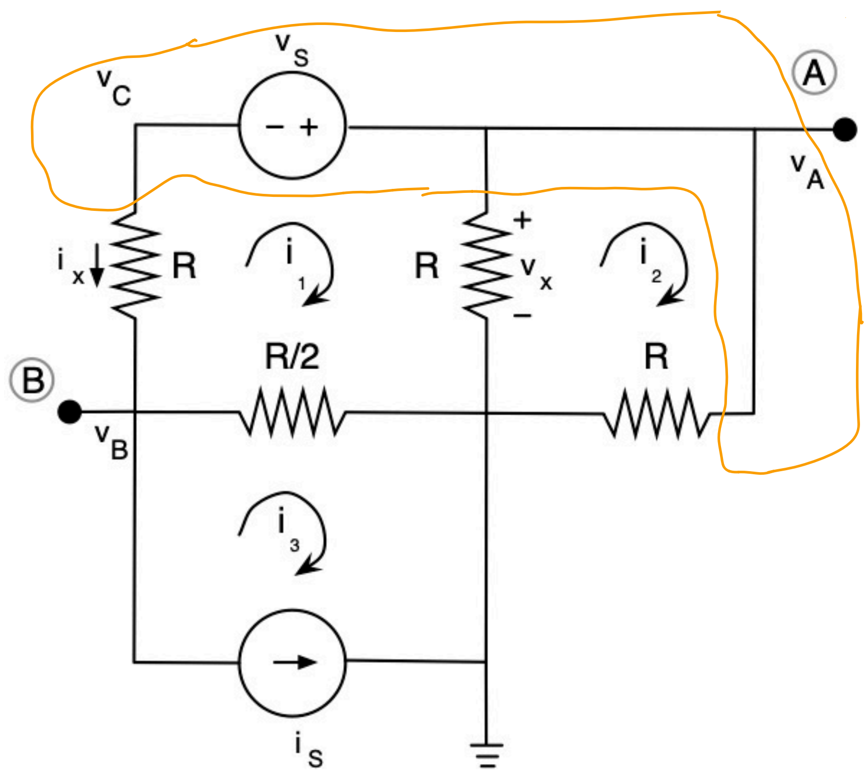
1.- Part I [Follow rubrics below depending on NV or MC solution]

[Node-voltage analysis]

To use node-voltage analysis, we need to take care of the presence of the voltage source. Since the statement says "do not modify the circuit", we cannot use source transformation. Also, ground has already been chosen in a node where we cannot use Method 2. So we have to use a supernode.

[+1 point]



Equation defining the supernode:

$$V_A - V_C = V_S$$ [+1 point]

Next, we write KCL for the supernode.

$$G(V_C - V_B) + G(V_A) + G V_A = 0$$ [+1 point]

(Here $G = \frac{1}{R}$ for convenience).

Finally, we write KCL for node B,

$$G(V_B - V_C) + 2G V_B = -i_S$$ [+1 point]

With these, we have 3 eqs. in 3 unknowns V_A, V_B, V_C.

[+1 point]

or

In matrix form,

$$\begin{pmatrix} 1 & 0 & -1 \\ 2G & -G & G \\ 0 & 3G & -G \end{pmatrix} \begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} = \begin{pmatrix} V_S \\ 0 \\ -i_S \end{pmatrix}$$

[Mesh-current analysis]

To use mesh-current analysis, we need ~~to~~ take care of the current source. Fortunately, the current source only belongs to one mesh. Therefore, Method 2 tells us that [+1 point]

$$i_3 = -i_S \qquad [+1\text{ point}]$$

Next, we write KVL for meshes 1 and 2.

KVL for mesh 1 takes the form:

$$-V_S + R(i_1 - i_2) + \frac{R}{2}(i_1 - i_3) + R i_1 = 0 \qquad [+1\text{ point}]$$

KVL for mesh 2 takes the form:

$$R i_2 + R(i_2 - i_1) = 0 \qquad [+1\text{ point}]$$

This gives us 3 eqs in 3 unknowns i_1, i_2, i_3.

Or if we substitute $i_3 = -i_S$ in the last 2 equations, then 2 eqs in 2 unknowns i_1, i_2.

