Linear Circuit Experiment (MAE171a)

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class information and lab handouts will be available on http://maecourses.ucsd.edu/labcourse/ Main Objectives of Laboratory Experiment:

modeling, building and debugging of op-amp based linear circuits for standard signal conditioning

Ingredients:

- modeling of standard op-amp circuits
- signal conditioning with application to audio (condensor microphone as input, speaker as output)
- implementation & verification of op-amp circuits
- sensitivity and error analysis

Background Theory:

- Operational Amplifiers (op-amps)
- Linear circuit theory (resistor, capacitors)
- Ordinary Differential Equations (dynamic analysis)
- Amplification, differential & summing amplifier and filtering

Outline of this lecture

- Linear circuits & purpose of lab experiment
- Background theory
 - op-amp
 - linear amplification
 - single power source
 - differential amplifier
 - summing circuit
 - filtering
- Laboratory work
 - week 1: microphone and amplification
 - week 2: mixing via difference and adding
 - week 3: filtering and power boost
- Summary

Linear Circuits & Signal Conditioning

Signal conditioning crucial for proper signal processing. Applications may include:

- Analog to Digital Conversion
 - Resolution determined by number of bits of AD converter
 - Amplify signal to maximum range for full resolution
- Noise reduction
 - Amplify signal to allow processing
 - Filter signal to reduce undesired aspects
- Feedback control
 - Feedback uses reference r(t) and measurement y(t)
 - Compute difference e(t) = r(t) y(t)
 - Amplify, Integrate and or Differentiate e(t) (PID control)
- Signal generation
 - Create sinewave of proper frequency as carrier
 - Create blockwave of proper frequency for counter
 - etc. etc.

Purpose of Lab Experiment

In this laboratory experiment we focus on a (relatively simple) signal conditioning algorithms: *amplification*, *adding/difference* and basic (at most 2nd order) *filtering*.

Objective: to model, build and debug op-amp based linear circuits that allow signal conditioning algorithms.

We apply this to an audio application, where the signal of a condenser microphone needs to amplified, mixed and filtered.

Challenge: single source power supply of 5 Volt. Avoid clipping/distortion of amplified, mixed and filtered signal.

Aim of the experiment:

- insight in op-amp based linear circuits
- build and debug (frustrating)
- compare theory (ideal op-amp) with practice (build and test)
- verify circuit behavior (simulation/PSPICE)

op-amp = operational amplifier

more precise definition:

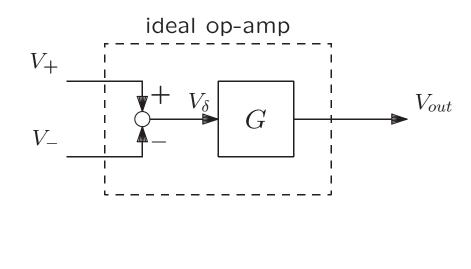
DC-coupled high-gain electronic voltage amplifier with differential inputs and a single output.

- DC-coupled: constant (direct current) voltage at inputs results in a constant voltage at output
- differential inputs: two inputs V_- and V_+ and the difference $V_{\delta}=V_+-V_-$ is only relevant
- high-gain: $V_{out} = G(V_+ V_-)$ where G >> 1.

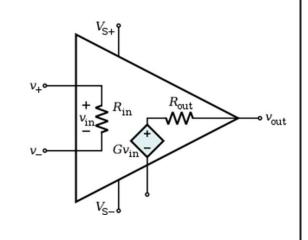
Ideal op-amp (equivalent circuit right):

- input impedance: $R_{in} = \infty \Rightarrow i_{in} = 0$
- output impedance: $R_{out} = 0$
- gain: $V_{\delta} = (V_+ V_-)$, $V_{out} = GV_{\delta}$, $G = \infty$
- rail-to-rail: $V_{S-} \leq V_{out} \leq V_{S+}$

Ideal op-amp (block diagram below)

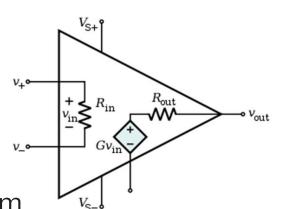






Infinite input impedance $(R_{in} = \infty)$ useful to minimize load on sensor/input.

Zero output impedance ($R_{out} = 0$) useful to minimize load dependency and obtain maximum output power.



Rail-to-rail operation to maximize range of output V_{out} between negative source supply V_{S-} and positive source supply V_{S+} .

But why (always) infinite gain G? Obviously:

$$V_{out} = \begin{cases} V_{S+} & \text{if } V_{+} > V_{-} \\ 0 & \text{if } V_{+} = V_{-} \\ V_{S-} & \text{if } V_{+} < V_{-} \end{cases}$$

not very useful with any (small) noise on V_+ or V_- .

Usefulness of op-amp with high gain G only by **feedback**!

