MAE40 - Linear Circuits - Winter 22
Final Exam, March 14, 2022

Instructions

(i) Prior to the exam, you must have completed the Academic Integrity Pledge at `https://academicintegrity.ucsd.edu/forms/form-pledge.html`

(ii) The exam is open book. You may use your class notes and textbook

(iii) Collaboration is not permitted. The answers you provide should be the result only of your own work

(iv) On the questions for which the answers are given, please provide detailed derivations

(v) The exam has 4 questions for a total of 40 points and 4 bonus points

(vi) You have from 11:30am to 2:30pm to complete the exam. Allow sufficient time to post your answers in Canvas (submission closes at 2:40pm).

(vii) If there is any clarification needed, post your question in the "Discussions" tab of the class Canvas webpage ("Clarifications on question statements of final")

Good luck!

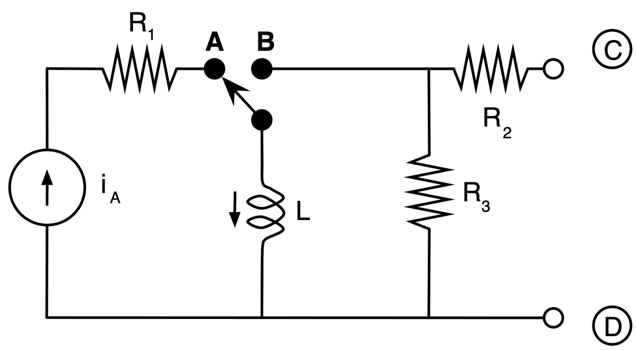


Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

Consider the circuit depicted in Figure 1. The value i_a of the current source is constant. The switch is kept in position **A** for a very long time. At $t = 0$, it is moved to position **B**.

Part I: [2 points] Find the initial condition $i_L(0)$ for the inductor.

Part II: [2 points] Transform the circuit in Figure 1 into the s-domain, using a voltage source to account for the initial condition of the inductor.

Part III: [2 points] For the circuit you obtained in Part II, find the open-circuit voltage transform as seen from terminals C-D. The answer should be given as a ratio of two polynomials.

Part IV: [2 points] For the circuit you obtained in Part II, find the short-circuit current transform as seen from terminals C-D. The answer should be given as a ratio of two polynomials.

Part V: [2 points] For the circuit you obtained in Part II, find the Thevenin equivalent in the s-domain as seen from terminals C-D (the impedance should be given as a ratio of two polynomials).

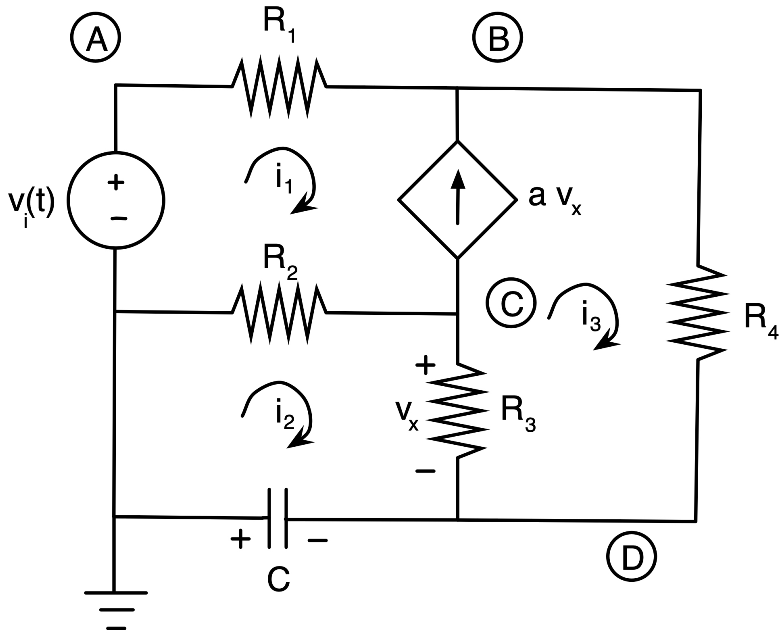


Figure 2: Nodal and Mesh Analysis Circuit for Question 2. a is a known positive constant.

2. **Nodal and Mesh Analysis on the s-domain**

   Part I: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its node-voltage equations. Use the reference node and other labels as shown in the figure. *Do not assume zero initial conditions.* Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

   Part II: [5 points] Convert the circuit in Figure 2 to the s-domain and formulate its mesh-current equations. Use the mesh currents shown in the figure. *Do not assume zero initial conditions.* Make sure your final answer has the *same* number of independent equations as unknown variables (notice the presence of the dependent source). No need to solve any equations!

   Part III: [Extra 2 points] Express the capacitor's voltage transform $V_C(s)$ in terms of the node voltages. Do the same in terms of the mesh currents.

