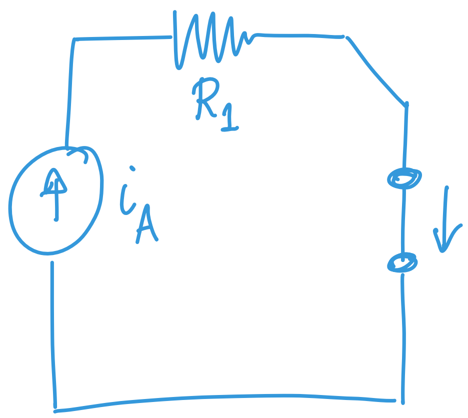
1._ Part I

Under DC excitations, we know the inductor behaves as a short circuit. Therefore, we have

[+1 point]



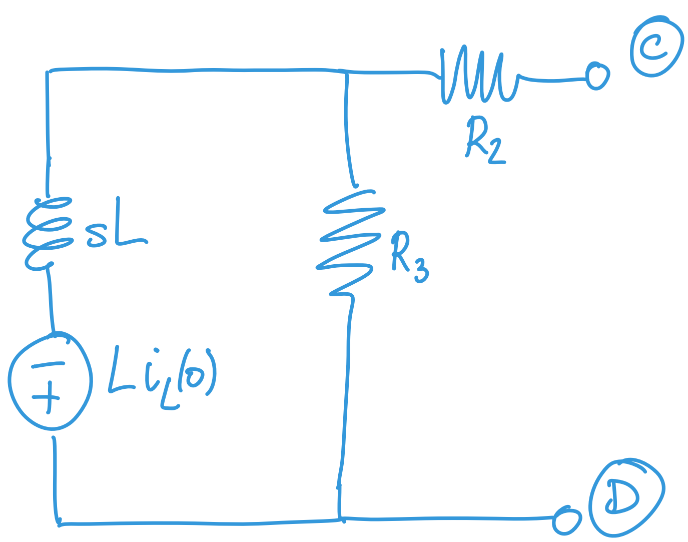
From the picture, we deduce that

$$i_L(0) = i_A$$

[+1 point]

Part II

Next, we transform the circuit into the s-domain using a voltage source to account for the initial condition.



[+2 points]

Part III

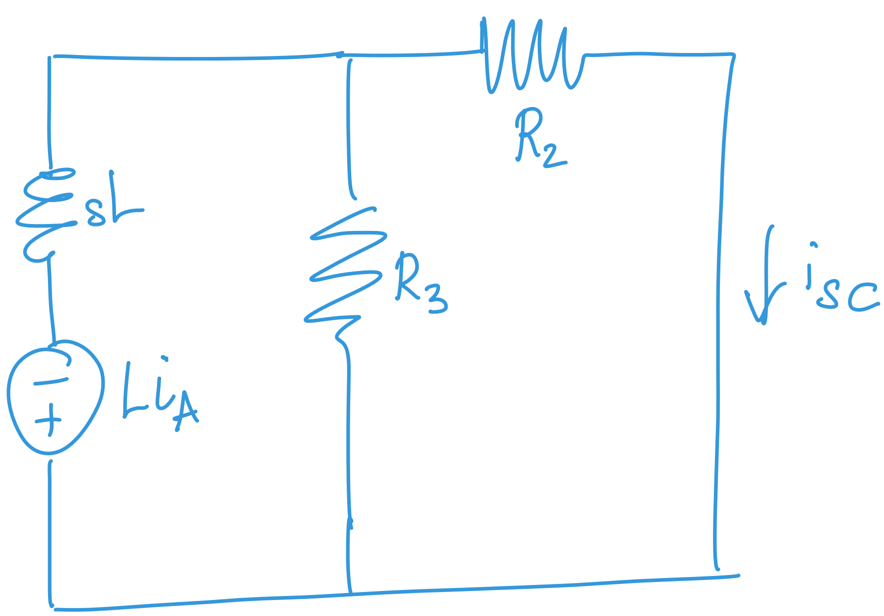
The open circuit voltage, as seen from C-D is also the voltage drop seen by the R_3 impedance (since there is no current through the R_2 impedance). Therefore, we can use voltage division to find [+1 point]

$$V_{OT} = V_{R_3} = \frac{R_3}{R_3 + sL}\left(-Li_L(0)\right) = \frac{-R_3 L i_A}{R_3 + sL}$$

