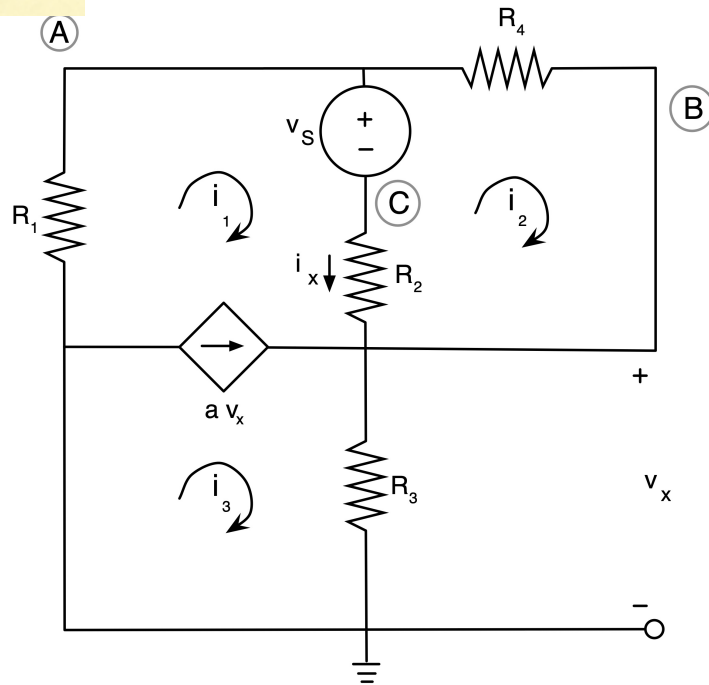# 1. — Part I

To use mesh-current analysis, we first need to take care of the presence of the current source. We do this using a <u>supermesh</u> (method 3), [+1 point]

$$i_3 - i_1 = a v_x$$ [+1 point]

KVL at the supermesh is

$$V_S + R_2(i_1 - i_2) + R_3 i_3 + R_1 i_1 = 0 \quad \text{[+1 point]}$$

The remaining KVL at mesh 2 takes the form

$$R_4 i_2 + R_2(i_2 - i_1) - V_S = 0 \quad \text{[+1 point]}$$

Finally, we need to take into account the presence of the dependent source. This means

expressing v_x as a function of the mesh currents.
Looking at the circuit, we have

[+1 point]

$$v_x = R_3 i_3$$

This gives us a total of 4 equations in 4 unknowns i_1, i_2, i_3, v_x. In matrix form ↖ Either or [+1 point]

$$\begin{pmatrix} -1 & 0 & 1 & -a \\ R_1+R_2 & -R_2 & R_3 & 0 \\ -R_2 & R_2+R_4 & 0 & 0 \\ 0 & 0 & -R_3 & 1 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \\ v_x \end{pmatrix} = \begin{pmatrix} 0 \\ -V_S \\ V_S \\ 0 \end{pmatrix}$$

Part II

[+0.5 point]

$$V_A = -R_1 i_1$$

[+0.5 point]

$$V_B = R_3 i_3$$

[+1 point]

$$V_C = R_2(i_1 - i_2) + R_3 i_3$$

Part III

[+1 point]
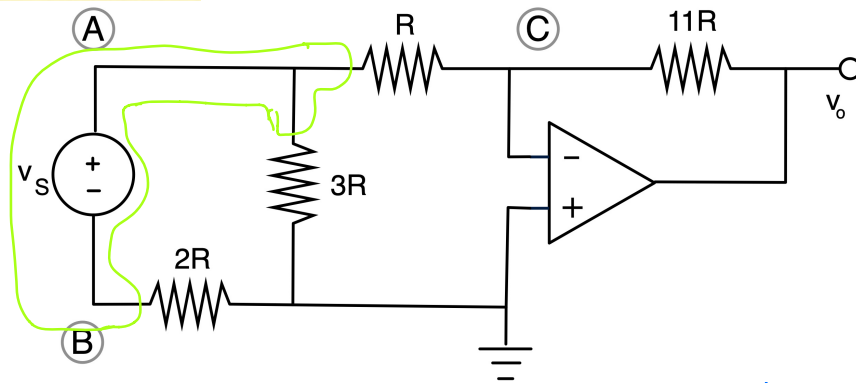
$$V_X = V_B$$

[+1 point]

$$i_x = \frac{1}{R_2}(V_C - V_B)$$

2. — Part I

The presence of the voltage source is a problem for node voltage analysis. We can create a supernode and set

[+1 point]