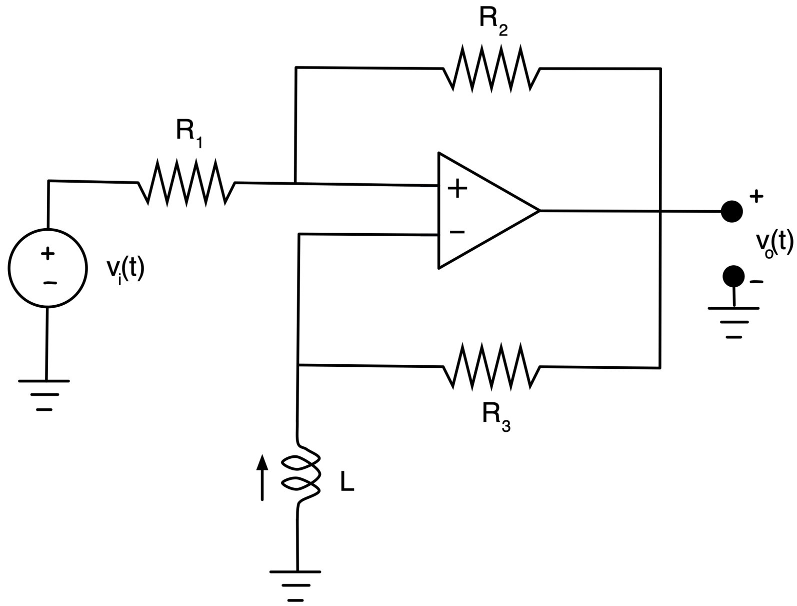


Figure 3: RL circuit for Laplace Analysis for Question 3.

3. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 3. The initial condition of the inductor is given by

$$i_L(0^-) = 1A.$$

Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the inductor in which the initial condition appears as a voltage source.

Part II: [3 points] Use nodal analysis to show that the output response transform $V_o(s)$ as a function of $V_i(s)$ is expressed as

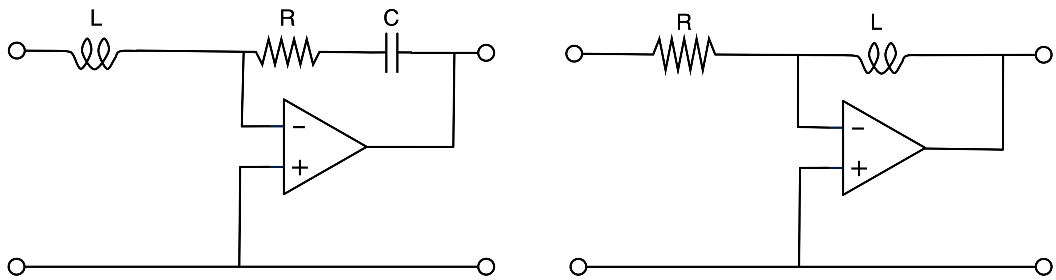
$$V_o(s) = -\frac{R_2(R_3 + sL)}{R_1R_3 - R_2Ls}V_i(s) + \frac{(R_1 + R_2)R_3L}{R_1R_3 - R_2Ls}$$

Part III: [3 points] Use partial fractions and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $v_i(t) = e^{-2t}u(t)\,V$, $L = 1\,\mathrm{H}$, and $R_1 = R_2 = R_3 = 1\,\Omega$ is

$$v_o(t) = \frac{1}{3}\big(e^{-2t} - 4e^{t}\big)u(t).$$

Part IV: [2 points] Decompose the output voltage of Part III as (i) the sum of the natural and forced response, and (ii) the sum of the zero-state and zero-input response.

4. **OpAmp Design and Chain Rule**



(a) Block 1 (b) Block 2

Figure 4: Circuits for Question 4.

Part I: [2 points] Find the transfer functions $T_1(s)$ and $T_2(s)$ for each block in Figure 4.

Part II: [2 points] If you were to connect the blocks in series in the following two configurations: (a) (block 1, block 2) and (b) (block 2, block 1), will you obtain the same transfer function? Why?

Part III: [3 points] An engineer used a configuration from Part II whose transfer function is

$$T(s) = T_1(s) \cdot T_2(s) = \frac{RCs + 1}{RCs}$$

However, the engineer was surprised to observe that when a source circuit with Thévenin equivalent $V_T(s)$ and $Z_T(s)$ was connected, the output transform was different from

$$V_o(s) = T(s)V_T(s)$$

which is what the engineer had expected. Can you explain why?

Part IV: [3 points] Eventually, the engineer gave up on the blocks above and came up with a different design for realizing $T(s)$. The engineer did so by employing resistors with value R and capacitors with values C, and a single op-amp. When the source circuit with Thévenin equivalent $V_T(s)$ and $Z_T(s)$ was hooked up to the design, the engineer finally got the expected output transform $V_o(s) = T(s)V_T(s)$. Can you reproduce the design?

Part V [Extra 2 points] Compute the steady-state response $v_o^{SS}(t)$ of your circuit in Part IV when $R = 1\,\Omega$, $C = 1\,\text{F}$, and $v_T(t) = 5\cos(3t + \frac{\pi}{2})$.