[+1 point]

Part IV

To find the short-circuit current transform, we draw the circuit



So the short-circuit current is the one seen by the impedance R_2. [+1 point]

Note that

$$V_{R_2 \| R_3} = \frac{R_2 \| R_3}{R_2 \| R_3 + sL}(-L i_A)$$

Therefore

$$i_{sc} = \frac{V_{R_2 \| R_3}}{R_2} = \frac{\frac{\cancel{R_2} R_3}{R_2+R_3}(-L i_A)}{\cancel{R_2} \cdot \left(\frac{R_2 R_3}{R_2+R_3} + sL\right)} =$$

$$= \frac{-R_3 L i_A}{R_2 R_3 + sL(R_2+R_3)}$$ [+1 point]

Part V

From Part III, we already know

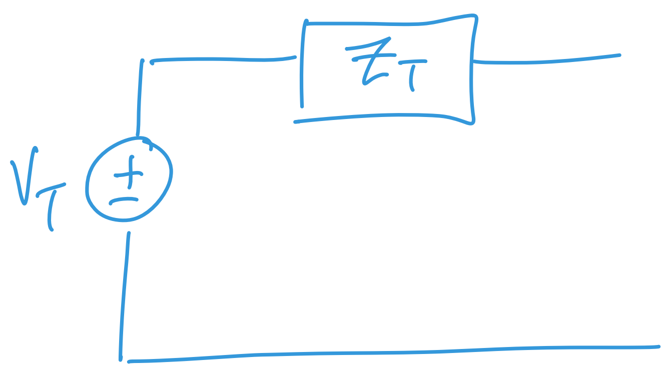
$$V_T = V_{OC} = \frac{-R_3 L i_A}{R_3 + sL}$$ [+1 point]

To find the equivalent impedance, we set

$$Z_T = \frac{V_{OT}}{i_{SC}} = \frac{\cancel{-R_3 L i_A}}{R_3 + sL} \cdot \frac{R_2 R_3 + sL(R_2 + R_3)}{\cancel{-R_3 L i_A}}$$

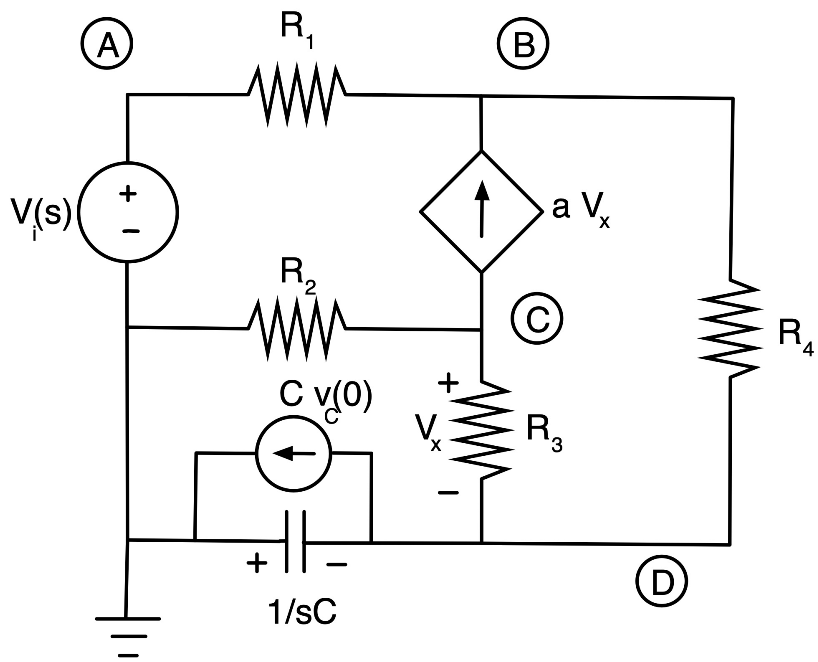
$$= \frac{R_2 R_3 + sL(R_2 + R_3)}{R_3 + sL}$$ [+1 point]