[+1 point]

$$V_A - V_B = V_S$$

KCL at the supernode is $\left(G = \dfrac{1}{R}\right)$

$$G(V_A - V_C) + \frac{G}{3}(V_A) + \frac{G}{2}(V_B) = 0 \quad \text{[+1 point]}$$

Finally, KCL at node Ⓒ is

$$G(V_C - V_A) + \frac{G}{11}(V_C - V_0) = 0 \qquad \text{[+1 point]}$$

(here, we have used ideal conditions, so $i_n = 0$)

Ideal conditions of the op-amp also specify

[+1 point]

$$V_C = V_n = V_p = 0$$

Substituting above, we obtain

$$V_A - V_B = V_S$$

$$\frac{4G}{3} V_A + \frac{G}{2} V_B = 0 \implies V_B = -\frac{8}{3} V_A$$

$$-G V_A - \frac{G}{11} V_0 = 0 \implies V_0 = -11 V_A$$

Solving for V_A,

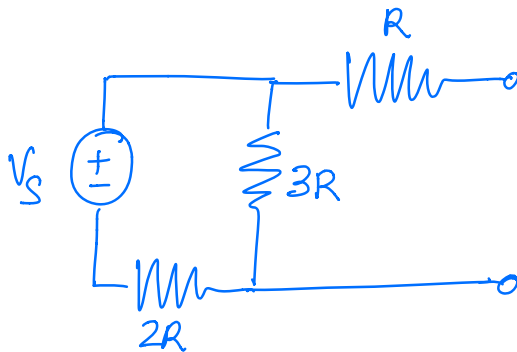$$\frac{11}{3} V_A = V_S \implies V_A = \frac{3}{11} V_S$$

$$V_B = -\frac{8}{11} V_S$$

$$V_0 = -3 V_S$$

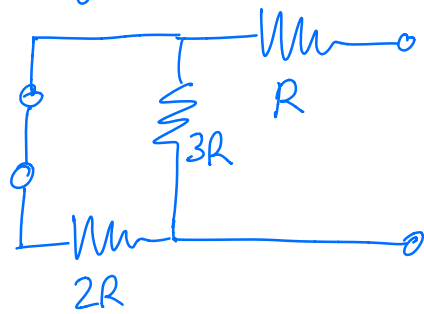So the first engineer was right.   [+1 point]

Part II

The source circuit described in the statement is

Let's compute its Thévenin equivalent

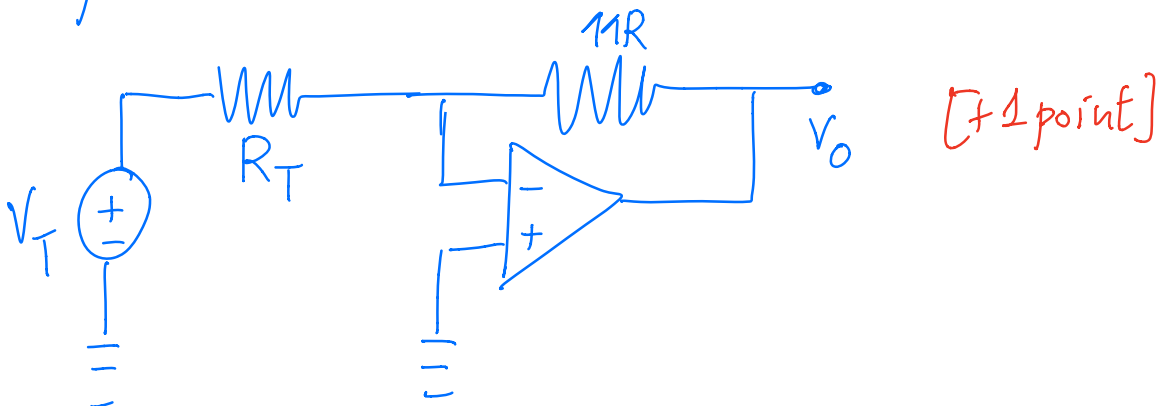[+1 point]

Turning the source off, we get

So $R_T = R + 2R \| 3R$

$$= R + \frac{6R^2}{5R}$$

$$= \frac{11}{5} R \quad \textcolor{red}{[+0.5\ \text{point}]}$$

On the other hand, V_T is the open-circuit voltage of the source circuit, i.e.,

$$V_T = V_{OC} = \frac{3R}{2R+3R} V_S = \frac{3}{5} V_S \quad \textcolor{red}{[+0.5\ \text{point}]}$$

Going back to the original circuit, we substitute the source circuit by its Thévenin equivalent to get

$\textcolor{red}{[+1\ \text{point}]}$

This is an inverting op-amp basic building

$\textcolor{red}{[+1\ \text{point}]}$

block, so

$$V_0 = -\frac{11R}{R_T} V_T = -\frac{11R}{\frac{11}{5}R} \cdot \frac{3}{5} V_S = -3V_S$$

which is the same answer we got in Part I.