In matrix form, or

[+1 point]

$$\begin{pmatrix} 0 & 0 & 1 \\ 5R/2 & -R & -R/2 \\ -R & 2R & 0 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -i_S \\ V_S \\ 0 \end{pmatrix}$$

Part II

i_1 is the current flowing through the leftmost R resistor. Therefore

$$i_1 = G(V_B - V_C)$$ [+1 point]

Likewise, i_2 is the current flowing through the rightmost R resistor,

$$i_2 = G V_A$$ [+1 point]

Finally,

$$V_B = \frac{R}{2}(i_3 - i_1),$$

so $2G V_B = i_3 - i_1$

$$i_3 = i_1 + 2G V_B = G(V_B - V_C) + 2G V_B$$
$$= 3G V_B - G V_C$$ [+1 point]

Part III

From the circuit,

$V_x = V_A$ [+1 point] and $i_x = G(V_C - V_B)$ [+1 point]

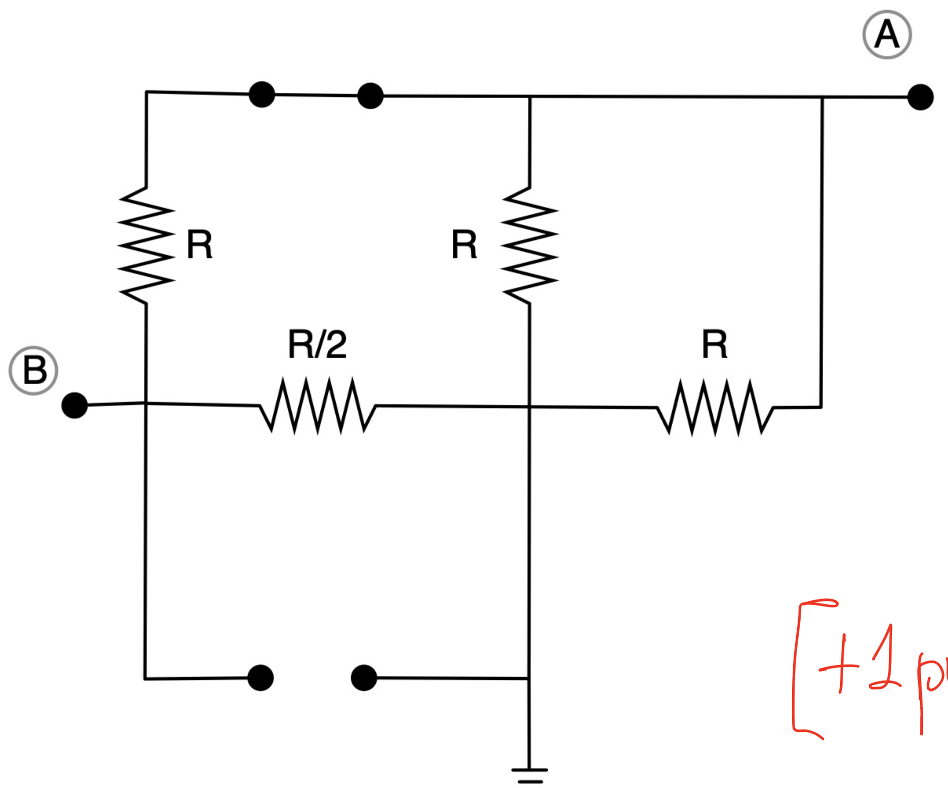
Part IV (extra)

If we were allowed to modify the circuit, then we could also take care of the voltage source by using source transformation since it is in series with the R-resistor on the left. (Method 1). This has the added advantage of decreasing the number of nodes by one.

[+1 extra point]

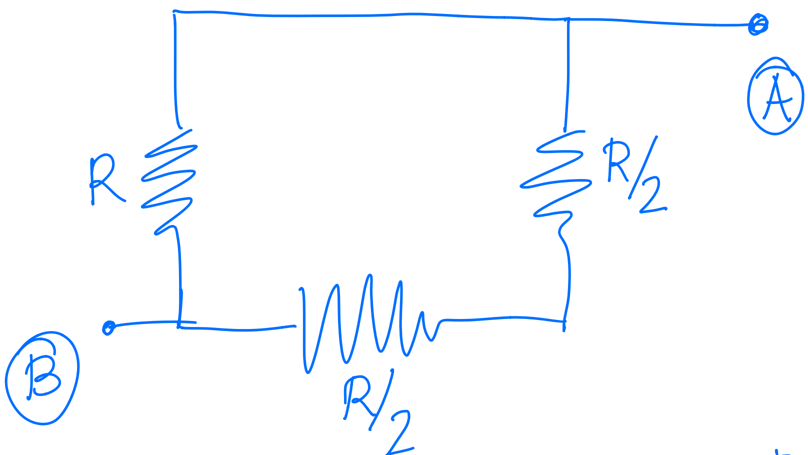
2.- Part I

As instructed, we turn off the sources to get the circuit below (the voltage source gets replaced by a short circuit, the current source gets replaced by an open circuit).