So the Thevenin equivalent looks like



2.- Part I

We convert the circuit to the s-domain, using a current source to take care of the initial condition (since we are going to use nodal analysis).



[+1 point]

Now, we only have 1 voltage source to take care of. With the choice of ground, we actually have

$$V_A = V_i(s)$$

[+1 point]

Therefore, we only need to write KCL for nodes B, C, and D.

KCL @ B,

$$\frac{1}{R_1}(V_B - V_A) + \frac{1}{R_4}(V_B - V_D) = aV_x$$ [+0.5 point]

KCL @ C,

$$\frac{1}{R_2}(V_C - 0) + \frac{1}{R_3}(V_C - V_D) + aV_x = 0$$ [+0.5 point]

KCL @ D,

$$\frac{1}{R_4}(V_D - V_B) + \frac{1}{R_3}(V_D - V_C) + sC\,V_D + CV_C(0) = 0$$ [+0.5 point]

Finally, we deal with the presence of the dependent source

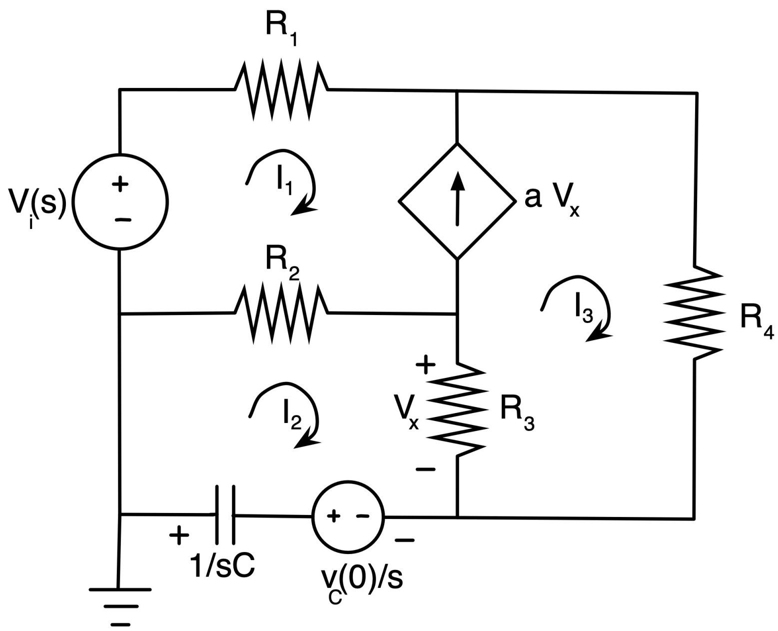
[+0.5 point]

$$V_x = V_C - V_D$$

This gives us a total of 5 eqs in 5 unknowns, V_A, V_B, V_C, V_D, V_x. [+1 point]

Part II

We convert the circuit to the s-domain, using a voltage source to take care of the initial condition (since we're going to use mesh analysis)



[+1 point]

We have one problem to take care of, the current source. Since it belongs to two meshes, we use a supermesh, defined by

$$I_1 - I_3 = -aV_x$$

[+1 point]

KVL for the supermesh is

$$R_1 \cdot I_1 + R_4 I_3 + R_3(I_3 - I_2) + R_2(I_1 - I_2) = V_i(s)$$

[+1 point]

We also write KVL for mesh 2

$$R_2(I_2 - I_1) + R_3(I_2 - I_3) - \frac{V_c(0)}{s} + \frac{1}{sC} I_2 = 0$$

[+1 point]

Finally, we deal with the presence of the dependent source,

$$V_x = R_3(I_2 - I_3)$$ [+1 point]

So we have 4 eqs in 4 unknowns, I_1, I_2, I_3, V_x.

