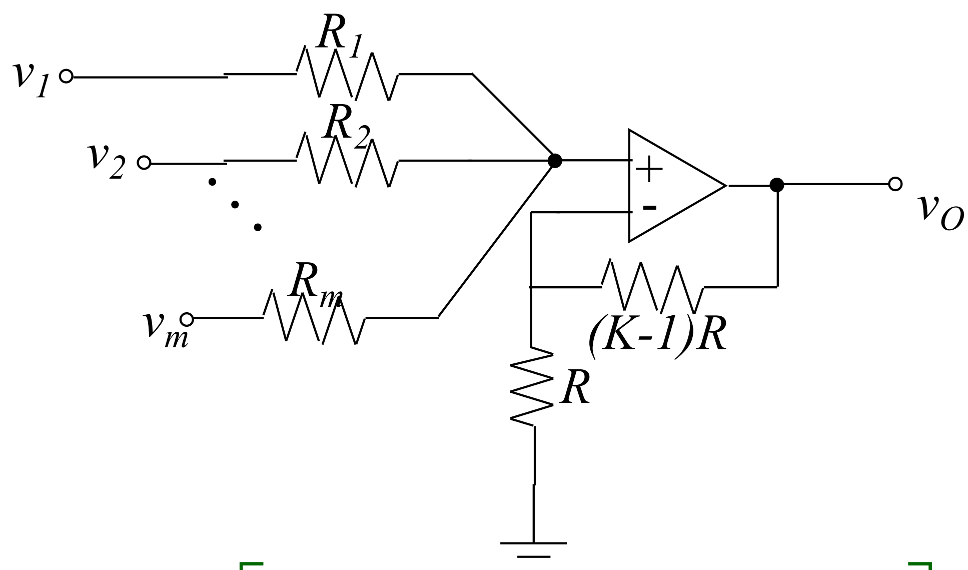
Non-inverting amp $v_p \rightarrow v_O$

$$v_O = Kv_p = \frac{100\times10^3 + 2.94\times10^3}{2.94\times10^3} v_p = 35 v_p$$

KCL at p-node with $i_p = 0$

$$\frac{v_1 - v_p}{2\times10^4} + \frac{v_2 - v_p}{10^4} + \frac{v_3 - v_p}{0.5\times10^4} = 0$$

$$3.5 v_p = 0.5 v_1 + v_2 + 2 v_3$$



Non-inverting summer

Fewer elements than inv-summer + inverter

$$v_O = K\left[\frac{R_{eq}}{R_1}v_1 + \frac{R_{eq}}{R_2}v_2 + \cdots + \frac{R_{eq}}{R_m}v_m\right] \qquad R_{eq} = R_1 \| R_2 \| R_3 \| \cdots R_m$$

Comparators – A Nonlinear OpAmp Circuit

We have used the ideal OpAmp conditions for the analysis of OpAmps in the linear regime

$$v_n = v_p, \quad i_n = i_p = 0 \quad \text{if} \quad A\left|v_p - v_n\right| \leq V_{CC}$$

What about if we operate with $v_p \neq v_n$?

That is, we operate outside the linear regime.

We saturate!!