Consider open-loop behavior:

 $V_{out} = GV_{\delta}$, where $V_{\delta} = V_{+} - V_{-}$

and create a feedback of V_{out} by choosing

 $V_{-} = KV_{out}$

to make

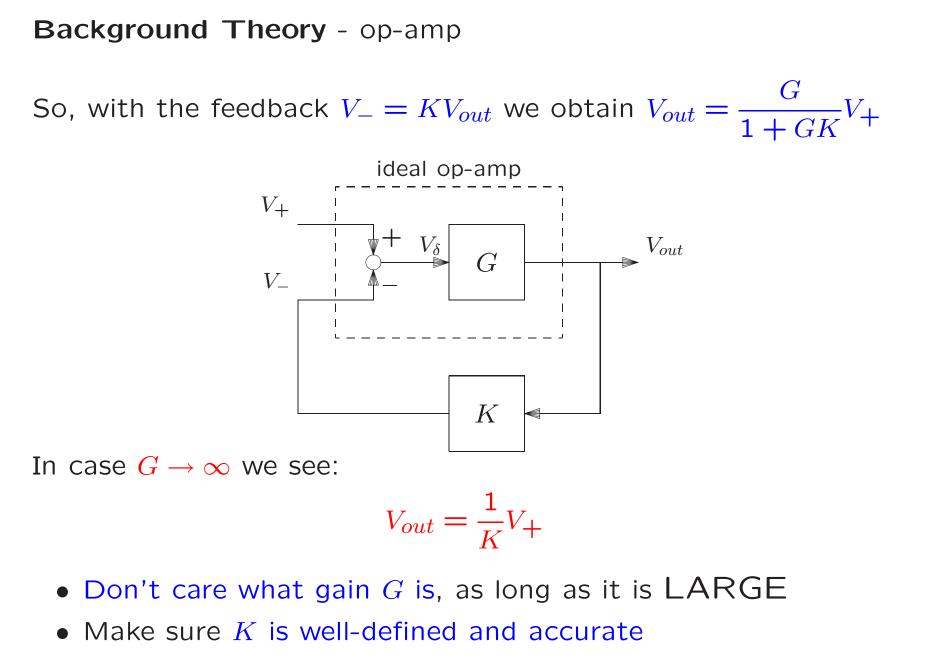
$$V_{\delta} = V_{+} - KV_{out}$$

Then

$$V_{out} = GV_{\delta} = GV_{+} - GKV_{out}$$

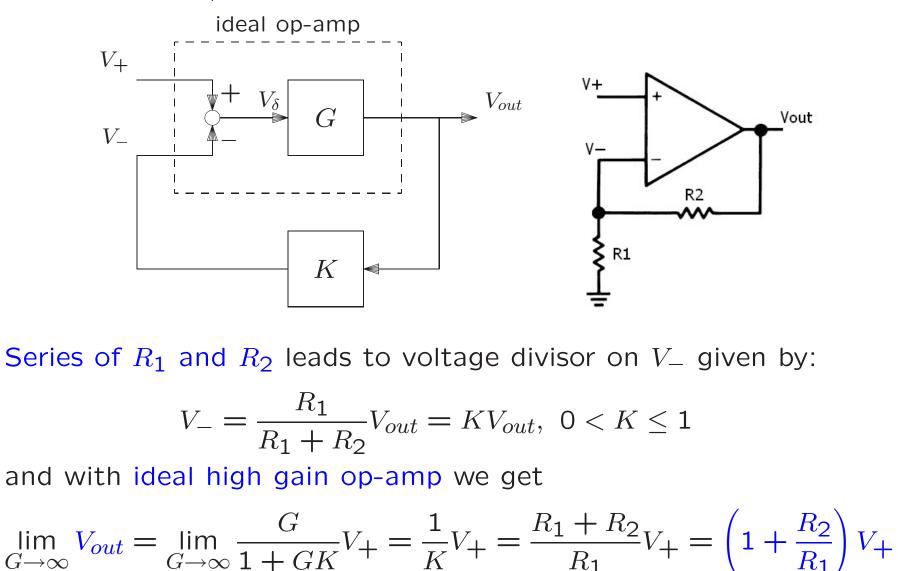
allowing us to write

$$V_{out} = \frac{G}{1 + GK} V_+$$



• If 0 < K < 1 then V_+ is nicely amplified to V_{out} by 1/K

Amplification 1/K by feedback K of ideal high gain op-amp:



Background Theory - non-inverting amplifier (voltage follower)

Our first application circuitry:

$$V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

So-called voltage follower in case

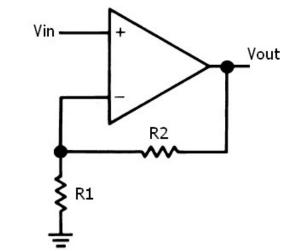
$$R_1 = \infty$$
 (not present) and $R_2 = 0$

where $V_{out} = V_{in}$ but improved output impedance!

Quick (alternative) analysis based on $V_+ = V_-$ and $i_+ = i_- = 0$:

- Since $i_{-} = 0$ and series R_1 , R_2 we have $V_{-} = \frac{R_1}{R_1 + R_2} V_{out}$
- Hence

$$V_{in} = V_{+} = \frac{R_1}{R_1 + R_2} V_{out} \Rightarrow V_{out} = \frac{R_1 + R_2}{R_1} V_{in} = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

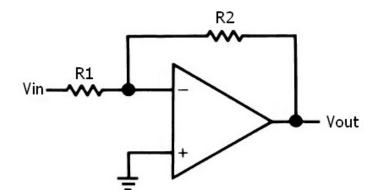


Background Theory - inverting amplifier

Similar circuit but now negative sign:

$$V_{out} = -\frac{R_2}{R_1} V_{in}$$

Quick (alternative) analysis based on $V_+ = V_-$ and $i_+ = i_- = 0$:



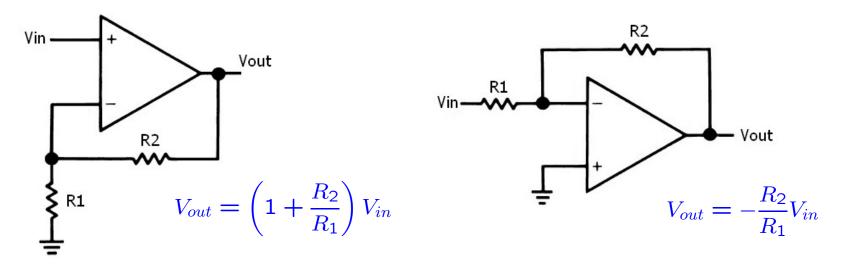
• With $V_{-} = V_{+} = 0$ and $i_{-} = 0$, Kirchhoff's Current Law indicates

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0$$

• Hence

$$\frac{V_{out}}{R_2} = -\frac{V_{in}}{R_1} \Rightarrow V_{out} = -\frac{R_2}{R_1} V_{in}$$

Background Theory - effect or rail (source) voltages



Formulae are for ideal op-amp with boundaries imposed by negative source supply V_{S-} and positive source supply V_{S+}

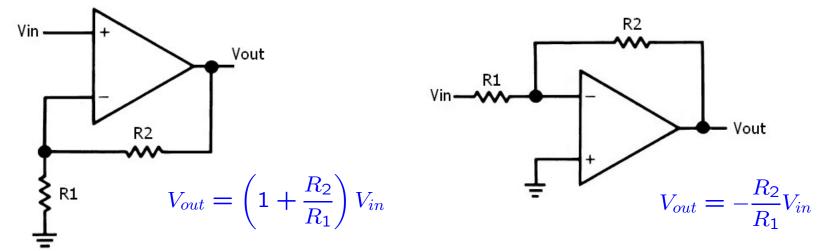
 $V_{S-} \leq V_{out} \leq V_{S+}$ (rail-to-rail op-amp)

Single voltage power supply with $V_{S+} = V_{cc}$ and $V_{S-} = 0$ (ground):

- Limits use of inverting amplifier ($V_{out} < 0$ not possible)
- Limits use of large gain R_2/R_1 ($V_{out} > V_{cc}$ not possible)

Design challenge: $0 < V_{out} < V_{cc}$ to avoid 'clipping' of V_{out} .

Background Theory - effect or rail (source) voltages



Single voltage power supply with $V_{S+} = V_{cc}$ and $V_{S-} = 0$ (ground) complicates amplification of

$$V_{in}(t) = a \sin(2\pi f t)$$

as $-a < V_{in}(t) < a$ (both positive and negative w.r.t. ground).

Example: audio application (as in our experiment).

To ensure $0 < V_{out} < V_{cc}$ provide offset compensation

$$V_{in}(t) = a\sin(2\pi ft) + a$$

to ensure $V_{in}(t) > 0$ and use non-inverting amplifier.

Background Theory - differential amplifier

Instead of amplifying one signal, amplify the difference:

$$V_{out} = \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{R_4}{R_1} V_2 - \frac{R_2}{R_1} V_1$$

Difference or differential amplifier is found by inverting amplifier and adding signal to V_+ via series connection of R_3 and R_4 . Analysis:

• With $i_+ = 0$ the series or R_3 and R_4 leads to $V_+ = \frac{R_4}{R_3 + R_4} V_2$

 $\frac{V_1 - \frac{R_4}{R_3 + R_4}V_2}{R_1} + \frac{V_{out} - \frac{R_4}{R_3 + R_4}V_2}{R_2} = 0$

R2

Vout

R1

R3

V1 -

• With $V_{-} = V_{+}$ and Kirchhoff's Current Law we have

Hence

$$V_{out} = \frac{R_2}{R_1} \cdot \frac{R_4}{R_3 + R_4} V_2 + \frac{R_4}{R_3 + R_4} V_2 - \frac{R_2}{R_1} V_1$$

or

$$V_{out} = \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{R_4}{R_1} V_2 - \frac{R_2}{R_1} V_1$$

Background Theory - differential amplifier

Choice
$$R_1 = R_3$$
 and $R_2 = R_4$ reduces
 $V_{out} = \frac{R_1 + R_2}{R_3 + R_4} \cdot \frac{R_4}{R_1} V_2 - \frac{R_2}{R_1} V_1$
to

$$V_{out} = \frac{R_2}{R_1} (V_2 - V_1)$$



Further choice of $R_1 = R_3$, $R_2 = R_4$ and $R_2 = R_1$ yields

$$V_{out} = V_2 - V_1$$

R2

Vout

R1

R3

V2

and computes the difference between input voltages V_1 and V_2 .

NOTE: $V_2 > V_1$ for a single voltage power supply with $V_{S+} = V_{cc}$ and $V_{S-} = 0$ (ground) to avoid clipping of V_{out} against ground. Background Theory - more advanced differential amplifiers