Part III

We have

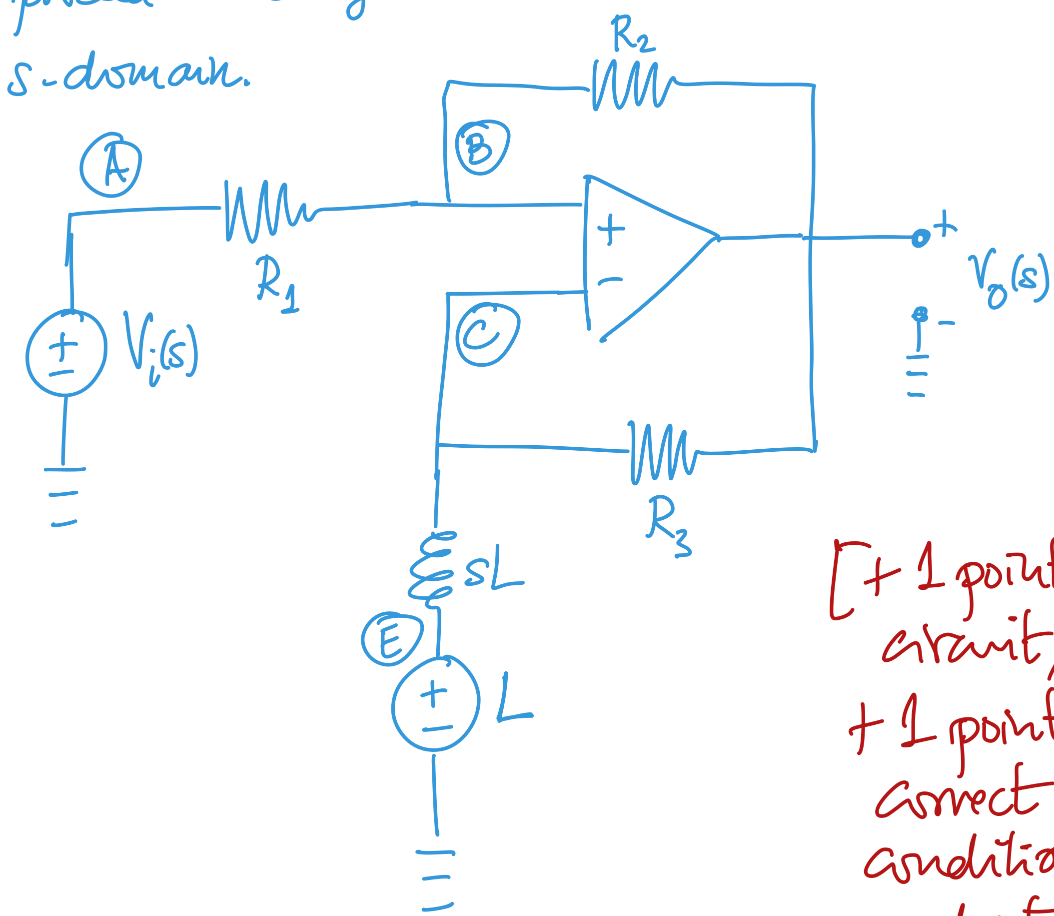
$$V_C(s) = 0 - V_D = -V_D$$ [+1 extra point]

We also have

$$V_C(s) = \frac{1}{sC}(-I_2) + \frac{V_C(0)}{s}$$ [+1 extra point]

3.– **Part I**

We are already given the information about the initial condition of the inductor, so we proceed directly to draw the circuit in the s-domain.



[+1 point for circuit, +1 point for correct initial condition of inductor]

Part II

We use the node labels in our plot above.