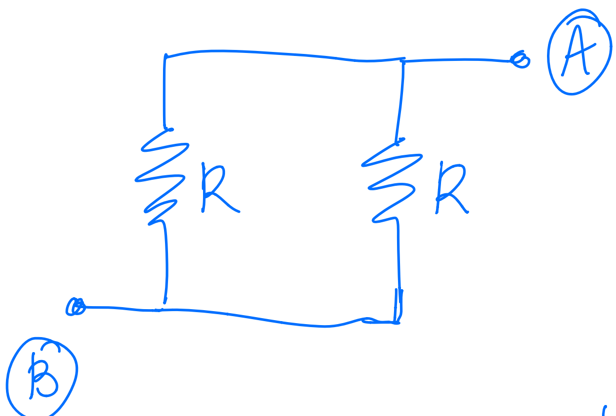


[+1 point]

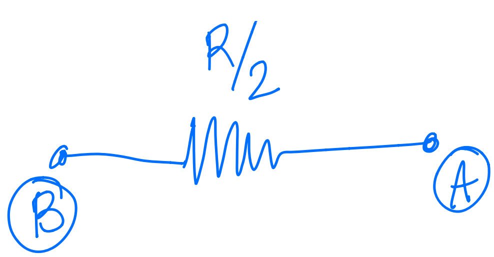
Now we use association of resistors to simplify, starting with the two R-resistors in parallel on the right.



Next, we combine the 2 $R/2$-resistors in series,



Finally, we combine the 2 R-resistors in parallel.

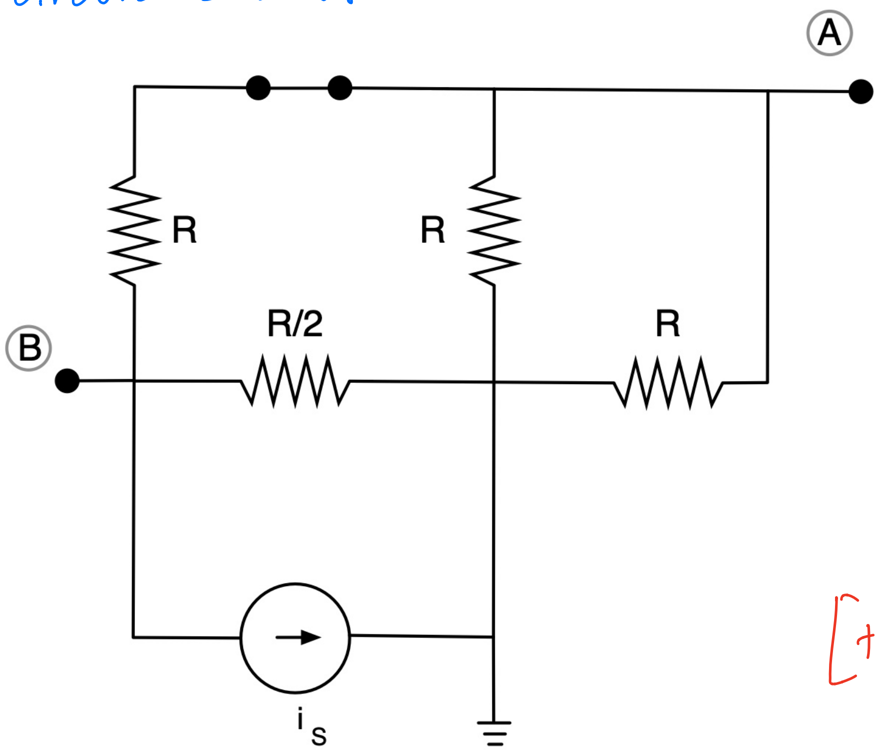


So $R_{EQ} = R/2$.