R2

Vout

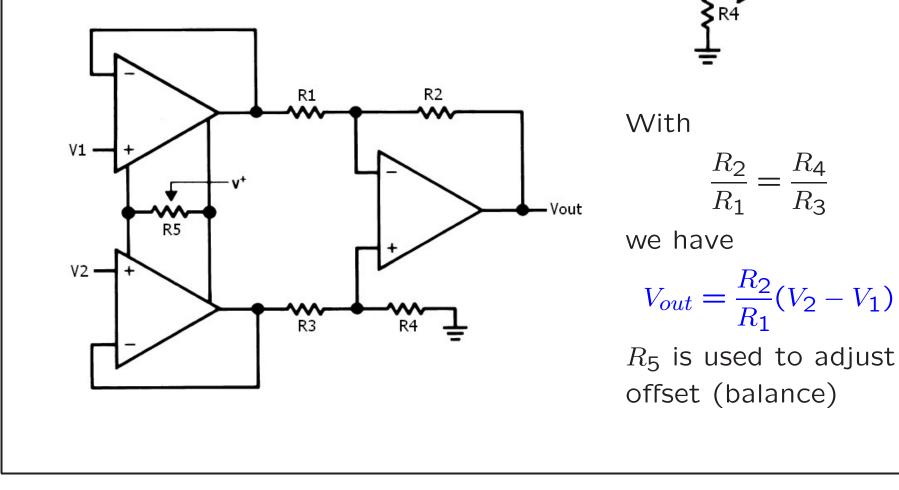
R1

R3

٧1

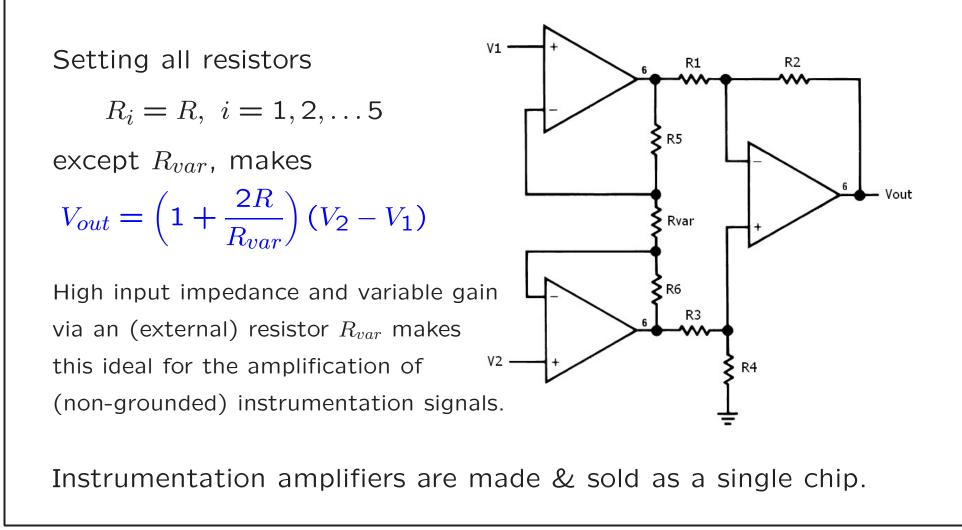
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Difference amplifier does not have high input impedance (loading of sensors). Better design with voltage followers:



Background Theory - more advanced differential amplifiers

Even better differential amplifier that has a variable gain is a so-called instrumentation amplifier:



Background Theory - inverting summing amplifier

Inverting amplifier can also be extended to add signals:

$$V_{out} = -R_4 \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

Analysis follows from Kirchhoff's Current Law for the – input of the op-amp:

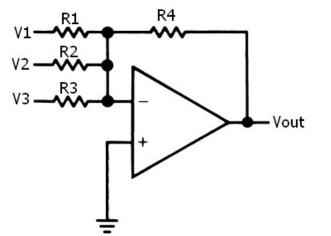
- With $V_- = V_+$ we have $V_- = 0$
- With $i_{-} = 0$ we have

$$\frac{V_{out}}{R_4} + \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = 0$$

Hence

$$V_{out} = -R_4 \cdot \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

creating a weighted sum of signals.



Background Theory - inverting summing amplifier

The choice
$$R_1 = R_2 = R_3$$
 reduces

$$V_{out} = -R_4 \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

to

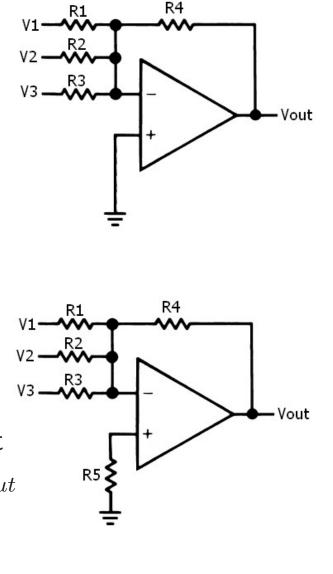
$$V_{out} = -\frac{R_4}{R_1} \left(V_1 + V_2 + V_3 \right)$$

simply amplifying the sum of the signals.

Oftentimes extra resistor R_5 is added:

$$\frac{1}{R_5} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

to account for possible small (bias) input currents $i_{-} \neq 0$, $i_{+} \neq 0$. This ensures V_{out} remains sum, without bias/offset.



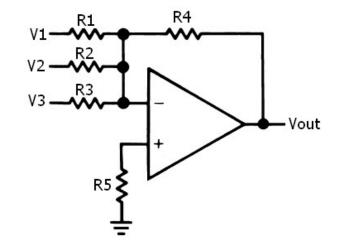
Background Theory - inverting summing amplifier

Inverting summing amplifier:

$$V_{out} = -R_4 \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

and for $R_1 = R_2 = R_3$:

$$V_{out} = -\frac{R_4}{R_1} (V_1 + V_2 + V_3)$$



has a limitation for single source voltage supplies:

Single voltage power supply with $V_{S+} = V_{cc}$ and $V_{S-} = 0$:

- Limits use of inverting summer ($V_{out} < 0$ not possible)
- Limits use of large gain R_4/R_1 ($V_{out} > V_{cc}$ not possible)

'clipping' of V_{out} will occur if sum of input signals is positive.