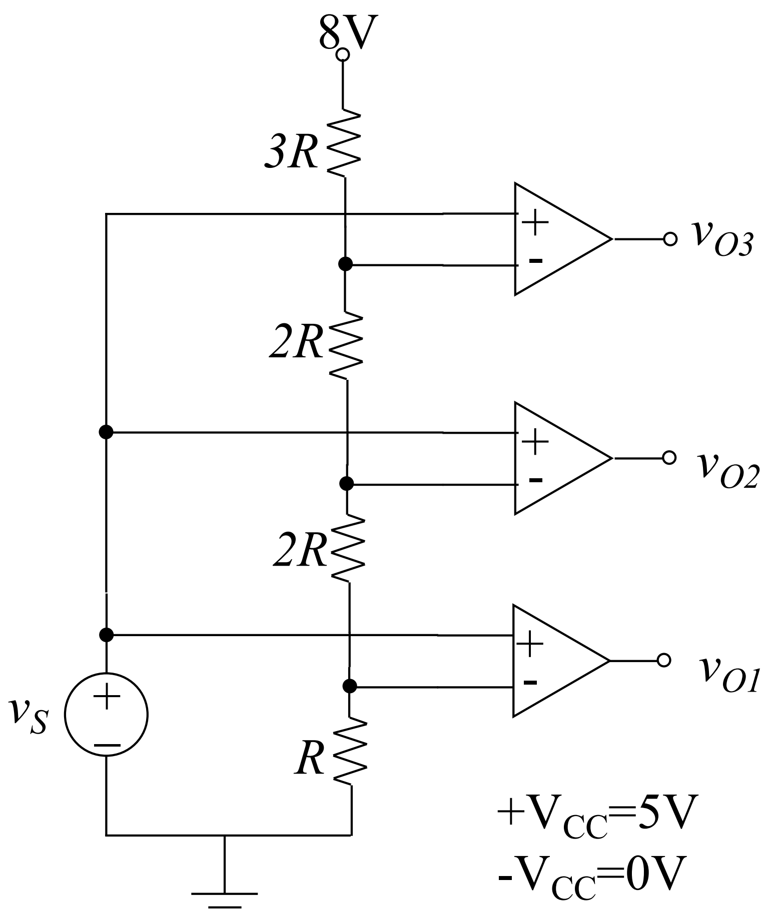
$$v_O = +V_{CC} \quad \text{if} \quad v_p > v_n$$

$$v_O = -V_{CC} \quad \text{if} \quad v_p < v_n$$

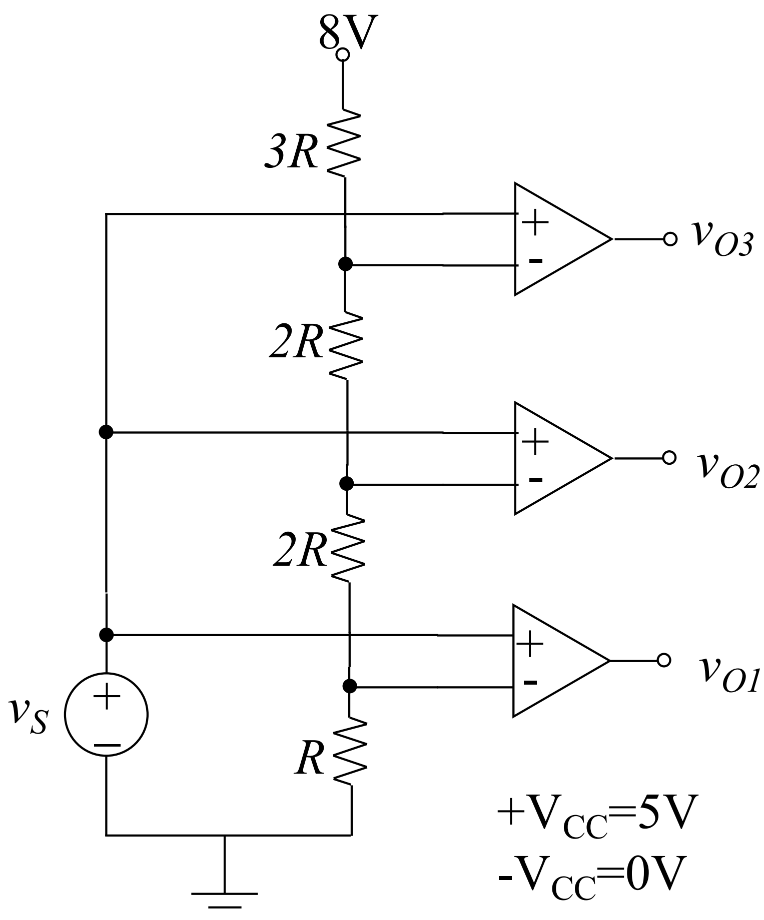
Without feedback, OpAmp acts as a comparator

There is one of these in every FM radio!