[+1 point]

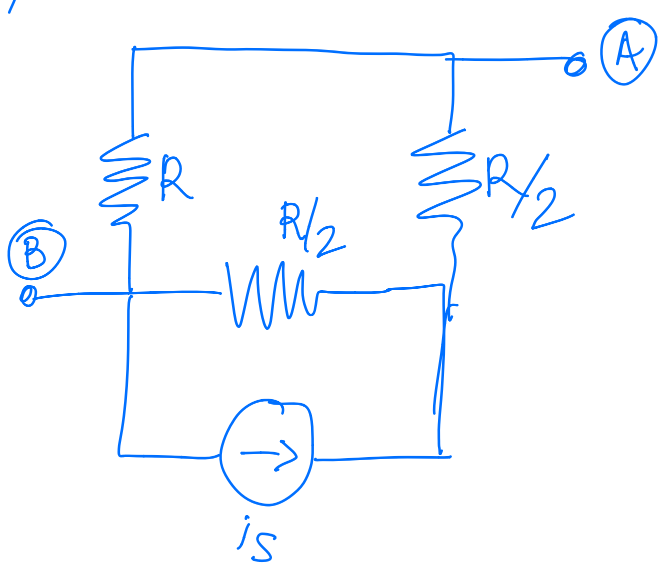
Part II

We turn off the voltage source to get the circuit below.

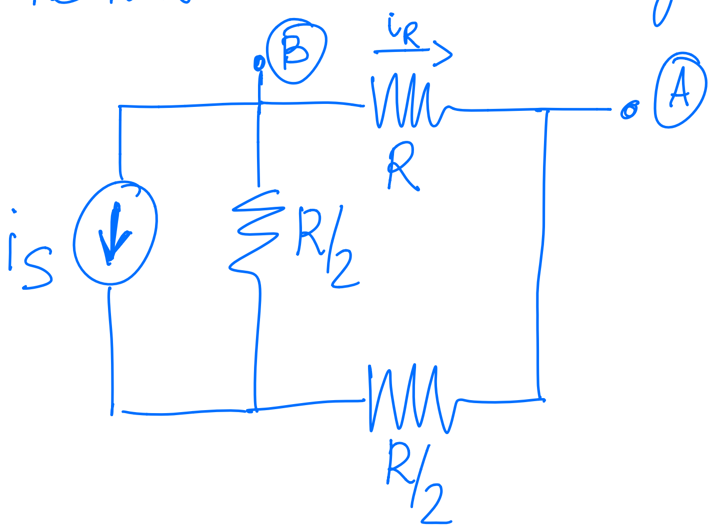


[+1 point]

Next, we combine the two resistors on the right.



To see things a bit more clearly, we redraw the circuit as follows



[Some students might instead have used source transformation and voltage division to compute the answer. That's also fine].

So we just need to determine the voltage drop across the R-resistor. We can do this by figuring out the current through the resistor and then using Ohm's law.

By current division,

$$i_R = \frac{\frac{2}{3}G}{\frac{2}{3}G + 2G}(-i_S) = -\frac{i_S}{4}$$

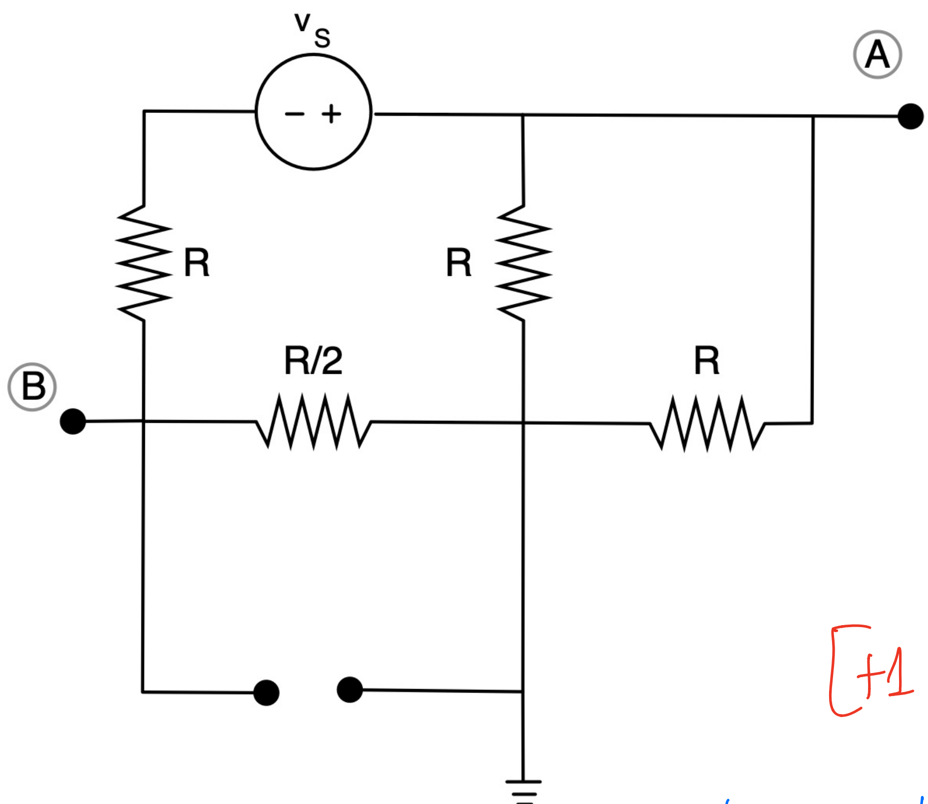
[+1 point]