Background Theory - non-inverting summing amplifier

Based on a non-inverting amplifier signals can also be summed:

$$V_{out} = \left(1 + \frac{R_4}{R_3}\right) \frac{R_1 R_2}{R_1 + R_2} \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

Analysis:

- due to $i_{-} = 0$ we have $V_{-} = \frac{R_3}{R_3 + R_4} V_{out}$
- Due to $V_+ V_-$ and $i_+ = 0$ with Kirchhoff's Current Law:

$$\frac{V_1 - \frac{R_3}{R_3 + R_4} V_{out}}{R_1} + \frac{V_2 - \frac{R_3}{R_3 + R_4} V_{out}}{R_2} = 0$$

Vout

R4

R3

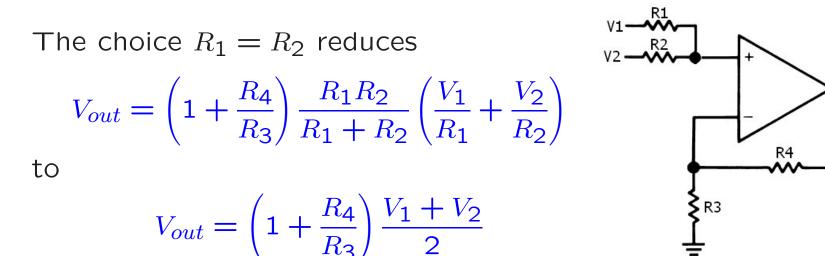
Hence

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) \frac{R_3}{R_3 + R_4} V_{out}$$

and

$$V_{out} = \left(1 + \frac{R_4}{R_3}\right) \frac{R_1 R_2}{R_1 + R_2} \left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

Background Theory - non-inverting summing amplifier



Vout

indicating amplifciation of the sum V_1 and V_2 if $R_4 \ge R_3$.

Further choice of also $R_1 = R_2 = R_3 = R_4$ leads to

 $V_{out} = V_1 + V_2$

indicating a simple summation of V_1 and V_2 .

Unlike inverting summing amplifier, no extra resistor can be added to compensate for bias input current. Not desirable: source impedance part of gain calculation...

So far, all circuits were build using op-amps and resistors.

When building filters, mostly capacitors are used as negative, positive or grounding elements.

Interesting phenomena: resistor value of capacitor depends on frequency of signal.

Analysis for capacitor: capacitance C is ratio between charge Q and applied voltage V:

$$C = \frac{Q}{V}$$

Since charge Q(t) at any time is found by flow of electrons:

$$Q(t) = \int_{\tau=0}^{t} i(\tau) d\tau$$

we have

$$V(t) = \frac{Q(t)}{C} = \frac{1}{C} \int_{\tau=0}^{t} i(\tau) d\tau$$

Application of Laplace transform to

$$V(t) = \frac{Q(t)}{C} = \frac{1}{C} \int_{\tau=0}^{t} i(\tau) d\tau$$

yields

$$V(s) = \frac{1}{Cs}i(s)$$

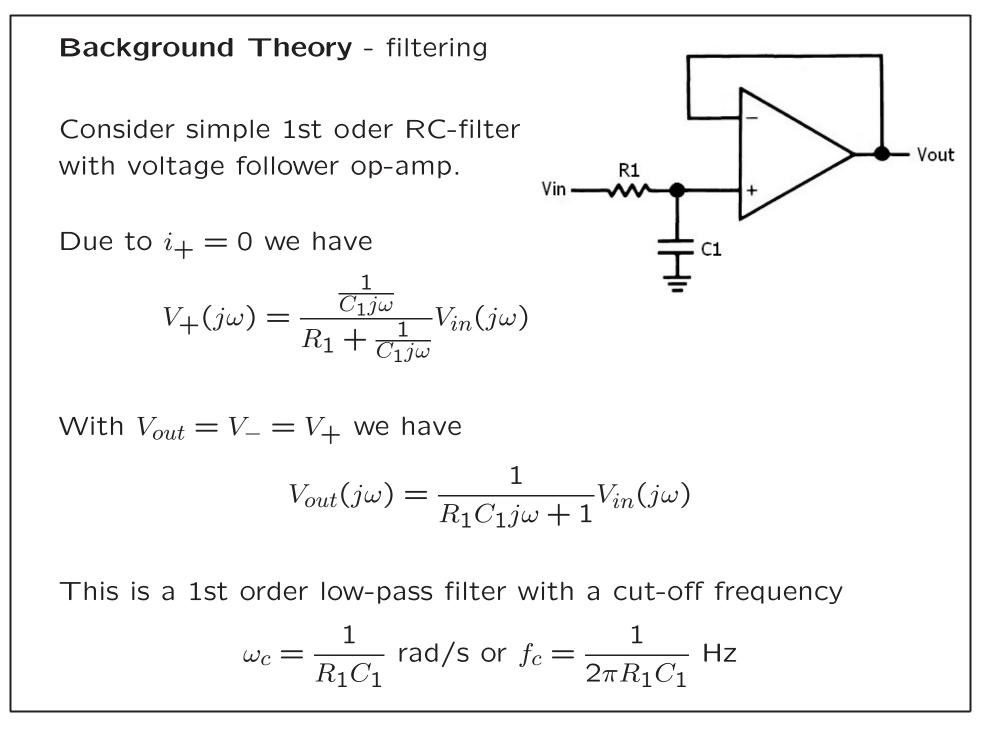
Hence we can define the impedance/resistence of a capacitor as

$$R(s) = \frac{V(s)}{i(s)} = \frac{1}{Cs}$$

With Fourier analysis we use $s = j\omega$ and we find the frequency dependent 'equivalent resistor value of a capacitor':

$$R(j\omega) = \frac{1}{jC\omega}$$

This value will allow analysis of op-amp circuits based on resistors (as we have done so far)



