MAE40 - Linear Circuits - Winter 21
Final Exam, March 18, 2021

Instructions

(i) Prior to the exam, you must have completed the Academic Integrity Pledge at `https://academicintegrity.ucsd.edu/forms/form-pledge.html`
(ii) The exam is open book. You may use your class notes and textbook
(iii) Collaboration is not permitted. The answers you provide should be the result only of your own work
(iv) On the questions for which the answers are given, please provide detailed derivations
(v) The exam has 4 questions for a total of 40 points and 4 bonus points
(vi) You have from 11:30am to 2:30pm to complete the exam. Allow sufficient time to post your answers in Canvas (submission closes at 2:40pm).
(vii) If there is any clarification needed, post your question in the "Discussions" tab of the class Canvas webpage ("Clarifications on question statements of final")

Good luck!

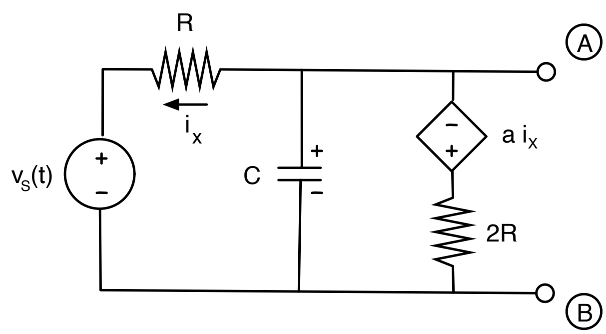


Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

   Here, $v_S(t) = e^{-t}u(t)$ and a is a known constant.

   Part I: [2 points] Assuming $v_C(0) = 1V$, transform the circuit in Figure 1 into the s-domain, using a voltage source to account for the initial condition of the capacitor.

   Part II: [3 points] For the circuit you obtained in Part I, find the open-circuit voltage transform as seen from terminals Ⓐ-Ⓑ. The answer should be given as a ratio of two polynomials.

   Part III: [3 points] For the circuit you obtained in Part I, find the short-circuit current transform as seen from terminals Ⓐ-Ⓑ. The answer should be given as a ratio of two polynomials.

   Part IV: [2 points] For the circuit you obtained in Part I, find the Norton equivalent in the s-domain as seen from terminals Ⓐ-Ⓑ (the impedance should be given as a ratio of two polynomials).

   Part V: [Extra 2 points] What is the capacitor voltage transform?

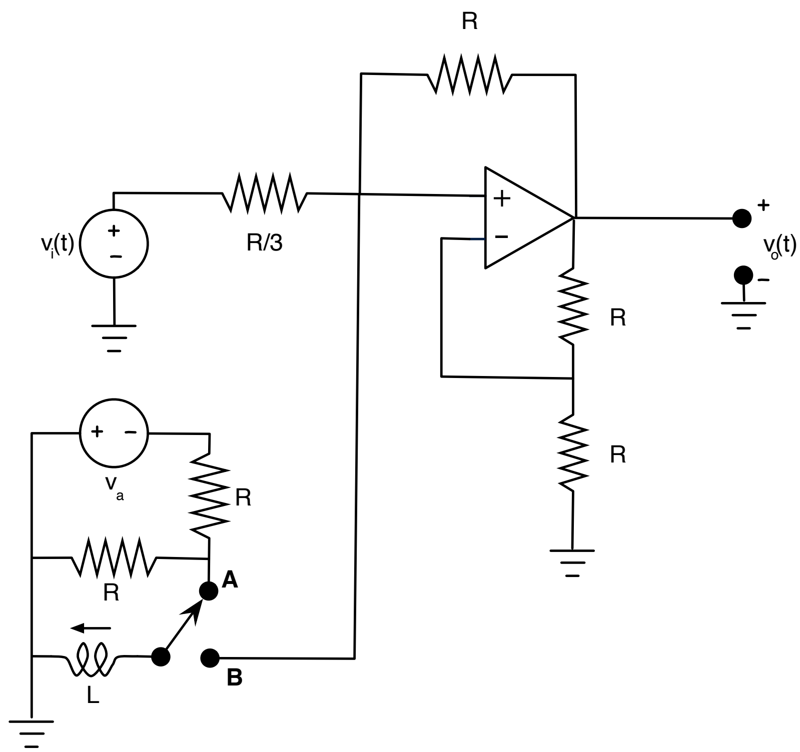


Figure 2: RL circuit for Laplace Analysis for Question 2.

2. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 2. The value v_a of the current source is constant. The switch is kept in position **A** for a very long time. At $t = 0$, it is moved to position **B**. Show that the initial condition for the inductor is given by

$$i_L(0^-) = -\frac{v_a}{R}.$$

[Show your work]

Part II: [5 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the inductor in which the initial condition appears as a voltage source. Use nodal analysis to express the output response transform $V_o(s)$ as a function of $V_i(s)$ and v_a.

Part III: [3 points] Use partial fractions and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $v_a = 1\,\mathrm{V}$, $v_i(t) = e^{-t}u(t)\,V$, $L = 1\,\mathrm{H}$, and $R = 1\,\Omega$ is

$$v_o(t) = \left(6e^{-t} - 2e^{-t/2}\right)u(t).$$

Part IV: [Extra 2 points] Decompose the output voltage of Part III as (i) the sum of the natural and forced response, and (ii) the sum of the zero-state and zero-input response.

3. **Frequency Response Analysis**

Consider the transfer function

$$T(s) = \frac{10}{s^2 + s + 1}$$

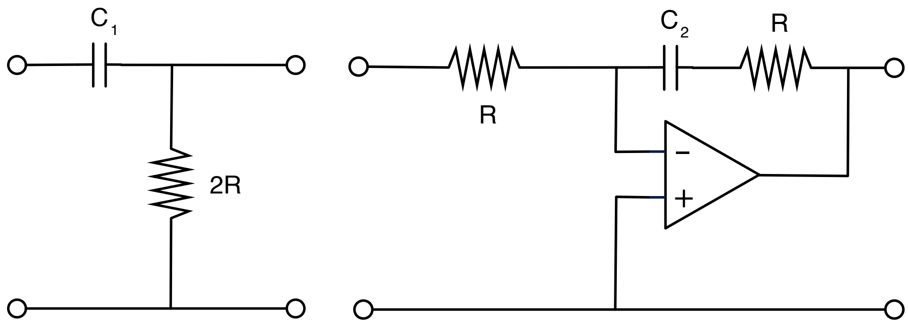
Part I [2 points] Compute the gain $|T(j\omega)|$ and phase $< T(j\omega)$ functions

Part II [4 points] What are the DC gain and the ∞-freq gain? What are the corresponding values of the phase function? What is the cut-off frequency?

Part III [2 points] Sketch plots for the gain and phase functions. What type of filter is this one? [Explain your answer]

Part IV [2 points] Using what you know about frequency response, compute the steady-state response $v_o^{SS}(t)$ to the input $v_i(t) = \sqrt{13}\cos(2t - \frac{\pi}{2})$.

4. **OpAmp Design and Chain Rule**



(a) Block 1 (b) Block 2

Figure 3: Circuits for Question 4.

Part I: [2 points] Find the transfer functions $T_1(s)$ and $T_2(s)$ for each block in Figure 3.

Part II: [2 points] If you were to connect the blocks in series in the following two configurations: (a) (block 1, block 2) and (b) (block 2, block 1), will you obtain the same transfer function? Why?

Part III: [3 points] An engineer used a configuration from Part II whose transfer function is

$$T(s) = T_1(s) \cdot T_2(s) = -\frac{s + \frac{1}{RC_2}}{s + \frac{1}{2RC_1}}$$

However, he was surprised to observe that when he hooked up a source circuit with Thévenin equivalent $V_T(s)$ and $Z_T(s)$, the output transform was different from

$$V_o(s) = T(s)V_T(s)$$

which is what he had expected. Can you explain why? If you could use one additional op-amp, how will you fix his design so that he obtains the intended output? Justify your answer.

Part IV: [3 points] Eventually, the engineer gave up on the blocks above and came up with a different design for realizing $T(s)$. He did so by employing the same type of components (resistors with values R and $2R$, and capacitors with values C_1 and C_2) and using a non-inverting op-amp circuit and an inverting op-amp circuit based on the factorization

$$T(s) = 2 \cdot \Big(- \frac{R + \frac{1}{C_2 s}}{2R + \frac{1}{C_1 s}} \Big)$$

Verify that this factorization is correct. When he hooked up the source circuit with Thévenin equivalent $V_T(s)$ and $Z_T(s)$, he finally got the desired output transform. Can you reproduce his design?