Therefore, by Ohm's law

$$(V_{AB})_1 = -R \cdot \left(\frac{-i_S}{4}\right) = \frac{R\,i_S}{4}$$ [+1 point]

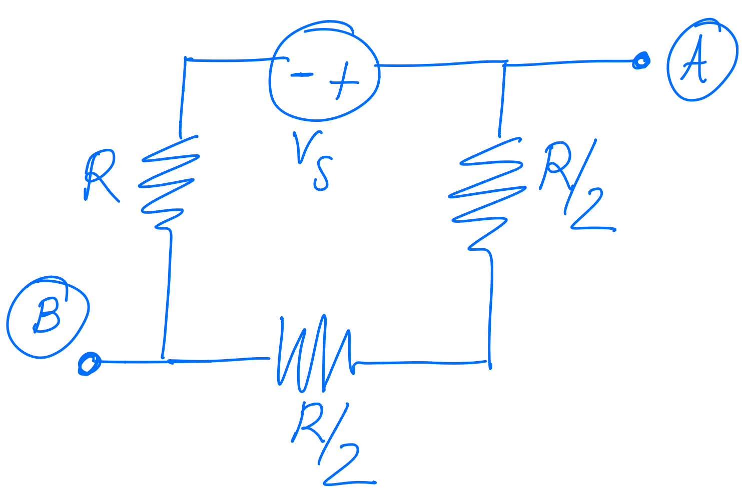
Part III

We turn off the current source to get the circuit below.

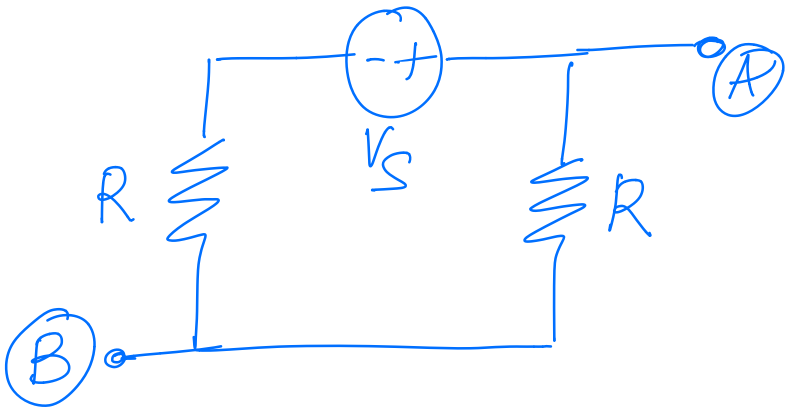


[+1 point]

We assume the 2 R-resistors on the right.



Next we combine the two $R/2$ resistors in series.



[+1 point]

Using voltage division, we deduce

$$(V_{AB})_2 = \frac{R}{R+R} V_S = \frac{V_S}{2}$$ [+1 point]

Part IV

By superposition, we have

$$V_T = V_{OC} = (V_{AB})_1 + (V_{AB})_2 = \frac{V_S}{2} + \frac{R}{4} i_S$$ [+1 point]

From Part I, we have $R_T = R_{EQ} = R/2$

Therefore the Thévenin equivalent is [+1 point]