"Analog-to-digital converter" - comparators



"Analog-to-digital converter" - comparators



$+V_{CC}$=5V
$-V_{CC}$=0V

Current laws still work

$$i_p = i_n = 0$$

Parallel comparison

Flash converter

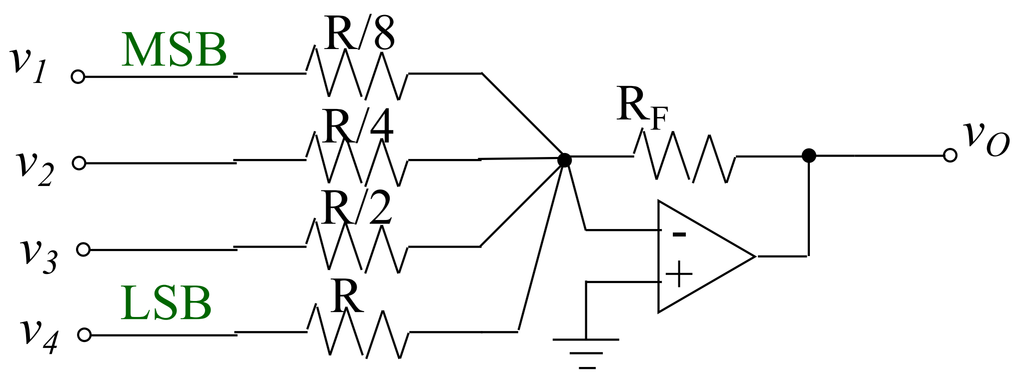
"3-bit" output

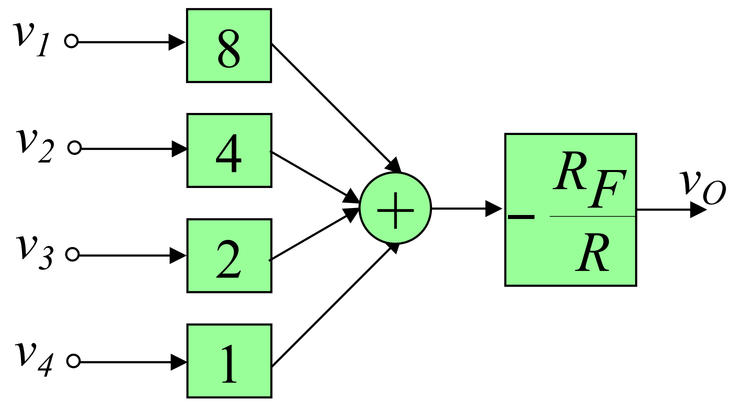
Not really how it is done

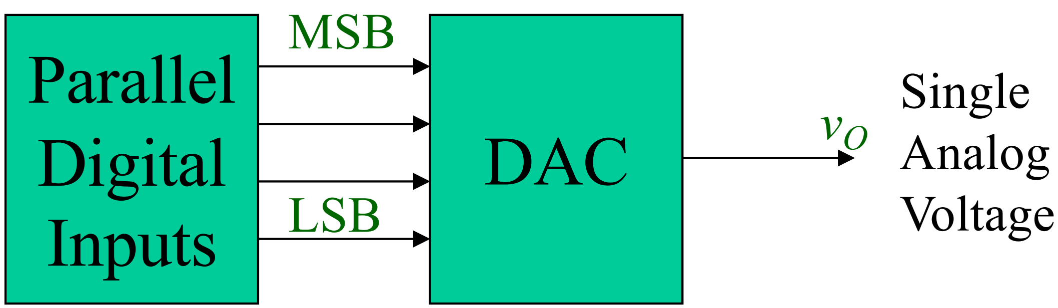
Voltage divider switched

Input	v_{O1}	v_{O2}	v_{O3}
$1>v_S$	0	0	0
$3>v_S>1$	5	0	0
$5>v_S>3$	5	5	0
$v_S>5$	5	5	5