Consider circuit of non-inverting amplifier where R_1 is now series of R_1 and C_1 . Equivalent series resistance is given by

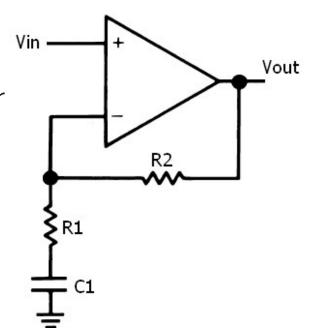
$$R_1 + \frac{1}{jC_1\omega}$$

Application of gain formula for non-inverting amplifier yields:

$$V_{out}(j\omega) = \left(1 + \frac{R_2}{R_1 + \frac{1}{jC_1\omega}}\right) V_{in}(j\omega)$$

We can directly see:

- For low frequencies $\omega \rightarrow 0$ we obtain a Voltage follower with $V_{out} = V_{in}$
- For high frequencies $\omega \to \infty$ we obtain our usual non-inverting amplifier $V_{out}(j\omega) = \left(1 + \frac{R_2}{R_1}\right) V_{in}(j\omega)$



Transition between low and high frequency can be studies better by writing $V_{out}(s) = G(s)V_{in}(s)$ where G(s) is a transfer function.

This allows us to write

$$V_{out}(s) = \left(1 + \frac{R_2}{R_1 + \frac{1}{C_1 s}}\right) V_{in}(s)$$

as

$$V_{out}(s) = \left(1 + \frac{R_2 C_1 s}{R_1 C_1 s + 1}\right) V_{in}(s) = \frac{(R_1 + R_2) C_1 s + 1}{R_1 C_1 s + 1} V_{in}(s)$$

making

$$G(s) = \frac{(R_1 + R_2)C_1s + 1}{R_1C_1s + 1}$$

The transfer function

$$G(s) = \frac{(R_1 + R_2)C_1s + 1}{R_1C_1s + 1}$$

has the following properties:

- single pole at $p_1 = -\frac{1}{R_1C_1}$ and found by solving $R_1C_1s+1=0$.
- single zero at $z_1 = -\frac{1}{(R_1+R_2)C_1}$ and found by solving $(R_1 + R_2)C_1s + 1 = 0$.
- DC-gain of 1 and found by substitution s = 0 in G(s). Related to the final value theorem for a step input signal $v_{in}(t)$:

$$\lim_{t \to \infty} V_{out}(t) = \lim_{s \to 0} s \cdot V_{out}(s) = \lim_{s \to 0} s \cdot G(s) \cdot \frac{1}{s} = \lim_{s \to 0} G(s)$$

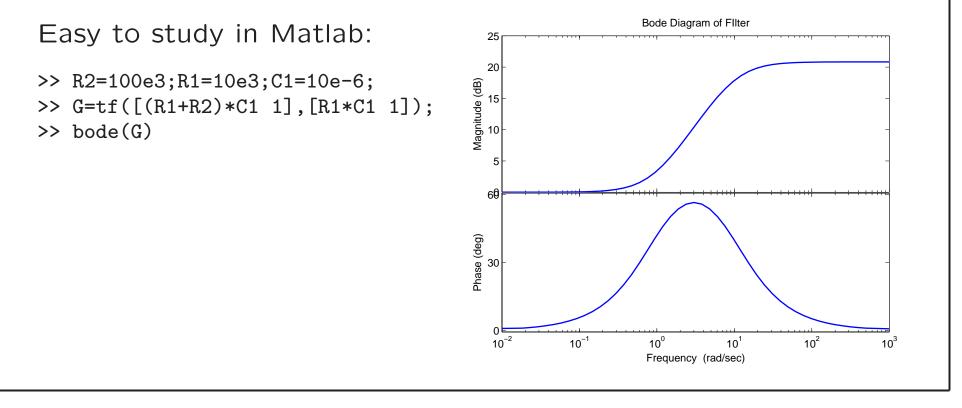
where $\frac{1}{s}$ is the Laplace transform of the step input $v_{in}(t)$.

• High frequency gain of $\frac{R_1+R_2}{R_1} = 1 + \frac{R_2}{R_1}$ and found by computing $s \to \infty$.

The transfer function

$$G(s) = \frac{(R_1 + R_2)C_1s + 1}{R_1C_1s + 1}$$

is a first order system where zero $z_1 = -\frac{1}{(R_1 + R_2)C_1} < p_1 = -\frac{1}{R_1C_1}$. This indicates G(s) is a lead filter.



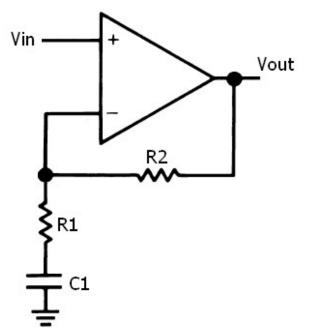
Amplification lead filter circuit with

$$V_{out}(j\omega) = \left(1 + \frac{R_2}{R_1 + \frac{1}{jC_1\omega}}\right) V_{in}(j\omega)$$

will be used to strongly amplify a small high frequent signal but maintain (follow) the DC-offset.