Right away, we see that

$$V_A = V_i(s) \quad \& \quad V_E = L$$ [+1 point]

This takes care of the two voltage sources, which as we know, are problems to set up node voltage eqs.

We next set up KCL for nodes B & C, but not for the output node of the op-amp, instead using ideal equations. [+0.5 point]

KCL @ B :

$$\frac{1}{R_1}(V_B - V_A) + \frac{1}{R_2}(V_B - V_0) = 0$$ [+0.5 point]

KCL @ C :

$$\frac{1}{sL}(V_C - V_E) + \frac{1}{R_3}(V_C - V_0) = 0$$ [+0.5 point]

Ideal conditions mean that

$$V_B = V_C$$ [+0.5 point]

Solving for V_0 in the first equation, we obtain

$$V_0 = \left(\frac{R_2}{R_1} + 1\right) V_B - \frac{R_2}{R_1} V_i$$

Doing the same in the second equation,

$$V_o = \left(\frac{R_3}{sL} + 1\right) V_C - \frac{R_3}{s}$$

Equating and solving for $V_B = V_C$, we get

$$\left(\frac{R_3}{sL} - \frac{R_2}{R_1}\right) V_B = \frac{R_3}{s} - \frac{R_2}{R_1} V_i$$

$$V_B = \frac{s R_1 L}{R_1 R_3 - R_2 L s}\left(-\frac{R_2}{R_1} V_i + \frac{R_3}{s}\right) =$$

$$= \frac{-s R_2 L V_i + R_1 R_3 L}{R_1 R_3 - R_2 L s}$$

Substituting into the first expression for V_o,

$$V_o = \frac{R_2 + R_1}{R_1} \frac{-s R_2 L V_i + R_1 R_3 L}{R_1 R_3 - R_2 L s} - \frac{R_2}{R_1} V_i =$$

$$= \frac{(R_2 + R_1)(-s R_2 L) - R_2 (R_1 R_3 - R_2 L s)}{R_1 (R_1 R_3 - R_2 L s)} V_i + \frac{(R_2 + R_1) R_3 L}{R_1 R_3 - R_2 L s}$$

$$= \frac{-\cancel{R_1}R_2Ls - \cancel{R_1}R_2R_3}{\cancel{R_1}(R_1R_3 - R_2Ls)} V_i + \frac{(R_1+R_2)R_3L}{R_1R_3-R_2Ls} =$$

$$= -\frac{R_2(R_3+Ls)}{R_1R_3-R_2Ls} V_i + \frac{(R_1+R_2)R_3L}{R_1R_3-R_2Ls} \quad \#$$

Part III

We substitute the values provided to get

$$V_o(s) = -\frac{(1+s)}{1-s}\frac{1}{s+2} + \frac{2}{1-s}$$ [+1 point]

We use partial fractions for the first expression,

$$\frac{s+1}{(s-1)(s+2)} = \frac{A}{s-1} + \frac{B}{s+2}$$

Using residues,

$$A = \lim_{s\to 1} (s-1)\cdot\frac{s+1}{(s-1)(s+2)} = \lim_{s\to 1}\frac{s+1}{s+2} = \frac{2}{3}$$

$$B = \lim_{s \to -2} (s+2) \frac{s+1}{(s-1)(s+2)} = \lim_{s \to -2} \frac{s+1}{s-1} = \frac{-1}{-3} = \frac{1}{3}$$

Therefore

$$V_0(s) = \frac{1}{3} \frac{2}{s-1} + \frac{1}{3} \frac{1}{s+2} + \frac{2}{1-s} =$$

$$= -\frac{1}{3} \frac{4}{s-1} + \frac{1}{3} \frac{1}{s+2}$$ [+1 point]

The inverse Laplace transform is given by

$$v_0(t) = \frac{1}{3} \left(e^{-2t} - 4 e^{t} \right) u(t)$$ [+1 point]