Digital-to-analog converter







Conversion of digital data to analog voltage value

Bit inputs = 0 or 5V

Analog output varies between v_{min} and v_{max} in 16 steps

Signal Conditioning

Your most likely brush with OpAmps in practice

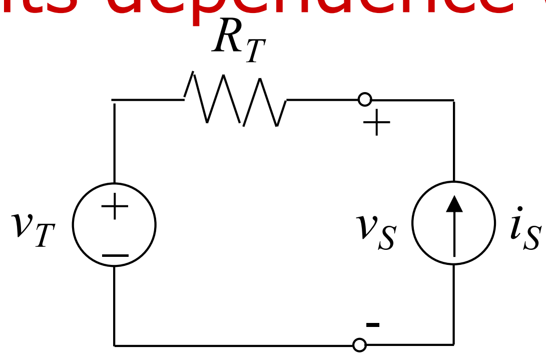
- Signal – typically a voltage representing a physical variable
  - Temperature, strain, speed, pressure
- Digital analysis – done on a computer after
  - Anti-aliasing filtering – data interpretation
  - Adding/subtracting an offset – zeroing
    - Normally zero of ADC is 0V
  - Scaling for full scale variation – quantization
    - Normally full scale of ADC is 5V
  - Analog-to-digital conversion – ADC
    - Maybe after a few more tricks like track and hold
- Offset correction: use a summing OpAmp
- Scaling: use an OpAmp amplifier
- Anti-aliasing filter: use a dynamic OpAmp cct

Thévenin and Norton for dependent sources

Cannot turn off the ICSs and IVSs to do the analysis

This would turn off DCSs and DVSs

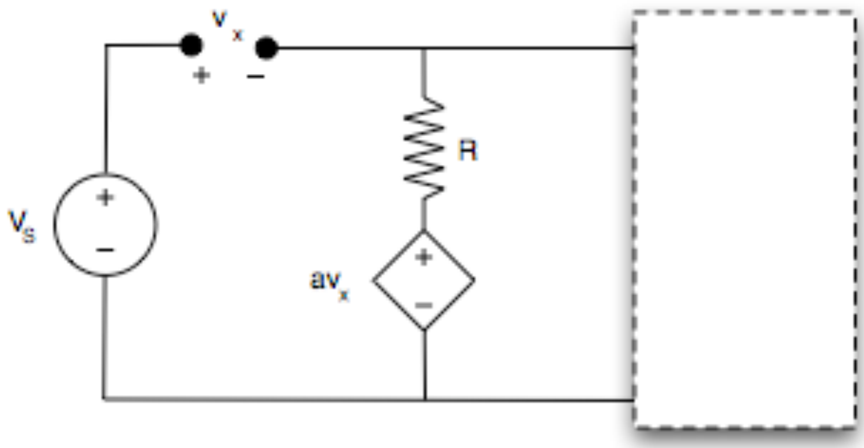
Connect an independent CS or VS to the terminal and compute the resulting voltage or current and its dependence on the source



Compute v_S in response to i_S: $v_S = v_T + i_S R_T$

Or just compute the open-circuit voltage and the short-circuit current

Thévenin and Norton for dependent sources



Thevenin equivalent circuit?

Open-circuit voltage

$$\left.\begin{aligned} v_{OC} &= v_S - v_x \\ v_{OC} &= v_R + av_x = av_x \end{aligned}\right\} \Rightarrow v_T = v_{OC} = \frac{a}{1+a} v_S$$

Thevenin resistance

$$R_T = \frac{v_{OC}}{i_{SC}} = \frac{1}{1+a} R$$

Short-circuit current

$$\left.\begin{aligned} 0 &= v_S - v_x \\ 0 &= -Ri_{SC} + av_x \end{aligned}\right\} \Rightarrow i_{SC} = \frac{a}{R} v_S$$

What would instead be the resistance obtained by turning off IVS?

Where to now?

Where have we been?

- Nodal and mesh analysis
- Thévenin and Norton equivalence
- Dependent sources and active cct models
- OpAmps and resistive linear active cct design

Where to now?

- Capacitors and inductors (Ch.6)
- Laplace Transforms and their use for ODEs and ccts (Ch.9)
- s-domain cct design and analysis (Ch.10)
- Frequency response (Ch.12) and filter design (Ch.14)
  - We will depart from the book more during this phase