Part III

With an input of $V_S = 4V$, if the op-amp is operating in the linear range, we should have $V_0 = -3 \cdot 4 = -12V$. Connecting a load at the output node does not change the voltage (because of the zero output resistance of the op-amp), so the current through the resistor load should be

$$i = -\frac{12}{6 \cdot 10^3} = -2 \cdot 10^{-3}A = -2\ mA$$

The fact that this is not the value means that the op-amp was not operating in the linear range, i.e., it <u>saturates</u> **[+1 extra point]** with the input $V_S = 4V$. To measure a current of $-1mA$, it means that $-V_{CC} = -6V$.

[+1 extra point]

3.—

Because there is current flowing through the inverting op-amp. This means that the second stage loads the first one, and the open-circuit voltage ($V_{OC} = V_S$) of the first stage is not the input voltage to the inverting op-amp.   [+2 points]
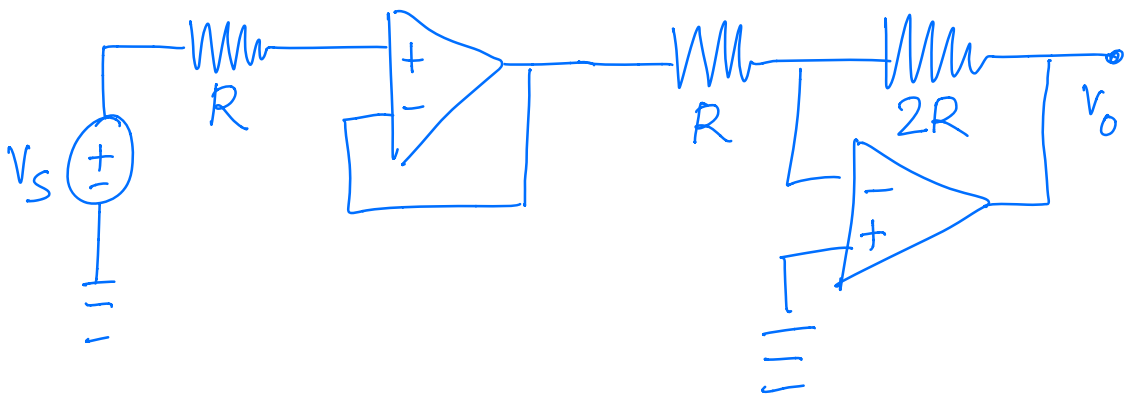
Instead, we will have

$$V_O = -\frac{2R}{R+R} V_S = -V_S \qquad \text{[+2 points]}$$

Part II

We could use a voltage follower, as follows
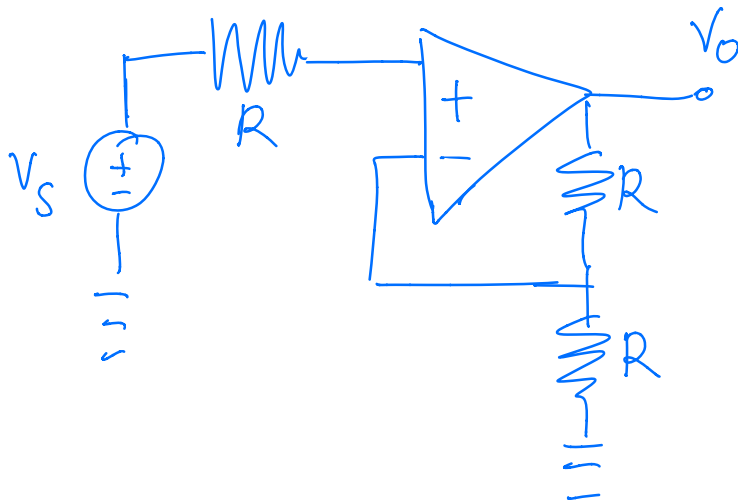

[+2 points]

The voltage follower has ∞-input resistance (so it does not load the first stage) and 0-output resistance (so it does not get loaded by the last stage). Consequently,

$$V_O = -\frac{2R}{R} \cdot 1 \cdot V_S = -2V_S$$

and the instructor's expectation becomes true.

[+2 points]

Part III

This can be done with a non-inverting op-amp.

$$V_O = \frac{R+R}{R} V_S$$
$$= 2V_S$$

[+2 points]