Part IV

(c) the input has a pole at $s = -2$. Therefore, the forced response is

$$(V_0)_{fr}(t) = \frac{1}{3} e^{-2t} u(t)$$ [+0.5 point]

and the natural response is

$$(V_0)_{nr}(t) = -\frac{4}{3} e^{t} u(t)$$ [+0.5 point]

(ii) if we turn the input to zero, then

$$(V_0)_{zi}(s) = \frac{2}{1-s}$$

So the zero-input response is

$$(V_0)_{zi}(t) = -2e^{t}u(t)$$ [+0.5 point]

Consequently, the zero-state response

$$(V_0)_{zs}(t) = \frac{1}{3}\left(e^{-2t} - 4e^{t}\right)u(t) + 2e^{t}u(t) =$$

$$= \frac{1}{3}\left(e^{-2t} + 2e^{t}\right)u(t)$$ [+0.5 point]

4.- Part I

Both blocks are inverting op-amps, so the transfer functions can be readily obtained from

$$T_1(s) = -\frac{\frac{1}{sC} + R}{sL} = -\frac{RCs + 1}{s^2LC}$$ [+1 point]

$$T_2(s) = -\frac{sL}{R}$$ [+1 point]

Part II

Yes, both (block 1, block 2) or (block 2, block 1) give rise to the same transfer function [+1 point]

$$T(s) = T_1(s) \cdot T_2(s) = -\frac{RCs+1}{s^2LC} \cdot \left(-\frac{sL}{R}\right) =$$

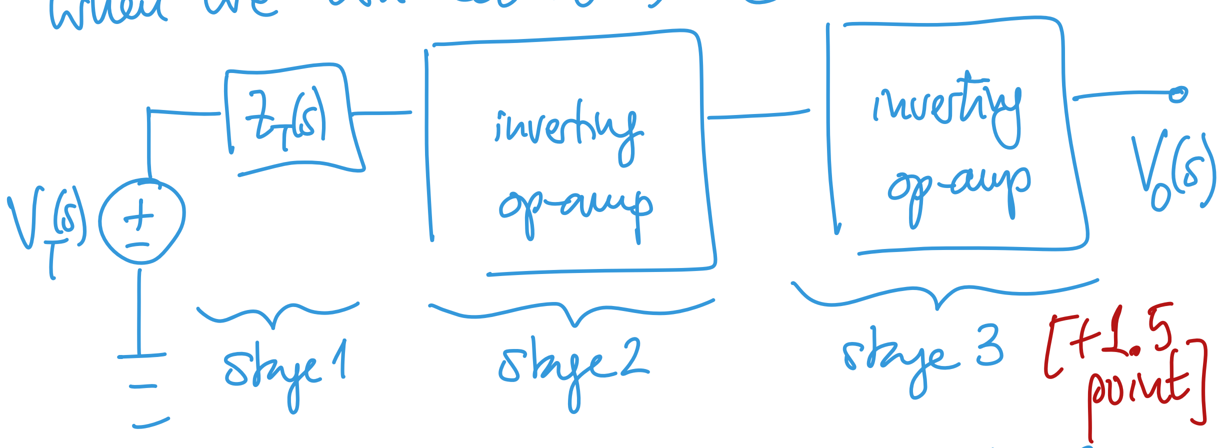
$$= \frac{RCs+1}{sRC}$$

This is because there is no loading in

either configuration. Both blocks have zero output impedance, and that makes sure that whatever the second stage is, it doesn't load the first stage. [+1 point]

Part III

Both configurations have the same transfer function. No matter what we use, when we connect it like



[+1.5 point]

we observe that there is loading: stage 2 will load stage 1. This explains why

$$V_o(s) \neq T_1(s)\, T_2(s) \cdot V_T(s)$$

[+1.5 point]