From the previous analysis we see:

- Gain at DC ($\omega = 0$) is simply 1.
- Gain at higher frequencies approaches $1 + \frac{R_2}{R_1}$



Another fine filter:

- 2nd order low pass
 Butterworth filter
- Pass-band frequency of 1kHz
- 2nd order 1kHz Butterworth filter is a standard 2nd order system

$$V_{out}(s) = G(s)V_{in}(s)$$

10.00 KΩ

Vin

141.4 nF

10.00 KO

Vout

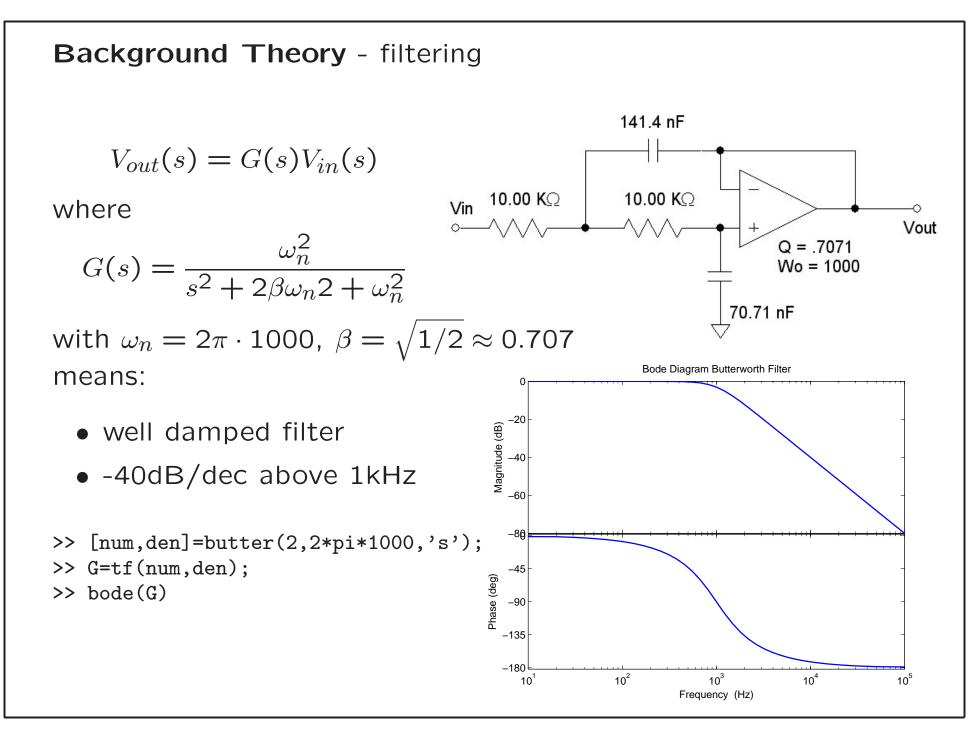
Q = .7071 Wo = 1000

70.71 nF

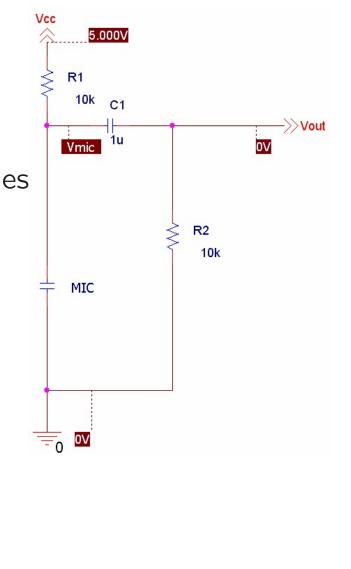
where

$$G(s) = \frac{\omega_n^2}{s^2 + 2\beta\omega_n 2 + \omega_n^2}$$

with $\omega_n = 2\pi \cdot 1000$, $\beta = \sqrt{1/2} \approx 0.707$.



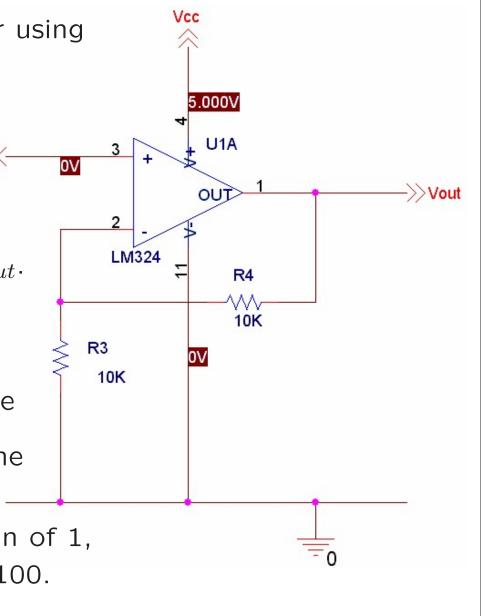
- Audio application: generate input signal via MIKE-74 electret microphone
- build a DC-bias circuit for the microphone to measure (sound) pressure variations
- Measure the DC-bias (offset) voltages
- Display and analyse the time plots generated by the microphone