Part IV

We want to design a circuit whose transfer function is

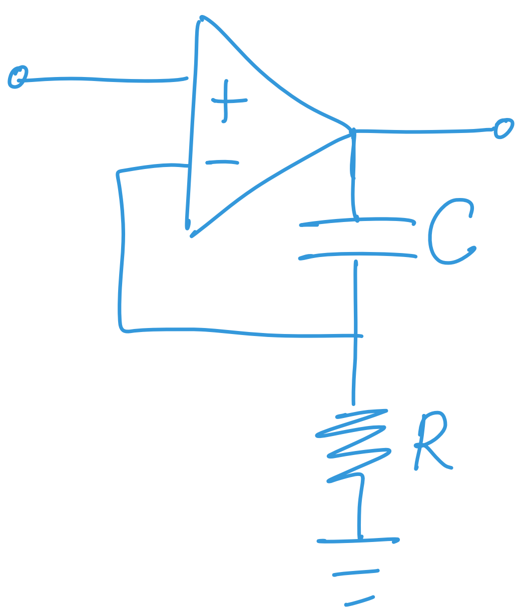
$$T(s) = \frac{RCs+1}{RCs}$$

with a single opamp. Note that

$$T(s) = \frac{R + 1/Cs}{R}$$

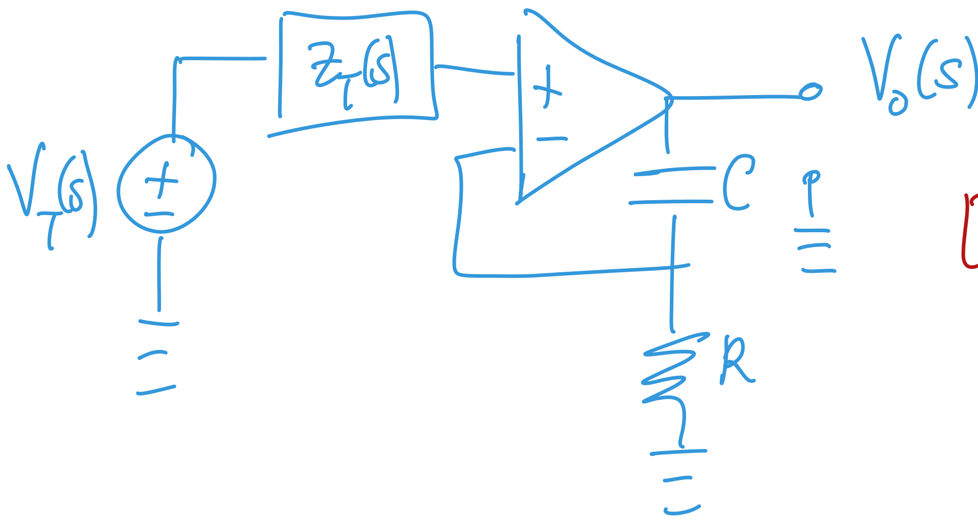
[+1 point]

So we can employ a non-inverting op-amp



[+1 point]

The advantage of this design is that it has 0 output impedance and ∞-input impedance. Therefore, when we connect it with the source circuit, we obtain



[+1 point]

$$V_o(s) = T(s) \cdot V_T(s)$$, which is the desired output.

Part V

We have $T(s) = \dfrac{s+1}{s}$

$$T(j\omega) = \frac{j\omega + 1}{j\omega}$$

$$|T(j\omega)| = \frac{\sqrt{1+\omega^2}}{\omega}$$

extra [+0.5 point]

$$\angle T(j\omega) = \arctan \omega - \frac{\pi}{2} \text{ rad}$$

extra [+0.5 point]

Given what we know from frequency response, we have

$$V_0^{ss}(t) = 5\,|T(j3)| \cdot \cos\left(3t + \frac{\pi}{2} + \arctan 3 - \frac{\pi}{2}\right)$$

$$= 5 \cdot \frac{\sqrt{10}}{3} \cos(3t + \arctan 3)$$

$\arctan 3 \simeq 1.249$ rad

$$\simeq 5.27 \cos(3t + 1.249)$$

extra (+1 point)