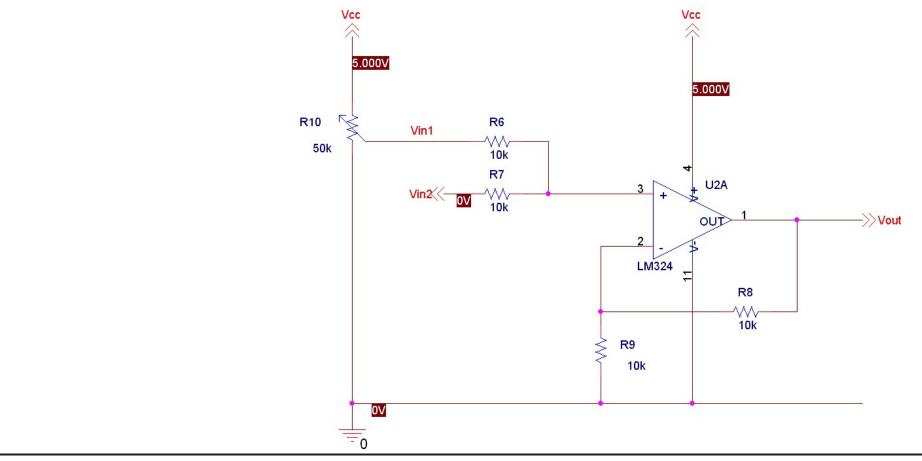
- Build non-inverting amplifier using quad LM324 op-amp
- Measure bias voltages
- Experiments: Vin determine gain for different resistor values and avoid clipping on output signal Vout.
- Measure the frequency response of your amplifier.

Connect amplifier to microphone

- Provide off-set of microphone signal to avoid clipping _____
- Create lead filter for DC-gain of 1, and high frequency gain of 100.



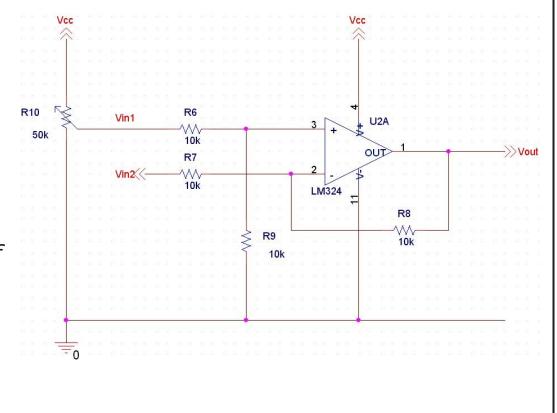
- Build none inverting summing amplifier
- Experiments: verification of operation (adding of signals)
- Bias voltage adjustment
- Experimental verification of bias effects.

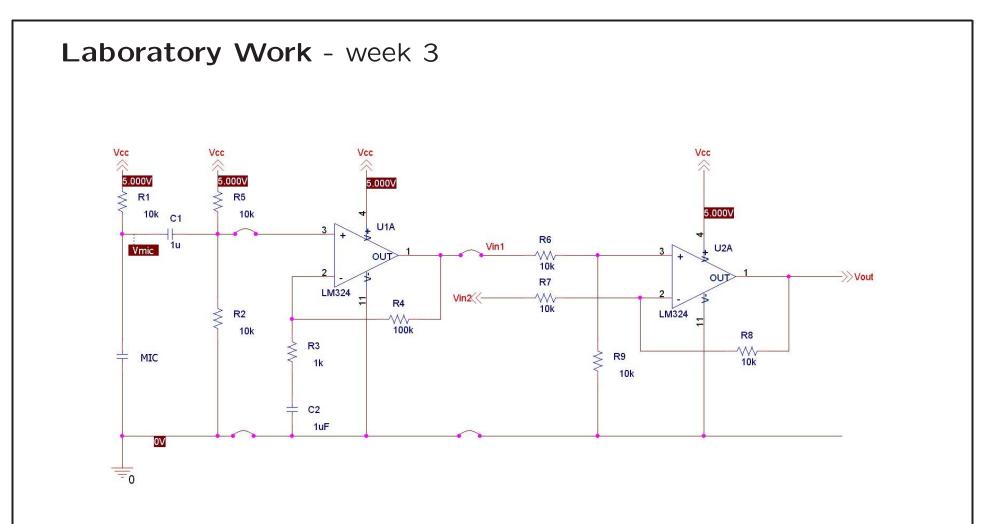


- Build differential/difference amplifier
- Experiments: verification of operation (difference of signals)
- Bias voltage adjustment
- Gain adjustments and experimental verification

Combine microphone & amplifier circuit from week 1 with difference amplifier to allow mixing of signals.

- Measure bias voltages
- Demonstrate mixing of sine wave signal and microphone signal without distortions





Starting point of week 3: completed amplified/mixed microphone signal.

 V_{out} will now be filtered and (optional) power boosted for speaker output.

- Create active low pass filter with cutt-off frequency of 1kHz.
- Demonstrate filter by measuring amplitude of output signal for sine wave excitation of different frequencies
- Connect filter to circuit of week 2 (microphone, amplifier and differential amplifier for mixing)
- Demonstrate filter by measuring microphone and filtered microphone signal
- Optional: add power boost to circuitry and connect speaker.

Summary

- (relatively simple) signal conditioning algorithms: *amplifica-tion*, *adding/difference* and basic (most 2nd order) *filtering*
- Challenge: single source power supply of 5 Volt. Avoid clipping/distortion of amplified, mixed and filtered signal.
- insight in op-amp based linear circuits by building and debuging
- compare theory (ideal op-amp) with practice (build and test)
- experimentally verify gain of circuitry
- for error/statistical analysis: measure gain for different resistor (of the same value)
- audio application on a single voltage power supply shows amplification of small signals and careful filter design to maintain DC (off-set)

GOOD LUCK